

***ELEMENTARY
THERMODYNAMICS***



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ELEMENTARY THERMODYNAMICS

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Third Edition

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PREFACE TO THE THIRD EDITION

This book, extracted from my *Thermodynamics*, is intended for short courses. Its feature is that its explanations are not shortened, but are as complete and clear as in a full length work. Thus this approach is expected to appeal to those who wish to offer a short theoretical course by omitting topics, rather than by covering hurriedly many topics. Even though the book is relatively short, too much material is probably included for the shortest courses, in which case, the teacher has some choice of topics to be covered.

While ideal gases are covered first (so as not to confuse the student with the consequences of the phenomena of evaporation and condensation), most of the basic theory has been covered at the outset; and there is a clearer indication of which principles and methods apply to gases and which to vapors. I think that the result of the reorganization and the changed viewpoint will be that the student leaves the short course with a more significant comprehension of the subject.

The problems are placed at the rear; somewhat more than enough for a course. The teacher may be interested in noting that many of the problems in *Problems on Thermodynamics* by Faires, Brewer, Simmang are appropriate to this book, and that moreover this problem book contains a set of steam tables and other useful reference material.

All comments, suggestions, and notices of errors will be gratefully received. Since it is always difficult to decide what matter to include in a book for a short course in thermodynamics, opinions on this subject will be welcomed.

VMF

SYMBOLS

With few exceptions, the ASA standard symbols have been used, but the choice of symbols has been influenced by those used in the ASHAE *Guide* and in Keenan and Kaye *Gas Tables*.

A	area; mass number.
a	acceleration.
B	volumetric percentage of a component in a mixture; represents a heat unit.
C	a constant.
C_p, C_v	molar specific heats at constant pressure and at constant volume.
c	clearance ratio in engines and compressors; specific heat, Btu/lb.-°F; c_p , specific heat at constant pressure; c_v , specific heat at constant volume; c_n , polytropic specific heat.
D	diameter.
d	a dimension; distance.
E	general symbol for energy; total stored energy of a system; E_e , electrical energy; E_c , chemical energy and energy chargeable against an engine in obtaining the thermal efficiency; E_f , energy dissipated in friction (not a different kind of energy); E_a , available energy; E_u , unavailable energy; E_t , total stored energy.
e	thermal efficiency; e_b , brake thermal efficiency; e_i , indicated thermal efficiency; e_c , combined or over-all thermal efficiency; e_n , nozzle efficiency.
F	force or total load; F_A , angle factor in radiation; F_E , emissivity factor.
g	acceleration of gravity; g_o , standard acceleration of gravity.
H	total enthalpy of w lb. of substance; H_m , enthalpy of a mixture.
h	specific enthalpy; h_f , specific enthalpy of saturated liquid; h_g , specific enthalpy of saturated vapor; h_{fg} , change of specific enthalpy during evaporation; h_{g2} , enthalpy of saturated vapor at state 2, etc.; \bar{h} , enthalpy of one mol of substance; h° , enthalpy in standard state (77°F).

h	film coefficient.
hp	horsepower, bhp , brake horsepower, $i hp$, indicated horsepower, fhp , friction horsepower
I	correction factor for initial velocity in nozzles
J	Joule's constant (≈ 778)
K	kinetic energy.
k	the ratio c_p/c_v
k	conductivity
L	distance, length, stroke of piston, represents unit of length, L'' , stroke in inches
M	Mach number
M	molecular weight, lb /mol, M_x , molecular weight of gas X , etc
m	momentum
m	mass in slugs, percentage steam bled in regenerative cycles, exponent in $pV^m = C$, which defines the pV relation at the end points of an irreversible process
N	number of anything, number of mols of a gas, number of power cycles per minute completed by an engine
n	revolutions per minute (rpm), polytropic exponent
P	potential energy, represents the unit of a pound
p	unit pressure (psi or psf), p_m , mean effective pressure (mep), p_{mi} , indicated mep, p_{mb} , brake mep, p_o , stagnation or impact pressure, p_r , relative pressure
Q	heat, Q_A , heat added, Q_R , heat rejected, Q_r , radiated heat, etc
q	heating value, q_l , lower heating value, q_h , higher heating value
R	specific gas constant, pv/T \bar{R} , universal gas constant
r	radius, ratios, reheat factor, r_k , isentropic compression ratio, r_o , cutoff ratio, r_p , pressure ratio, r_e , expansion ratio
S	total entropy
s	specific entropy, s_f , specific entropy of saturated liquid, s_g , specific entropy of saturated vapor, s_{fg} , change of specific entropy during vaporization, s_{g2} , specific entropy of saturated vapor in state 2, etc
T	absolute temperature, usually degrees Rankine, T_o , stagnation temperature
t	temperature usually in degrees Fahrenheit, time, t_o , stagnation temperature
U	total internal energy, over-all transmittance
u	specific internal energy, u_f , for saturated liquid, u_g , for saturated vapor, u_{fg} , change during vaporization, u_{f2} , for a saturated liquid in state 2, etc
\bar{u}	internal energy of one mol of substance
V	total volume, V_D , displacement volume
v	specific volume, v_r , relative volume

\bar{v}	volume of one mol.
v	velocity; speed.
W	work; W_I , indicated work; W_B , brake work; W_K , combined work; W_f , flow work; W_p , pump work.
w	mass in pounds; mass flow per unit time; w_f , specific fuel consumption; w_b , brake steam rate; w_i , indicated steam rate; w_k , combined steam rate.
x	quality of a two-phase system.
y	percentage (or fraction) of liquid in a two-phase system.
z	altitude; potential energy of one pound, ft-lb./lb.
Z	atomic number.
α (alpha)	constant in specific heat equation; absorptivity.
β (beta)	constant in specific heat equation; coefficient of thermal expansion; bulk modulus.
γ (gamma)	constant in specific heat equation; coefficient of performance; specific weight; angle.
δ (delta)	angle.
ϵ (epsilon)	effectiveness; emissivity.
η (eta)	efficiency ratios; engine efficiency; combustion efficiency; η_b , brake engine efficiency; η_c , compression efficiency (adiabatic if not qualified); η_d , discharge coefficient; η_i , indicated engine efficiency; η_k , combined engine efficiency; η_m , mechanical efficiency; η_n , nozzle velocity coefficient; η_p , propulsive efficiency, pump efficiency; η_r , effectiveness of regenerative process, efficiency of reaction blades; η_s , turbine stage efficiency; η_v , volumetric efficiency.
θ (theta)	represents unit of temperature; angle.
μ (mu)	degree of saturation; absolute viscosity.
π (pi)	3.1416 . . .
ρ (rho)	density.
σ (sigma)	Stefan-Boltzmann constant.
τ (tau)	time; represents a unit of time.
ϕ (phi)	relative humidity; angle; used to mean <i>function of</i> .
ω (omega)	humidity ratio; angle.
Δ (delta)	indicates a difference or a change of value, Δt = change of temperature or difference in temperatures, in accordance with the context.

ABBREVIATIONS

A/F	air fuel ratio		
AIChE	American Institute of Chemical Engineers	ICE	internal combustion engine
ASA	American Standards Association	ID	inside diameter
ASME	American Society of Mechanical Engineers	imep	indicated mean effective pressure
ASHAE	American Society of Heating and Air Con- ditioning Engineers	kg	kilogram
atm	atmospheres a unit of pressure	kw	kilowatt
BDC	bottom dead center	LMTD	logarithmic mean tem- perature difference
bhp	brake horsepower	ln	natural logarithm (base e)
bmep	brake mean effective pressure	LOCE	law of conservation of energy
Btu	British thermal unit	log	logarithm to the base 10
cfm	cubic feet per minute	LP	low pressure
cgs	centimeter gram second system	mep	mean effective pressure
cm	centimeter	mev	million electron volts
cpm	cycles per minute	mph	miles per hour
cps	cycles per second	OD	outside diameter
da	dry air	psf	pounds per square foot
dg	dry gas	psi	pounds per square inch
ev	electron volt	psia	pounds per square inch absolute
F/A	fuel air ratio	psig	pounds per square inch gage
fpm	feet per minute	rpm	revolutions per minute
fps	feet per second	rps	revolutions per second
fps ²	feet per second second	TDC	top dead center
gpm	gallons per minute	TEL	tetraethyl lead
HP	high pressure	SAE	Society of Automotive Engineers
p	horsepower bhp brake horsepower, ihp indi-	v	vapor

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***ELEMENTARY
THERMODYNAMICS***

1

THERMODYNAMIC PROPERTIES

1. Introduction. *Thermodynamics is that branch of physics which treats of various phenomena of energy, and especially of the laws of transformations of heat into other forms of energy, and vice versa.* Examples of such transformations are: the process of converting heat into work, of converting electricity into heat (because of a resistance), and any chemical reaction (heat of reaction). In engineering, the science is conventionally divided into two parts: chemical thermodynamics, which is devoted largely to reactions and solutions, and the thermodynamics concerned with the production and use of work and power. This division is fortuitous, as the areas merge at many points. Moreover, there are aspects of the science that may not be included in either of the foregoing parts, notably the thermodynamics of atomic processes, those processes in which the structure of the atom is changed.

This book is the story of power, the demand for which is increasing at geometric rates. Using the basic principles, we shall study the theory of internal combustion engines, gas turbines, air compressors, steam turbines, refrigeration, air conditioning, and a few miscellaneous topics of related interest such as the flow of fluids and heat transfer. Insofar as possible, we shall apply the theory to actual engineering situations shortly after the theory has been developed. This plan of study should engender the reader's interest and result in a more intimate understanding of the implications of the theory. It is assumed in the writing that the reader is familiar with the contents of an introductory book on engineering physics and that he can handle simple calculus.

Thermodynamics takes the macroscopic or large-scale view rather than the microscopic (1).* The science has been developed without regard to

* Italicized Arabic numerals designate a reference at the end of the book. In many

the structure of matter. A pressure gage indicates the *average* pressure of a gas at rest in a container. Since a finite quantity of a gas is composed of a tremendous number of molecules about 4.5×10^{20} in one cubic inch at atmospheric temperature and pressure each moving with its independent velocity it may occur (microscopically) that the pressure at a point is momentarily quite high (or low) because by chance a number of high velocity (or low velocity) molecules have just happened to strike that point. However, the area of that part of the instrument which the molecules actually strike is so large compared with the area of the point, that the instrument gives a statistical answer that is, it measures the force exerted by the pounding of a large number of molecules but of a relatively small percentage of the total number. The actual number striking the instrument is large enough to provide an adequate statistical sample. Hence we safely conclude that the same pressure would be indicated at any and all other positions in the container (effect of gravity negligible). Similarly the temperature which indicates something of the level of energy in a body is a *statistical measurement obtained from the action of a large sample of molecules (the macroscopic view)* and it tells nothing of the energy of a single molecule (the microscopic view—really submicroscopic).

In engineering applications we often find it expedient to neglect some macroscopic effects. The temperature of a substance in a container is virtually a certain value but we may knowingly neglect the effect of a localized spot where the temperature is significantly higher or lower. Since the engineering of thermodynamics is the art of applying the science of thermodynamics, one's judgment and experience are used to decide whether or not such macroscopic differences may be safely ignored.

At the outset of our study we should become familiar with certain concepts at the foundation of the subject.

2 The System and the Working Substance A *system* is that portion of the universe an atom or a galaxy or some certain quantity of matter, which we specifically wish to study. *It is a region enclosed by specified boundaries or by imaginary but definite mental boundaries.* We isolate a system or region in our minds because we wish to study transformations of energy occurring

cases the reference will be an acknowledgement of the use of an idea or other information. The interested reader should find elaboration or a related discussion in the work cited.

* As Humpty Dumpty said to Alice in Wonderland: "When I use a word it means just what I choose it to mean—neither more nor less." It is only natural that as a science develops there should be troubles with the meanings of words. On the whole the science of thermodynamics is well developed but unfortunately there are several words which do not yet have a single precise and generally accepted definition. In general the reader should expect to find now and then the same word being used by various writers to mean different things and different words to mean the same thing and he should be alert to this situation. E. F. Obert in an ASEE paper on *Teaching the concepts of state properties boundaries systems etc.* (2) has highlighted variations in the definitions of some important thermodynamic terms.

within the boundaries, and the passage of energy (if any) across the system's boundaries, to or from the surroundings, which passage may or may not be accompanied by the movement of matter across the boundaries. The region all about the system, outside of the boundaries, is called the *surroundings* or *environment*. The *free body* in analytic mechanics is a system, in which the mode of analysis is based on Newton's laws of motion. The principal mode of analysis in thermodynamics is on the basis of energy laws. *The working substance in thermodynamics is a fluid in which energy can be stored and from which energy can be removed.* The purpose of storing and removing energy is to bring about desired energy transformations, such as transforming heat into work. A large part of this study is devoted to learning about this particular transformation. Examples of working substances are: *steam* in a steam turbine, *air* in an air compressor, *air-and-fuel mixture* in an internal combustion engine, and *water* in a hydraulic turbine.

A working substance, or a certain mass of working substance, may be (and in this book usually is) taken as the system. However, the word *system* has a more general connotation. [The system which the mind has surrounded with imaginary boundaries may be an electric motor, a storage battery, or a shaft and propeller combination on an airplane.] The mind can place the boundaries of the system wherever desired, as easily in one position as another. If a block is sliding down a rough incline, we may take the block alone as the system, or the block and the incline together. In another situation, the system may be the turbine (which would include effects at the bearings) or it may be the region within the turbine through which the working substance passes. In the case of a reciprocating engine, the system may be taken as the fluid in the cylinder; it may be the fluid plus the piston and piston rod (which would enclose the frictional effects at glands and cylinder walls); it may be the whole engine, or the whole power plant. Whatever the system is, it is important to maintain a precise mental picture of its limits or *boundaries*. While considering a particular problem, the reader should make a conscious effort to define the boundaries and then to keep them in mind.

In a closed system, matter does not cross the boundaries. Energy may, however, flow into or out of this system. *An open system is one where matter passes across its boundaries.* Energy may also pass across the boundaries, either with the flow of mass or separately. The mass of matter in an open system may vary with time or be constant. A special case with which we shall frequently deal is one in which the rate of flow of mass into the system is constant and equal to that leaving the system (constant mass in the system), called a *steady flow system*.

3. The Pure Substance. We shall consider that a *pure substance* is a single substance which retains an unvarying molecular structure or a fixed solution of homogeneous substances, each of which retains an unvarying molecu-

lar structure (3) Thus a system of pure oxygen is a pure substance, as is dry air (in the gaseous state) which is largely a solution of oxygen and nitrogen with fixed percentages of each component. A system of ice water, and water vapor may be considered to be a pure substance, since the molecular structure in all parts is the same (substantially so, although there are other water molecules than H_2O). Examples of systems which are not pure substances are a mixture of oil and water, because they will not go into solution, liquefied dry air in contact with its vapor because, since the condensing temperatures (boiling points) of oxygen and nitrogen are different, the relative percentages of oxygen and nitrogen in the liquid and in the vapor would change—that is, the solutions before and after partial condensation would not be the same, also, a system which includes any chemical process such as combustion would not be a pure substance during the process, because the molecular structures before and after the process are different.

In general a pure substance may exist in any of three phases: the solid phase, the liquid phase and the gaseous phase. Under certain conditions (§ 230) all three phases may coexist. *Melting* is the change of phase from solid to liquid. Heat must be added to cause melting. The change in the reversed direction, liquid to solid, is *freezing* or *solidifying*. The change of phase from the liquid to the gaseous phase is called *vaporization*, and the liquid is said to *vaporize* (or *boil*). The change from vapor (gaseous phase) to liquid is *condensation*, and during the process the vapor is said to be *condensing*.

Not all substances pass through these three phases, some normally pass directly from the solid to the gaseous phase, a change of phase called *sublimation*. Moreover many substances which ordinarily pass through the three phases may sublime under certain conditions. For example a piece of ice exposed to the atmosphere at temperatures below $32^\circ F$ will sublime, and given time, will pass entirely into the atmosphere as water vapor (steam). Dry ice, which is solid carbon dioxide, sublimates as you would ordinarily use it, while it receives heat.

In our study of thermodynamics we shall usually be dealing with a pure substance in fluid form. *A fluid is either gas or liquid, or both*. The significance of the pure fluid to our study is that *any particular condition of a fluid at rest is completely defined by two independent properties*, provided there are no effects from motion, gravity, capillarity, electricity or magnetism (3). This brings us to the matter of how to define the condition of a pure substance.

4 Properties and State In order to compute changes of energy which have occurred in a system or working substance we must be able to express the behavior of the system in terms of descriptive characteristics, which are called *properties*. Properties which are familiar to the reader from prior

study include pressure p , temperature T , density ρ , and specific volume v , each of which is to be discussed shortly.

The condition (state) of a pure substance in liquid or gaseous form is defined by two independent properties. If these two properties are stipulated, the *state* of the substance is set. By this, we mean that all other properties of the substance have certain particular values whenever the substance is in this particular state. There are a number of properties of interest and utility, but the working list for the heat-power engineer includes those mentioned above, together with specific internal energy, specific enthalpy, and specific entropy. (These terms are studied later.) Now repeating the idea of the first sentence of this paragraph, we may say that if the pressure and specific volume of a certain pure substance are stipulated, the temperature will be a certain value, as will the specific values of internal energy, enthalpy, and entropy. No matter what happens to the substance (no chemical change, though), be it compressed, heated, expanded, or cooled, if it is returned to the stipulated pressure and volume, the other properties also return to values identical, respectively, with their original values. See Fig. 1.

We sometimes speak of two kinds of properties, intensive and extensive. *Intensive properties* are independent of the mass (extent) of the substance. *Extensive properties* are total values, such as the total volume, and total internal energy. We note that the total volume, for example, is a property of the total mass of the system and we shall have frequent occasion to deal with extensive properties of this kind. On the other hand, the intensive property temperature is independent of the mass,

To describe a pure fluid system completely, we specify the substance (steam, nitrogen, etc.) and its mass, in addition to the two independent properties.

Consider for a moment the repeated qualification, *independent* properties. As you know, the density is the reciprocal of the specific volume; hence, these properties are not independent of each other. During the boiling of a liquid, the pressure and temperature of the liquid-vapor mixture are not independent; the boiling temperature is a certain value for a particular substance, depending upon the value of the pressure (as we shall learn in detail later).

From your study of mathematics you have learned that two coordinates (the values of x and y) locate (or define) a point which is known to be in a given plane (the xy plane). Three coordinates locate a point in space. So it is with the properties of a pure substance. We look upon these proper-

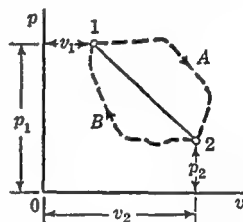


Fig. 1. A substance whose state is represented by point 1 has a temperature T_1 . If the pressure and volume are varied as defined by the path 1-A-2-B-1, returning to their original values, the temperature also returns to its original value T_1 .

ties as being coordinates locating a point (*defining a state*) and we may picture this point, or any number of state points, on various planes, for example, the pressure-volume plane (Fig 1), the temperature-entropy plane, etc. Any two properties may be used. Because, as described, two properties locate a point in a plane, these properties are called *point functions*.

5. Systems of Units. Isaac Newton* made the momentous statement to the effect that the acceleration of a body is directly proportional to the force acting on it and inversely proportional to its mass, and systems of units have been based on it ever since. Rearranging this statement for a proportionality constant of unity, we say

$$\text{Force} = \text{mass} \times \text{acceleration},$$

$$(a) \quad F = ma,$$

in which force is defined in terms of mass, and mass in terms of force. We know that acceleration has the dimensions of length per unit of time squared, L/τ^2 . Recall that a dimension is an attribute of something in general terms, thus, length L is also an attribute of volume L^3 . Units are dimensional characteristics expressed in terms of defined quantities. The length of a foot is accurately defined (in terms of the International meter). Other common English units of length are inches, rods, yards, and miles. Defined units of time are seconds, minutes, hours, days, etc. In English units, a unit quantity of acceleration is one foot per second second (ft per sec² or fps²).

Now we may say from equation (a), that a unit force is one which produces unit acceleration in a body of unit mass. A "consistent" system of units may be built up on this statement. If a unit mass is taken as a pound we call the corresponding unit force a *poundal*, and we say

$$(b) \quad F = ma, \quad \text{poundal} \rightarrow \text{lb}_m \times \frac{\text{ft}}{\text{sec}^2},$$

[CORRESPONDING TO A POUND MASS]

where the sign \rightarrow is taken to mean *has the units (or dimensions) of*. The engineer is accustomed to using the pound as the unit of force, in which case the unit of mass is a slug,

$$(c) \quad m = \frac{F}{a}, \quad \text{slug} = \frac{\text{lb}_f \cdot \text{sec}^2}{\text{ft}},$$

* Sir Isaac Newton (1642-1727) is often credited with being the greatest scientist of all times. Born of farmer parents, he was soon exercising his mechanical aptitude, devising a water clock and a sun dial during his years in grammar school. Two years after graduating from Cambridge, he had discovered the binomial theorem, started inventing calculus, experimented with color, and speculated on gravity. A few of his achievements: the reflecting telescope, the composite nature of sunlight, a science of optics, the invention of a thermometer (this is before thermodynamics became a science), and the most monumental—the laws of gravitation. He himself credited his scientific successes to hard work and patient thought.

where m is the mass in slugs. A consistent system of units may also be derived on the basis of a pound force.

When an engineer says, "We used six pounds of steam," or "The weight of the body is six pounds," it is most likely that he means a mass of six pounds. When he says, "The push of the spring on the piston is six pounds," he means a force of six pounds. There is ordinarily little confusion until a problem arises in which the engineer is consciously involved with systems of units. To rationalize the matter, consider the following statements:

1. One *must* know the difference between mass and force. A *pound mass is an absolute quantity of matter*. By *absolute quantity*, we mean that one pound of matter is one pound of matter regardless of where it is located—in outer space at zero gravity, for example.

2. A pound mass located at a point of standard gravity g_0 is subjected to a force of gravity of one pound. To find the mass of a body, we measure the force of gravity at a point of standard gravity g_0 and compare this force with that on a body of known mass. Or, we compare the unknown mass with a known mass on balance scales.

3. In classical mechanics, "weight of" means *the force of gravity on*, but as previously mentioned, engineers generally mean an absolute quantity of matter (mass) when they say weight (but not in stating potential energy—more on this later.)

4. There are two units of mass commonly used in consistent English systems, a pound and a slug ($\rightarrow \text{lb}_m\text{-sec.}^2/\text{ft.}$). We can and do on occasion use ounces, tons, kilograms, grams, etc., for mass.

5. There are two units of force in consistent use in the English systems, a pound and a poundal ($\rightarrow \text{lb}_m\text{-ft./sec.}^2$). The poundal is not used very often. We can and do on occasion use ounces, kips, tons, kilograms, grams, dynes, etc., for force.

6. The same unit, pound, is used for both force and mass. This should not cause any more confusion than using the foot-pound or inch-pound to measure both moment and energy. However, it is essential to know the concepts of moment and energy and of force and mass.

In the study of mechanics, the pound is the unit of force, and therefore in order to maintain a consistent system of units, the slug ($\rightarrow \text{lb}_m\text{-sec.}^2/\text{ft.}$) is the unit of mass. In thermodynamics, by virtue of tradition and the fact that the pound mass cancels from the energy equations, we use a mixed or inconsistent system of units. That is, we use the pound for the unit of both force (and pressure p , lb. per sq. ft., or psf) and mass. No harm is done, as we shall see. In general, the mathematics of the science of thermodynamics can be developed independently of the quantity of matter (mass) because it deals with properties. However, in studying heat transfer, we again must be consistent in units; the need for consistency will be explained in Chapter 25.

Standards of measure are maintained by governments, so that the magnitude of a pound does not change. At a point where the force of gravity is not the standard value of $g_0 = 32.174 \text{ fps}^2$, this standard quantity of matter, one pound, would not "weigh" one pound on a *spring balance*,

because a spring balance measures the force of gravity. Since gravity is less than standard at high altitudes, a pound of mass *weighs* less than one pound there. Consider Newton's law with respect to a given mass and note that the force on the mass is proportional to the acceleration. Let w_0 lb_f be the force of gravity on this mass at a locality of standard gravity (it will also be the mass in pounds because it is being weighed at standard gravity), and let w lb_f be the force of gravity on this mass at some other locality where the acceleration of gravity is g , then

$$(d) \quad \frac{w_0}{w} = \frac{g_0}{g}, \quad \text{or} \quad w = \frac{g}{g_0} w_0 = gm,$$

where $m = w_0/g_0$ (\rightarrow lb_f-sec²/ft) the mass in *slugs* [equation (c)]. Also from (d), we see that the mass in slugs is $m = w/g$ where w is the force of gravity on the mass in pounds and g is the acceleration of gravity at the same locality which may be anywhere within the gravitational field of the earth. Said another way. If the mass m of a body in slugs is multiplied by the local acceleration of gravity g , the result is the local force of gravity w lb on the body also.

$$(e) \quad \text{Mass in slugs} = \frac{\text{mass in pounds}}{\text{standard acceleration of gravity, } g_0},$$

because gravitational force w_0 lb and mass in pounds are identical at standard gravity.

Since there is little variation in the force of gravity on the surface of the earth we may consider g equal to g_0 for practically all engineering purposes, and for slide-rule calculations we may use $g \approx g_0 \approx 32.2 \text{ fps}^2$. We shall assume that the reader has mastered the concepts of mass and force, and therefore we shall not use subscripts (lb_f and lb_m) to designate whether force or mass is intended except occasionally in analyzing units. When it is said that there are so many pounds of substance the reader should understand that an absolute quantity of matter is intended (unless it is clear from the context that the force of gravity is meant). From here on, with few exceptions, let

$$w = \text{mass in pounds} \quad m = \text{mass in slugs}$$

It will be rewarding to pay close attention to units throughout this study. We shall rationalize the units of various quantities as they are discussed and return to the subject in the chapter on heat transfer.

6 Pressure Pressure, which is the force per unit area, is one of the most useful thermodynamic properties because it is easily measured directly (Extremely accurate measurements of anything are difficult.) In terms of the kinetic theory of matter, the pressure of a fluid is due to the change of momentum of the molecules when they strike the boundaries of the system.

(walls of the container). It is a macroscopic observation, as mentioned in § 1.

In practice, pressure above and below atmospheric are determined by means of a pressure gage (Fig. 2) or a manometer. The dial of a pressure gage is marked to read the *gage pressure*, usually in pounds per square inch (psi). The gage pressure is the *difference* between the pressure inside of a vessel and the atmospheric pressure outside. Thus, to find the *absolute pressure* when this pressure is above atmospheric, add the atmospheric pressure to the gage reading; that is,

$$\begin{aligned} \text{(f) Absolute pressure (psia)} \\ &= \text{atmospheric pressure} \\ &\quad + \text{gage pressure (psig).} \end{aligned}$$

The units of pressure in each of the foregoing terms must of course be the same. Hence it will be convenient to recall the relations of the various units of pressure in common use by the engineer. Barometers (for measuring atmospheric pressure)* and vacuum gages (for measuring pressures below atmospheric) usually give the pressure in inches of mercury. Manometers may give the pressure in inches of mercury, in inches of water, or in terms of the height of column of any fluid that may be used.

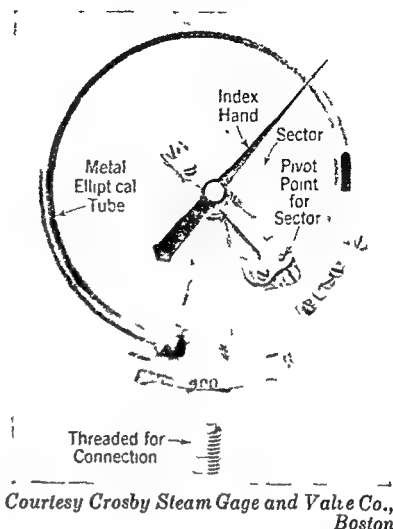


Fig. 2. Movement for a Bourdon Pressure Gage. This picture shows the movement in one type of pressure gage, known as the single-tube gage. The fluid enters the tube through the threaded connection. As the pressure increases, the tube with an elliptical section tends to straighten, the end that is nearest the linkage moving toward the right. The linkage causes the sector to rotate. The sector engages a small pinion gear. The index hand moves with the pinion gear. The whole mechanism is of course enclosed in a case, and a graduated dial, from which the pressure is read, is placed under the index hand.

* After Evangelista Torricelli (1608–1647) discovered the pressure of the atmosphere, Otto von Guericke (1602–1686) invented the air pump and set about producing a vacuum. He finally succeeded after having made two hemispheres, known as the Madgeburg hemispheres, which were capable of withstanding atmospheric pressure. Before a large audience of notables, von Guericke placed his hemispheres together and soon had most of the air pumped from the inside. A horse was hitched to each hemisphere, and try as they might, the two of them could not pull the hemispheres apart. Additional horses were hitched until there were thirty in all and still the hemispheres could not be pulled apart. The people, who knew nothing of the pressure of the atmosphere, were astounded when von Guericke broke the vacuum and the hemispheres fell apart. Had not von Guericke been a public official, renowned for his wisdom and kindness, his magic might have resulted in no good for him. Other scientists of the time were persecuted, even killed, for less.

A manometer (Fig 3 any fluid in the column) measures the pressure by balancing the force of gravity gm on the part d of the fluid column plus atmospheric pressure against the pressure p in the system. If precision is needed, the manometer reading taken at a location where gravity is not standard should be corrected in accordance with equation (d). If the column is mercury the pressure in inches of mercury taken at a point of standard gravity (for a temperature of about 60°F) is converted to pounds per square inch as follows

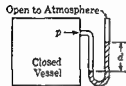


Fig 3 Manometer

$$(g) \quad (\text{Inches Hg}) \left(0.491 \frac{\text{psi}}{\text{in Hg}} \right) \rightarrow \frac{\text{lb}}{\text{sq in}}$$

Corrections for gravity and temperature may be made when desirable. Since the pressure p in the equations of this book will usually be in pounds per square foot (psf), a conversion from psi to psf is a regular requirement

$$(h) \quad \left(p \frac{\text{lb}}{\text{in}^2} \right) \left(144 \frac{\text{in}^2}{\text{ft}^2} \right) \rightarrow \frac{\text{lb}}{\text{ft}^2}$$

Even when the conversion unit cancels from the equation it is advisable to write it down (and then cancel it), because the *habit* of conversion is important.

If the absolute pressure is less than atmospheric pressure the gage reading is sometimes spoken of as the *vacuum pressure*, or the *vacuum*. In this instance, the absolute pressure is found from

$$(i) \quad \text{Absolute pressure (psia)} \\ = \text{atmospheric pressure} - \text{gage pressure (psig)}$$

The gage still measures the *difference* between the pressure inside the system and the pressure of the atmosphere. Standard atmospheric pressure is 29.92 in Hg or 14.696 psia (use 14.7 for slide rule work). Remember to convert gage pressure to absolute pressure.

7 Specific Volume and Density The density ρ of any substance is its mass per unit volume

$$(j) \quad \text{Density} = \frac{\text{mass}}{\text{volume}} \rightarrow \frac{\text{lb}}{\text{ft}^3} \text{ or } \frac{\text{slugs}}{\text{ft}^3}$$

If the mass w is measured in pounds and the volume V in cubic feet then density is $\rho = w/V$ lb per cu ft. The specific volume v is the volume of a unit mass, say, cubic feet per pound and is the reciprocal of the density, $v = V/w = 1/\rho$. Unless otherwise stated, the density will be in pounds per cubic feet.

8 Temperature. The temperature of a body is its thermal state considered with reference to its power of communicating heat to other bodies (4). Our

sense of feel responds to temperature. Roughly speaking, objects at lower temperatures than body temperature (98.6°F) feel cool or cold; objects at higher temperatures feel warm or hot. However, for engineering and scientific work, the temperature must be designated more precisely with reference to some scale. A scale of temperature is an arbitrary thing. The most common reference points are the *ice point* (the temperature at which pure water freezes) and the *boiling point* for water at standard atmospheric pressure. By way of an example of a scale, if the temperature zero is assigned to the ice point and 100 to the boiling point, the scale obtained is the centigrade or Celsius scale used in scientific work. If the temperatures assigned to the ice and boiling points are 32 and 212, respectively, we obtain the Fahrenheit scale, used in engineering and everyday measurements in this country. Thus, between the freezing and boiling points, there are 100 degrees on the centigrade scale and 180 degrees on the Fahrenheit scale ($180/100 = 9/5$), giving the relations

$$\begin{aligned} \text{(k)} \quad & t_c = \frac{5}{9}(t_f - 32), \\ \text{(l)} \quad & t_f = \frac{9}{5}t_c + 32, \end{aligned}$$

where t_c and t_f are the temperatures on the centigrade and Fahrenheit scales, respectively. Because there is more than one in use, the scale of a temperature is always stated: as 212°F or 100°C , where F means Fahrenheit and C centigrade.*

Thermodynamics requires the use of absolute temperature (or thermodynamic temperature), which is measured from a point of absolute zero. More will be said on this subject later (§ 97). At this time, we shall simply accept the location of absolute zero on the Fahrenheit scale as -459.69° (below 0°F). Use 460 in slide-rule work. Absolute temperatures T on the Fahrenheit scale are called *degrees Rankine* ($^{\circ}\text{R}$);

$$\text{(m)} \quad T^{\circ}\text{R} = t^{\circ}\text{F} + 459.69 \approx t^{\circ}\text{F} + 460.$$

For example, 60°F is the same temperature as 520°R , approximately. Absolute temperatures on the centigrade scale are called *degrees Kelvin*, written $^{\circ}\text{K}$, and absolute zero is -273.16°C . Thus,

$$\text{(n)} \quad T^{\circ}\text{K} = t^{\circ}\text{C} + 273.16.$$

For example, 40°C is the same temperature as 313°K , approximately. Notice the symbolization; lower case t for Fahrenheit (or centigrade) and upper case T for Rankine (or Kelvin). Engineers in English-speaking countries generally use Fahrenheit and Rankine degrees.

* Galileo invented a thermometer in 1592, but it did not have a well-founded scale. Gabriel Fahrenheit of Amsterdam, Holland, was the first (in 1720) to devise an instrument that indicated temperature in degrees, choosing the ice and boiling points of water as 32° and 212° , respectively. The centigrade scale was introduced in 1742 by Anders Celsius (1701–1744), a Swedish astronomer and professor at Uppsala.

Temperatures are measured in a number of ways, always through a change in some other characteristic of the substance to be measured

1. *Change in volume* Nearly everyone is familiar in a qualitative way, at least, with the phenomenon of substances (say, mercury or a gas) that expand with increase in temperature. If the amount of expansion in a particular case is correlated with the freezing and boiling points of water, and the change in volume divided into 100 or 180 equal parts, the instrument could be used to "read" temperatures. Liquids used include (5) mercury (-38°F to about 600°F —or 900°F with nitrogen above the mercury), alcohol (-100°F to about 300°F), and pentane (-300°F to about 70°F). Glass begins to soften at about 900°F .

2. *Change in pressure* If a gas is confined to a constant volume its pressure will go up as its temperature increases, and the change in pressure can be correlated with temperature change. See Fig. 2.

3. *Change in electrical resistance* The electrical resistance of metals increases almost in direct proportion to temperature increase. Thus the measured change in resistance of a particular piece of wire can be converted to temperature change. Metals used include nickel, copper (0°F to 250°F), and platinum (at high temperature and in high precision instruments).

4. *Change in electrical potential* The device that measures temperature by the electromotive force (emf) is called a *thermocouple**. It operates by virtue of the phenomenon occurring when two wires of different materials are joined together at their ends, with different temperatures existing at the two junctions. Under such a condition, an electromotive force exists which is very nearly directly proportional to the temperature difference between the two junctions. The potentiometer which measures the electromotive force can have a scale which reads temperature directly. One junction of the thermocouple is kept at a reference temperature which may be the room temperature. This is one of the favorite means of measuring temperature. Combinations of metals used include (5) copper and constantan (-300 to 650°F), iron and constantan (-300 to 1500°F), chromel and alumel (-300 to 2200°F).

5. *Optical changes* The instrument for this measurement is called an *optical pyrometer*. It is sighted at the hot body whose *brightness* is compared with the brightness of an adjustable and calibrated source of light within the instrument. When the brightness of the instrument light has been adjusted to be the same as the body whose temperature is desired, the instrument "reads" the body temperature. There are several types of optical pyrometers.

Obtaining precise measurements of any property is not easy, and even in ordinary engineering work where great precision is unnecessary, readings should be made with care. Also, one must know of the characteristics and eccentricities of the instrument being used in order to avoid taking false readings. In short, if you are concerned with the actual taking of data, you should do further study, for instance, references (5), (6), (7), (8), (9),

* Thomas J. Seebeck (1770-1831), who discovered the thermocouple (1821), was born in Estonia. Although he was educated as a Doctor of Medicine (Göttingen College, Germany), he chose to lecture and experiment in the physical sciences. For his invention of the thermocouple, he was given a citation by the Paris Academy of Sciences.

and others, including handbooks. We shall deal further with temperature after having built a background for better understanding.

9. Process. *If any property (or all properties) of a substance (§ 4) changes, the substance is said to have undergone a process; there has been a change of state. Actual processes involve changes in all or nearly all properties, but we study thermodynamics via ideal counterparts in which often one of the properties remains constant. For example, if a change of state occurs during which the pressure does not change (1-2, Fig. 4), the working substance is said to undergo a **constant pressure process** (isobaric); if the volume of a particular mass remains constant (ab, Fig. 4), but other properties change, the process is called a **constant volume process** (isometric). Processes will be studied later in some detail in several chapters.*

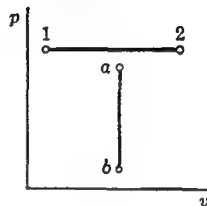


Fig. 4. Processes.

10. Cycles. *When a fluid in a particular state passes through a series of processes and returns to its initial state, it undergoes a cycle. The processes may be ones which have names, as constant volume and constant pressure, or they may be series of state change without names, as in Fig. 1. The path of a process may be any random path 1-A-2-B-1; it is only necessary that the substance return to state 1 in order to have completed a cycle. Cycles are studied in detail after a study of processes.*

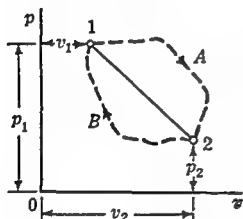


Fig. 1. Repeated.

11. Closure. Much of what is said in this chapter is a review of topics which have been studied in previous courses. However, do not let this deceive you. It will likely be necessary in this course to know this material better than you have ever known it before. Inasmuch as most failures in school subjects are due to a lack of understanding of the English language, and inasmuch as an understanding of succeeding chapters depends on knowing the meanings of the technical words in this chapter, review this chapter and each chapter hereafter with the aim of being sure that you understand the new words, most of which will be in **bold-face italics** for easy identification.

It is necessary for the beginner in thermodynamics to study regularly and earnestly each assignment. In many subjects, it may seem feasible, if not desirable, for you to study intensively at irregular intervals in order to perform acceptably where your work is graded. However, on average, such tactics are sure to result unfortunately in this subject. At the outset of the course, you are confronted with conceptions, most of them new conceptions, which you will use off and on throughout the remainder of the work. More-

over, in many instances, these conceptions build up, one on the other, so that without a clear understanding of the earlier work it is not only impracticable but also impossible to achieve a clear understanding of the later work. Study each assignment diligently and learn as much as you can before a class explanation is given. Then use the classroom period to buttress your original efforts and to clear up the hazy parts. You must not just understand the words of your teacher, but *you must make the ideas he explains a part of your working knowledge*. You will find profitable frequent *quick reviews* made on your own initiative.

While the subject of thermodynamics is not an easy one neither is it difficult if taken *and mastered* bit by bit. Moreover, students who want to become engineers find the course attractive and many of them voluntarily remark that thermodynamics is exceptionally interesting.

2

CONSERVATION OF ENERGY

12. Introduction. Energy (E) is inherent in all matter. Indeed, there is scientific evidence that suggests that matter and energy are fundamentally the same thing, or at least that mass may be converted into energy and energy into mass. According to Einstein, whose hypothesis (1905) was one step in the development of atomic power, the relation between mass and energy is $(\text{energy}) = (\text{mass})(\text{velocity of light})^2$, where a consistent system of units is used. Since the velocity of light squared is a very large number, the energy of a particular mass is very large; for example, one pound of matter is equivalent to nearly 39×10^{12} Btu—but this is not to say that it will ever be possible to convert a particular mass of any substance entirely into energy. (See Chapter 7.)

By *energy*, we mean a certain something which appears in many different forms, forms which are related to each other insofar as conversion can be made from one form of energy to another. This is admittedly not a definition. We are unable to define the general term *energy* (E) in a simple way, but we can define with precision the various forms in which it appears. This chapter considers the forms of energy with which we shall be concerned in this study and it names some of the other forms.

13. Measuring Energy. The total amount of energy that a system contains cannot be determined. We are accustomed therefore to measuring energy above some arbitrary datum. This practice is quite satisfactory, since in engineering we need know only the *change* in energy. Thus, we consider the potential energy of one pound of water to be the change in potential energy that this pound of water would undergo in falling from the reservoir to the power plant. The level of the power plant is the datum level. In like manner, other forms of energy are measured above some so-called *datum state*.

Energy is a scalar quantity, not a vector quantity Velocity, a vector quantity has direction as well as magnitude Energy has only magnitude The energy of a system of bodies is simply the sum of the energies of the individual bodies The total energy of a single system or substance is the sum of the magnitudes of the various forms of energy (such as mechanical kinetic energy, molecular energy, chemical energy) which the system or substance possesses

You are familiar with many units of energy from your study of physics defined before the law of conservation of energy was accepted Experimenters with heat defined the British thermal unit (Btu) as the amount of heat to raise the temperature of one pound of water 1°F This Btu is slightly different at different temperatures, hence the mean Btu $\frac{1}{180}$ of the heat required to raise the temperature of one pound of pure water from 32° to 212°F under standard atmospheric pressure, 14.696 psia has been widely used Of course, experimenters with work had the foot pound as a unit of energy, and then the early workers in electricity came up with their units of energy When it was realized that heat and work and electricity were manifestations of the same concept, energy a tremendous amount of experimentation was done to relate the foot-pound to the Btu Since experiment cannot by its nature give a single unique answer (experiment can say that the odds are so many that a value is between certain limits) we finally decided simply to *define* the relations between the units just as an American yard is defined in terms of an international meter (metric system) maintained at Sèvres France As currently defined then, 1 Btu is equivalent to 778.172 ft-lb Because of Joule's* work in establishing the relation between heat and work, we call the conversion constant Joule's constant and designate it by J ,

$$J = 778.172 \text{ ft lb /Btu};$$

use 778 in slide rule work See the Appendix for other conversion constants

14 Mechanical Potential Energy The mechanical potential energy or *gravitational energy* of a body is its energy due to its position or elevation with respect to the earth A weight at some distance z above the floor will give up a certain amount of energy upon impact if it is allowed to fall freely to the floor The change of mechanical potential energy of the body at the instant of impact is equal to the product of the force of gravity on the body [mg see equation (d), p 8] and the vertical distance z through which the

* James Prescott Joule (1818-1889) an English scientist was educated privately by a tutor being at one time a student assistant to Dalton (§ 193) His researches were significant in the budding science of electricity as well as in the science of thermodynamics in which he established two important fundamental principles the equivalence of heat and work and the dependence of the internal energy change of a perfect gas upon temperature change (§ 57) As a result of this work the modern kinetic theory of heat superseded the caloric theory of heat Joule once remarked I believe I have done two or three little things but nothing to make a fuss about

force acts. For a body with finite dimensions, z is the distance moved by the body's center of gravity. Thus, the mechanical potential energy P , due to elevation and the change of potential energy $\Delta P = P_2 - P_1$, are

$$(1A) \quad P = mgz = \frac{w_o}{g_o} gz \text{ ft-lb.},$$

$$(1B) \quad P = \frac{g}{g_o} z \text{ ft-lb./lb.}$$

$$(1C) \quad \Delta P = mg(z_2 - z_1) \text{ ft-lb.}, \text{ or } \frac{g}{g_o} (z_2 - z_1) \text{ ft-lb./lb.},$$

where m is the mass in slugs

$$[\rightarrow \text{lb.-sec.}^2/\text{ft.}; mgz \rightarrow (\text{lb.-sec.}^2/\text{ft.})(\text{ft./sec.}^2)(\text{ft.}) = \text{ft-lb.}]$$

and g is the local acceleration of gravity; w_o may be considered as either the mass in pounds or the force of gravity at a location of standard gravity g_o . In actual engineering systems, we generally deal with a rate of flow; for instance, w lb. per sec., V cu. ft. per min. (cfm), in which case, the change of potential energy is in foot-pound per second or per some other unit of time. The context makes clear the time unit involved. Inasmuch as the acceleration of gravity varies little on the surface of the earth, it is generally accurate enough in engineering problems to consider $g \approx g_o$, and get

$$(a) \quad P \approx wz \quad \text{and} \quad \Delta P = P_2 - P_1 \approx w(z_2 - z_1) \text{ ft-lb.},$$

$$\text{or} \quad \Delta P \approx \frac{w(z_2 - z_1)}{J} \text{ Btu},$$

where w is the mass in pounds of the body whose potential energy P or change ΔP is desired, and where z ft. is the elevation of the body's center of gravity above the desired datum level. Equation (a) is generally accurate enough, but note particularly the true value in equation (1). We say that a body has mgz ft-lb. of energy stored in it when we mean that it has mgz ft-lb. of potential energy with respect to the datum z ft. away.

A change of elevation is a change of state. For a fluid to have a change in the property z , it must move. Thus, to define a change of state of a body which changes elevation, specify not only the changes in two independent thermodynamic properties, such as p and v , but also the change in z . Observe that mechanical potential or gravitational energy exists by virtue of the force of attraction between two bodies, the earth and the body which is said to have potential energy.

15. Mechanical Kinetic Energy. A body of mass m (any unit, but only slugs result in ft-lb.) all of whose parts are moving with a speed of v fps possesses kinetic energy K , as it is derived in mechanics, (48), of

$$(2A) \quad K = \frac{mv^2}{2} \left(\rightarrow \frac{\text{lb.-sec.}^2}{\text{ft.}} \times \frac{\text{ft.}^2}{\text{sec.}^2} \right) \text{ ft-lb.},$$

with the earth as the reference body Since m slugs is the mass w in pounds divided by the standard acceleration of gravity g_0 [equation (e) p 8] we have the energy stored in a body in the form of mechanical kinetic energy as

$$(2B) \quad K = \frac{wv^2}{2g_0} \text{ ft lb} \quad \text{or} \quad \frac{v}{2g_0} \text{ ft lb /lb} \quad \text{or} \quad \frac{wv^2}{2g_0 J} \text{ Btu}$$

and the change of kinetic energy is

$$(2C) \quad \Delta K = K_2 - K_1 = \frac{w}{2g_0} (v_2^2 - v_1^2) \text{ ft lb} \quad \text{or} \quad \frac{w}{2g_0 J} (v_2^2 - v_1^2) \text{ Btu}$$

where v is the speed in feet per second If you care to remember it

$$2g_0 J \approx 50\,000$$

This equation (2) is applicable to moving fluids liquids or gases as well as to rigid bodies Notice that $v/(2g_0)$ is in foot pounds per pound of mass where the force pound in the foot pound should not cancel the mass pound because the units would then lose their significance To define a change of state of a fluid which changes speed specify not only the changes in two independent thermodynamic properties such as p and v but also the initial and final speeds In thermodynamic practice w is likely to be a rate of flow in which case the time unit is made clear by the context

16 Internal Energy Matter is composed of an aggregation of molecules which are moving continuously but haphazardly Since the molecules have mass we conceive of them as having a kinetic energy called *internal kinetic energy* analogous to that of any more tangible body in motion

The total internal kinetic energy arises principally from (1) the translational motion of the molecules (2) the rotational motion of the molecules and (3) a vibratory motion of the atoms within the molecules. Some molecules consist of one atom (for example helium and argon) The molecular kinetic energy of such substances called *monatomic substances* is almost wholly due to the translational motion of the molecules first because the vibration of atoms within the molecule relative to one another is impossible since there is only one atom and second because the kinetic energy of rotation is negligible since the mass of the molecule (atom) is concentrated so close to the axis of spin (that is the moment of inertia of the molecule with respect to the axis of rotation is negligible)

In a diatomic molecule which is one made up of two atoms (for example oxygen O_2 and nitrogen N_2) the atoms may move to and fro relative to each other in a vibration while the molecule is undergoing a translation and a rotation Moreover the atoms are removed somewhat from an axis of spin so that the rotational energy may be appreciable These conditions are true of polyatomic molecules in general Thus especially at high temperatures the internal kinetic energy of molecules of actual gases with more

than one atom includes a significant amount of rotational and vibratory energy. (See Fig. 19.)

In addition to the internal *kinetic* energy, substances have an internal *potential* energy; that is, an internal energy due to the relative positions of its molecules and the attraction of these molecules for one another. We know that a change in the mechanical potential energy of a body occurs when the elevation of the body relative to the earth as a datum is changed; the force of gravity is acting to aid or hinder this motion. The change of potential energy results from a force of attraction between at least two bodies which change position with respect to one another. Since there is an attractive force between molecules, we can see, by analogy with the concept of mechanical potential energy, that if anything happens to increase or decrease the average distance between molecules, there will be a change in the *internal potential energy*, which is that portion of the internal energy due to the configuration of the molecules. As an illustration of such a change, imagine one pound of water being boiled. As water, it occupies about 0.016 cu. ft. If the pound of water is entirely evaporated at atmospheric pressure and the steam collected, it will be found that the one pound of steam occupies about 27 cu. ft., a volume about 1700 times that occupied by the water. Since the number of molecules in the pound of water is virtually the same as the number in the pound of steam, it follows that the molecules must be much more widely separated in the steam than in the water. To move these molecules apart against their attractive forces requires a large amount of energy, which is retained in the steam as part of the steam's store of energy. When and if the steam is condensed, this same amount of internal potential energy will be given up.

The sum of the internal *kinetic* energy and the internal *potential* energy is called the *internal energy*,* which is energy *stored* within a body or substance by virtue of the *activity* and *configuration* of its molecules and of the vibration of the atoms within the molecules. We do not know how to find the absolute quantity of internal energy in any substance; however, what must be known in engineering is not the absolute quantity but the change of internal energy. Above any convenient datum,

u Btu per lb. represents the specific internal energy (1 lb.),

$U = wu$ Btu represents the internal energy of w lb.,

$\Delta u = u_2 - u_1$ and $\Delta U = U_2 - U_1$, the change of internal energy.

In thermodynamic practice, w is often a rate of flow, in which case the time unit is made clear by the context.

17. Work. For work W to be done, there must be a force acting on a body that moves. The *work* of a force, F , may be defined as the *displace-*

* Some writers use the term *internal energy* to mean the total energy stored within a system (β). In this book, it will always mean the molecular energy as defined. This usage has become well entrenched.

ment dx of a body (considered as a particle) multiplied by the component F_x of the force in the direction of the displacement $dW = F_x dx$ ft lb This definition provides a basic unit of energy the *foot pound* when the force is measured in pounds and the displacement in feet

It is important to observe that work is energy *in transition* that is it exists only when a force is overcoming a resistance (which may be inertia only in an ideal system) and it is being done only when a force is moving through a distance When the point of application of the force ceases to move there is no work Contrast the concept of work with that of internal energy The internal energy is energy which the body contains On the other hand a body never contains work The body or system may have the capacity to do work or work may be done on the system For example work flows into a system but after it is in it is not work it may have been converted into internal energy (We study what happens in detail later) Similarly as derived in mechanics work done on a body may be converted into kinetic energy and vice versa

Work is a form of energy which can be converted 100% into other forms in the absence of friction of any kind and for that reason it is the most valuable form For example it can all be converted into electricity or kinetic energy and electricity and kinetic energy can be fully converted into work in ideal frictionless systems In fact we could with logic take electricity as a form of work (10) since it too is fully available energy in transition but not so with kinetic energy which is comparable in kind with internal energy in that it is *stored* in a body or system Work can be converted 100% into heat (§ 19) friction or no friction friction hastens the conversion of work into heat

Since work can be measured at various points in a machine adjectives must be applied in order to say what work is intended Many thermodynamic computations give the *ideal work* W of the fluid that is the work which the fluid would do (or have done on it) if there were no friction or other losses during the process Ideal work is done *by* or *on* the fluid

In a machine which manufactures work the actual work which the fluid does is always less than the ideal and the actual work of the fluid is greater than the work delivered by the shaft of the engine In a reciprocating engine such as a Diesel engine the actual work of the fluid in the cylinder [system (1) Fig 5] is called the *indicated work* W_I when it is taken for a complete cycle of events in the cylinder because it may be determined with an indicator (§ 119) Note that energy W_I is crossing the boundary of system (1) The actual work which passes a boundary at C is less than W_I because of friction at the piston rings and cylinder walls at the packing glands at B and at the pin near C In the engine system (2) the work W_B which crosses the boundary (that is delivered by the shaft) is smaller than W_I because of friction at the crank pin and at the crankshaft bearings (and other miscellaneous frictional losses for example the fanning of the air

by the flywheel and crank). This work delivered by the shaft is commonly called *brake work* W_B because it was first measured by a brake which absorbed all the output of the engine via friction. Other names for brake work include *shaft work* and *delivered work*.

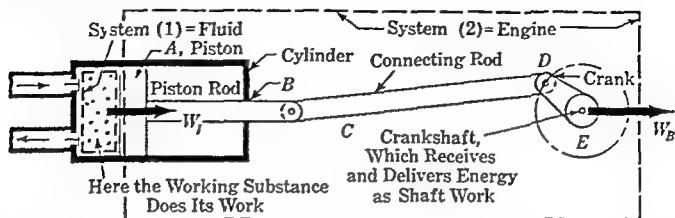


Fig. 5. Work in a Reciprocating Engine. Observe that shaft work (or any kind of work) is energy in transition in the sense that it exists by virtue of the movement of an element of a machine against a resistance or, in general, of the movement of a force through a distance. Shaft work may be transformed into kinetic energy and stored in the attached rotating parts to a limited extent. System (2) is outlined by the large dotted rectangle.

Since work may be done either by or on the system, it is convenient to use plus and minus signs to designate its direction of "flow." Let

Work done by the system be positive (as a steam turbine); an outflow of energy;
Work done on the system be negative (as an air compressor); an inflow of energy.

The symbol W stands for work done by or on one pound or any number of pounds of substance; the mass involved is defined by the context. Also, there may be a time unit involved. Typical units for W , which will be clear from the context, include ft-lb., ft-lb. per min., Btu, Btu per sec., hp, hp-hr., etc.

18. Work of a Nonflow System. Let the system be a quantity of an expansible fluid, such as a gas or vapor, enclosed within a cylinder and piston (Fig. 6). This is a *closed system* in which *nonflow processes* may occur. The volume of the fluid is V_1 and its pressure is p_1 . If we consider the state of the fluid on the pV plane (meaning that the coordinates are p and V), the particular coordinates p_1 and V_1 locate the point 1 (Fig. 6). If the working substance expands and moves the piston against a variable resistance, work will be done by the fluid. In a typical practical process of this sort, the pressure drops and the state of the substance changes as suggested by

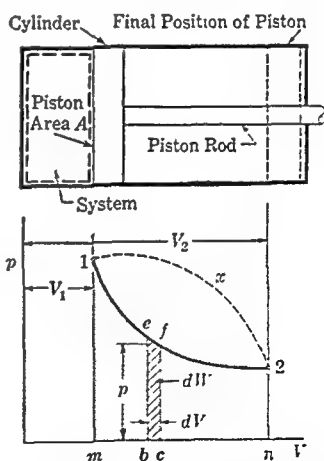


Fig. 6. Closed System—Work of Expansion. The boundaries of the system expand and contract with the movement of the piston.

the curve 1-e-f-2, which curve is called the *path of the state point*. This curve represents a process of some kind. Since the pressure and therefore the force on the piston is varying it is necessary to integrate $\int F_x dx$ in order to find the work. Consider a change of state from *e* to *f* (Fig 6) so small that the pressure is essentially constant during the change. The force acting on the piston will be the pressure times the area of the piston, $F_x = pA$. The distance that the piston moves is a differential quantity dL , and the work for this infinitesimal motion of the piston is force times distance,

$$dW = (pA)dL = p(A dL) = p dV,$$

where $A dL$ is a differential volume equal to dV . The total work done in the reversible nonflow process is

$$(3) \quad W = \int_1^2 p dV \text{ ft-lb,}$$

[REVERSIBLE, NONFLOW PROCESS]

where the units must be p lb per sq ft (psf) and V cu ft to give work in foot pounds. We shall go into the matter of a reversible process later, so

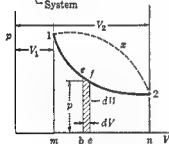
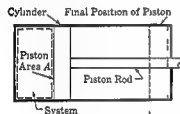


Fig 6 Repeated

let it suffice here to say that there is no friction in a *reversible process*, mechanical or fluid, everything is on an ideal basis and the work of the fluid $\int p dV$ appears as shaft work.

Notice the differential area *befc* (Fig 6) whose width is dV and height p . The magnitude of this area is $p dV$ and the sum of all these differential areas is the total area under the curve 1-e-f-2 when the integration is made from state 1 to state 2. Thus we have the area equal to $\int p dV$ and $W = \int p dV$, therefore, the area "under the curve of the process on the pV plane represents the work done during a nonflow reversible process.

The work done and the area under the curve on the pV plane depend upon the location of the path 1-e-f-2 (Fig 6), that is upon the relation between p and V [$p = f(V)$]. Although states 1 and 2 are particular points and p_1 , p_2 , V_1 , and V_2 have particular values the change of state may occur along any other path 1-x-2. Thus the work done has changed because the path of the state point has changed. *the work depends upon the process*. It is not a property, as pressure and volume and internal energy. Since such properties are dependent only on the location of the state point, they are called *point functions*. All the properties listed in § 4 are point func-

tions, depending only on the state of the substance. It is correct to say that the change of volume $\Delta V = V_2 - V_1$, but the work done cannot be said to be $W_2 - W_1$; W is not a *point* function.

When the function $p = f(V)$ is known or assumed, the integration can be made. For example, Boyle's law is that $pV = C = p_1V_1 = p_2V_2$, etc., when the temperature of an ideal gas is constant; C is a constant. Substituting $p = C/V$, into equation (3), we get the work during a frictionless process at constant temperature as

$$(b) \quad W = \int_1^2 p \, dV = C \int_{V_1}^{V_2} \frac{dV}{V} = C \ln \frac{V_2}{V_1} = p_1V_1 \ln \frac{V_2}{V_1}.$$

[GOOD FOR REVERSIBLE PROCESS ONLY WHEN BOYLE'S LAW HOLDS]

The units in equation (b) are foot-pounds or foot-pounds per some unit of time, the time unit being associated with V , as cfm. The convention of signs defined for W in § 17 was chosen because it accords with the results of the integral $\int p \, dV$; that is,

$\int p \, dV$ is *positive* when nonflow work is done *by* the system,
 $\int p \, dV$ is *negative* when nonflow work is done *on* the system.

This integration may be made for any kind of expansible substance, given the pV relation.

19. Heat. We shall use the word *heat* Q to mean *energy in transition (moving) from one body or system to another because of a temperature difference between the bodies or systems.** This transfer of energy occurs by conduction, by radiation, and by convection. We have confidence on the basis of personal experience that heat flows from a *hot* body to a *cold* body. Also, it is easy to accept the notion that if two bodies are at the same temperature, heat does not "flow." When two bodies are in thermal communication (that is, ineffective insulation between them) and in states such that no heat goes from one body to the other, they are said to be in *thermal equilibrium*. It logically follows that if two bodies are in thermal equilibrium with a third body, the two are in equilibrium with each other; this statement is known as the Zeroth law of thermodynamics (1). Thus, when two bodies are in thermal equilibrium with the same environment they are in equilibrium with each other and there will be no net interchange of heat between them.

Conduction of heat occurs because the faster moving molecules in the hotter part of a body communicate (by impacts) some of their energy to adjacent molecules. Since temperature is a measure of the molecular activity, we are reminded that heat flows from the hotter to the cooler portion of a body, that the more active molecules lose some of their energy to

* In layman's parlance and in some physics books, *heat* is used to mean molecular energy, that which we here call *internal energy*. If you are accustomed to this notion of heat, you must make an effort to replace it with the foregoing definition.

the less active molecules Heat is conducted through solids, liquids, and gases

Radiant heat consists of electromagnetic waves waves of the same nature as light and radio waves We may consider that the radiant energy which enters or leaves a system is included in the symbol Q , but it is better to give other forms of radiated energy a distinguishing symbol, reserving Q for heat Typical wave lengths of radiant energy in meters are as given in parentheses (12) cosmic rays (10^{-14} and shorter) gamma rays (10^{-13} – 10^{-11}), X-rays (10^{-11} – 10^{-8}), ultraviolet (10^{-8} – 10^{-7}), visible light (4×10^{-7} – 7×10^{-7}), infrared or *heat* (10^{-6} – 10^{-4}), microwaves, radar (10^{-2} – 10^{-1}), television and FM radio (1–10), short-wave radio (10 – 10^3), AM radio (10^2 – 10^4), maritime communications (10^3 – 10^4)

All bodies radiate heat (§ 416), so a transfer of heat by radiation occurs because a hot body emits more heat than it receives or a cold body receives more heat than it emits

Convection occurs because a moving fluid picks up energy from a warm body and delivers energy to a colder body For example the air surrounding the furnace of a hot air heating system receives heat by radiation and conduction This heated air being lighter, will rise and circulate through the house, giving up energy by radiation and conduction to keep the house and contents warm When this series of events happens, we say that heat has been convected although the energy is not classified as heat while it is being transported but only while it is being received or discharged In the systems which we shall study, the convected energy will generally not be considered as heat This energy quantity will be cared for by energies associated with the stream

Observe that heat, like work, is energy in transit, it is a concept of that something which moves across a boundary into or out of a system by virtue of a driving potential which we call *temperature* Heat Q which flows into a system may result in an increase of internal energy of the system or in work being done by the system but after it enters the system it is no longer heat Work is 100% available for conversion into other forms of energy, whereas heat cannot continuously be converted 100% into work even in the most perfect engine imaginable, this is a significant distinction between these two kinds of energy in transition

It will be made clearer soon (§§ 35–37) that heat, like work, is a path function By its nature, Q is a change, we never say $Q_2 - Q_1$ because such a difference holds only for point functions (state properties) The units of Q are any convenient energy units—in this book usually Btu or Btu per unit of time, either for one pound or any number of pounds

20 Flow Work. If a substance is flowing into or out of a system being studied, a certain energy quantity is involved, as the substance crosses the boundaries of the system, called the *flow work* or *flow energy* Imagine a

fluid, liquid or gas, flowing in a pipe (Fig. 7). Let some small quantity of this substance be on the point of crossing the boundary and entering the system. In order for it to get into the system, work must be done on it in an amount sufficient to move it against the resistance (at pressure p) offered by the system. The resisting force F is pA , and the work done against this resistance in pushing a quantity of fluid of length L across the boundary is $FL = pAL = pV$, where $V = AL$ is the volume of fluid pushed across the boundary. An energy quantity equal to pV thus crosses the boundary and enters the system. Similarly, there is an energy quantity pV leaving the system while the substance passes outward across the boundary. This idea is generalized for a movement of any volume V across any boundary against constant resistance when we may say that the energy of flow crossing such boundary is pV , where p and V are values at the boundary.

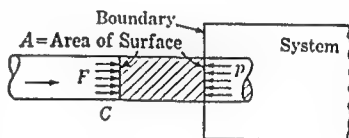


Fig. 7. Flow Work.

This energy is a point function, $W_f = pV$, because it depends on particular values of p and V at some boundary; therefore for boundaries 1 and 2, we may say

$$\begin{aligned} (c) \quad \Delta W_f &= W_{f2} - W_{f1} = p_2 V_2 - p_1 V_1 \text{ ft.-lb.}, \\ \Delta W_f &= W_{f2} - W_{f1} = p_2 v_2 - p_1 v_1 \text{ ft.-lb./lb.}, \end{aligned}$$

where p is the absolute pressure in *pounds per square foot*, $V = wv$ cu. ft. is the volume of the total mass of substance crossing a boundary (usually during a particular time, as cfm), and v is the specific volume (for unit mass per unit time).

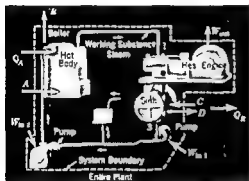
Since the energy to cause flow in a pipe does not create itself, we may think of the flow work W_f as originating, for example, in a pump which is located upstream, or in the potential energy of a reservoir. However, it is generally *not* pump work (but it may be); it exists because energy is being expended somewhere, somehow, to cause a movement of a fluid across a boundary.

21. Other Forms of Energy. Manifestations of energy occur in many forms with which we shall have little or no concern in this book. However, the laws of thermodynamics apply to all forms of energy and are often useful in specialized fields of study. Recapitulating first, we have defined the energy forms:

- P , potential energy; *stored*; change is $P_2 - P_1$;
- K , kinetic energy; *stored*; change is $K_2 - K_1$;
- U , u , internal energy; *stored*; change is $U_2 - U_1$ or $u_2 - u_1$;
- W_f , flow work; *in transition*; change (between boundaries) is $W_{f2} - W_{f1}$;
- W , work; *in transition*; change is path function, $\int p dV$ in ideal nonflow;
- Q , heat; *in transition*; change is a path function.

Then there is electricity E_e (which we shall call *combined work* W_K on occasion), chemical energy E_c (resulting from a change in molecular structure—as in combustion of fuels), electromagnetic emanations (light, radio, brain waves (?), etc), acoustic energy (sound waves), electrochemical energy—a source of electricity (storage battery), nuclear energy (resulting from a change in the structure of the atom's nucleus and a conversion of mass into energy), energy stored due to surface tension, and others. In subsequent discussions, all energy forms mentioned in this paragraph are assumed to be absent unless specifically included, for example, when it is said that an equation applies to all reversible processes, it is understood, in the absence of a contrary statement, that the processes are concerned only with the energy quantities listed through Q at the bottom of p 25

22 Classification of Systems Systems may be classified in different ways. The definitions in this article will serve our purpose. As we have



Courtesy Power New York City

Fig 8 Steam Power Plant The source of the heat added to the steam is in the entering fuel and air stream at A in the form of chemical energy. Operation: Water is evaporated into steam in the boiler, steam flows to the prime mover (engine) and some of its energy is converted into work W_{out} leaving the system, exhaust steam enters the condenser and is condensed to water by circulating cooling water, in and out at C and D , where heat is rejected from the cycle, the two pumps pump the water back into the boiler and it repeats the cycle. The energy required by the pumps is (W_{in1} and W_{in2}). The net work of the cycle is $W = W_{out} - W_{in}$.

as the mass circulates through the cycle. The total energy within such a system is its stored energy E , the sum of all potential, kinetic, and internal energies.

A closed system of a working substance which is not flowing, as for

learned, a closed system is one in which mass does not cross its boundaries energy may (and does in the systems we study here) cross the boundaries—in the form of work and/or heat. Within a closed system the working substance may or may not have motion. If, for example, all of the working substance, steam and water in a power plant is taken as a system, the system is a closed one. See Fig 8, but note that the system referred to here is inside the various pipe lines and containers. However, the working substance is continuously moving, and at any instant it will have stored in it some total kinetic energy and some total potential energy as well as internal energy. Each of these energy quantities as they apply to a particular small mass of substance varies from point to point.

instance the system in Fig. 6, p. 21, is a *nonflow system*, and the processes which such a system undergoes are called *nonflow processes*. Excluding turbulence, which may exist in a nonflowing system, the total stored energy is the total internal energy ($P = 0$).

An *open system* is one in which mass crosses its boundaries; transit energy, work and heat, may or may not be crossing the boundaries at the same time. There may be several mass streams, as in a steam power plant where the mass streams are (1) the entering fuel, (2) the entering air to burn the fuel, both at *A*, Fig. 8, (3) the products of combustion leaving at *B*, (4) the circulating cooling water for condensing the exhaust steam from the turbine, entering at *C*, leaving at *D*. There are a number of common examples of systems involving one substance; for example, a steam turbine (Fig. 9) where the steam enters the system at 1 through a valve and leaves at 2. In the turbine, the steam speeds up in going through nozzles and gains kinetic energy which is converted into work as the steam passes through the turbine blades.

The type of open system which we shall study most frequently is a *steady flow system*, which has the following characteristics: (1) the rate of flow (pounds per second, for example) of each mass stream entering and leaving the system is constant; this means that there is neither accumulation nor diminution of mass within the system; (2) there is neither accumulation nor diminution of energy within the system (the rate of flow of heat Q and work W are constant); for example, the turbine of Fig. 9 delivers work and radiates

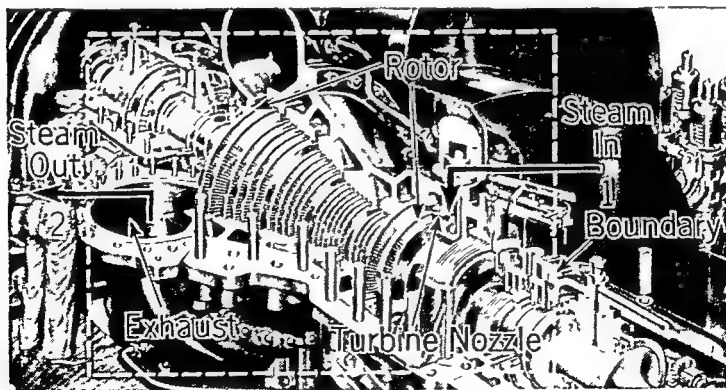


Fig. 9. Turbine as a System.

heat at a constant rate while operating under steady flow conditions; (3) the state of the working substance at any point in the system remains constant. For most practical purposes, a reciprocating engine may be analyzed as a steady flow system when the state of the working substance varies uniformly and periodically through the same series of states; for example, an

internal combustion engine may be considered as a steady state system when it burns the same amount of fuel during each cycle, when the temperature of each batch of exhaust is always the same, when the heat carried away by the cooling water is the same during each cycle of events, etc

Unsteady flow systems are of several forms, each one characterized by accumulation or diminution of either mass or energy, or both, within the system. For example (1) fluid may flow outward only from a system (as a system of air in a tank from which air flows to "pump up" a tire), (2) fluid may flow inward only (as in refilling the air tank with the discharge valve closed), (3) fluid may be flowing both inward and outward but at different rates so that the amount of fluid within the system increases and decreases, or (4) the engine is warming up (increasing power) or cooling off (decreasing power). By way of illustration of all these ideas consider an automotive engine. When it is started cold much of the energy from the fuel is used to heat the metal parts and the cooling water. While the car is getting under way, varying quantities of fuel and air enter the engine. If the car speed is maintained constant on level ground and if the environment (temperature and wind speed, principally) remains constant the engine eventually reaches steady flow operation—even though the temperature at a particular vicinity within the system (inside the cylinder for instance) fluctuates, each cycle of fluctuation is the same as the preceding ones. If the car now goes up hill at the same speed more work must be done by the engine more working substance must flow into the engine and the engine "gets hotter". If the car goes down hill at the same speed less work is done by the engine less fuel is used, and the engine runs "cooler" and not in steady state. While steady flow and steady state are evidently difficult to achieve in a moving automobile, tests and analyses of automotive engines are generally made under virtual steady flow conditions notwithstanding. It is easy to approach steady flow in a stationary internal combustion engine on an overall basis.

23 Continuity of Mass

Implied in the foregoing discussion is the *law of the continuity of mass*, also called *conservation of mass*, which says that mass is indestructible. Within the bounds of our experience, this law holds except in atomic processes where mass is converted into energy. Also, there is speculation that mass may be created in the "vacant"

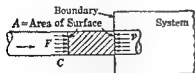


Fig 7 Repeated

space of the universe—one atom of hydrogen per year in some several million cubic feet of space.*

In Fig 7, let the average velocity of the fluid at section *C* in the pipe line

* See Fred Hoyle, *The Nature of the Universe* for an interesting account written for the layman.

be v fps; then the quantity of fluid flowing is $A v$ cu. ft. per sec., where A sq. ft. is the cross-sectional area. If the specific volume is v cu. ft. per lb., then the mass rate of flow is $w = A v / v$ lb. per sec. If the area of the cross section changes, the law of the continuity of mass shows that

$$(4A) \quad w = \frac{A_1 v_1}{v_1} = \frac{A_2 v_2}{v_2} \text{ lb./sec.}$$

$$(4B) \quad w = A_1 v_1 \rho_1 = A_2 v_2 \rho_2 \text{ lb./sec.,}$$

for v fps, A sq. ft., v cu. ft. per lb., and ρ lb. per cu. ft. Equation (4) says that the mass of flow past boundary 1 is equal to the mass passing boundary 2—assuming no accumulation or diminution of mass between sections 1 and 2.

24. Conservation of Energy. The *law of the conservation of energy* states that *energy can be neither created nor destroyed*.^{*} It is a law based on physical observations and is not subject to mathematical proof. In its application to energy transformations on this earth, there is no known exception, except as mass is converted into energy, and vice versa, and it is therefore an accepted principle and a reliable guide.

Historically, the *first law of thermodynamics* stated that work and heat are mutually convertible (see § 82), but the tendency now is to broaden the statement to include all forms of energy: *one form of energy may be converted into another*.

25. Energy Equations for Closed Systems. The law of conservation of energy may be stated in several different working forms. For example, for any kind of a system, it should be evident, if energy is neither created nor

^{*} This law is not an idea which burst suddenly upon the scientific world. After scientists began working with energy, it was years before the law was comprehended. Benjamin Thompson (Count Rumford), 1753–1814, who has been called an arrogant and insufferable genius, really discovered the equivalence of work and heat in the course of manufacturing cannon (1797) by boring solid metal submerged in water. He was intrigued by the water boiling because of the mechanical work of boring, yet no heat had been added to the water. He convinced himself, but not the world, that the then accepted caloric theory of heat (a theory which supposed heat to be a substance without mass) did not explain all known phenomena of heat, and that work and heat were in some manner related phenomena. In his words, "Is it possible that such a quantity of heat as would have caused five pounds of ice cold water to boil could have been furnished by so inconsiderable a quantity of metallic dust merely in consequence of a change in its capacity for heat?" Other experimenters later discovered more evidence, until some fifty years after Rumford's cannon experiments, Joule, with assistance from Lord Kelvin, showed conclusively that mechanical work and heat are equivalent. (We are now just a few years from Joule. See the footnote on p. 16. Considering the age of the earth in billions of years and the age of man as about 1,000,000 years, thermodynamics is a new science—as is all science). Rumford was teaching school in Rumford, Mass. (now Concord, N. H.) when he met, wooed, and won a wealthy widow. He was sympathetic with the "other" side at the time of the Revolutionary War and decided that it would be smart to leave Boston with the British, which he did, deserting his wife and daughter. Though he wanted to, he never returned to America, but he gained fame and honors in Europe and England.

or solving for Q ,

$$(7B) \quad Q = \Delta U + \Delta W_f + \Delta K + \Delta P + W,$$

[STEADY FLOW ENERGY EQUATION]

where all terms are in the same units and all Δ values are the second state property minus the first state property. Since the foot-pound is a relatively small unit for most of our purposes, we use the Btu more often and Joule's constant J to show the conversion where necessary. Also, (7) may represent one pound or any number of pounds of fluid flowing during some elapsed time. You should fix in mind the meaning of (7), the method of determining the value of each term (or the change in value, such as ΔU), and learn these things so well that you will remember and *recall* them as long as you practice engineering. We know that the potential energy is

$$P = \frac{(wg/g_s)z}{J} \text{ Btu}$$

and ΔP is given by $\Delta P = P_2 - P_1 = (wg/g_s)(z_2 - z_1)/J \approx w(z_2 - z_1)/J$ Btu, that the kinetic energy $K = wv^2/(2g_s J)$ Btu and

$$\Delta K = \frac{w(v_2^2 - v_1^2)}{2g_s J},$$

that the flow work is $W_f = pV/J$ Btu and $\Delta W_f = (p_2V_2 - p_1V_1)/J$ Btu, or $(p_2v_2 - p_1v_1)/J$ Btu per lb. The convention of signs is the same as explained for the nonflow equation (6).

The working substance of some systems may be at a higher temperature than the environment. In such event, heat Q will naturally flow outward. Or the flow will be inward if the working substance is at a lower temperature than the environment. In certain situations, heat is intentionally added or abstracted concurrently with the natural passage of heat, as in a steam boiler. In most systems where heat undesirably moves in or out, the amount of such heat is often negligibly small compared with other energy quantities, hence, we often think of Q as a planned transfer.

27 Enthalpy A property which is found to be frequently useful to engineers and scientists is one called *enthalpy* (H, h) pronounced en-thal-py. It is a composite property applicable to all fluids and is defined by

$$(8A) \quad h = u + \frac{pv}{J} \text{ Btu/lb} \quad \text{and} \quad H = U + \frac{pV}{J} \text{ Btu for } w \text{ lb},$$

where $H = wh$. It is seen to be a point function (or property) because it is defined in terms of point functions only. The change in specific enthalpy is

$$(8B) \quad \Delta h = \Delta u + \frac{\Delta pv}{J},$$

$$h_2 - h_1 = u_2 - u_1 + \frac{p_2v_2}{J} - \frac{p_1v_1}{J},$$

and in terms of differentials,

$$(8C) \quad dh = du + \frac{d(pv)}{J} = du + \frac{p dv}{J} + \frac{v dp}{J}.$$

Enthalpy* being a point function, $\int dh = h_2 - h_1$, from state 1 to state 2. Since the absolute quantity of internal energy is not known, the absolute value of enthalpy is unknown. However, it usually matters little, because what we wish to know is the *change* of enthalpy. Enthalpy may be measured above any convenient datum.

28. Steady Flow Equation with Enthalpy. Inasmuch as the steady flow equation (7) includes the terms

$$u + W_f = u + \frac{pv}{J} \quad \text{and} \quad U + W_f = U + \frac{pV}{J},$$

we may replace these two terms with their equivalents, h and H . As we shall learn later, it will be as easy or easier to determine the change in enthalpy as it is to find the change in internal energy (not to mention pv). It is also true that we shall seldom be concerned with the change in potential energy ΔP ; it is not to be considered unless specifically included or unless it obviously belongs. The change of potential energy in a hydraulic power plant is the significant change, but in heat engines, elevation differences are relatively minor; or $P_1 \approx P_2$. With these two revisions, the steady flow equation becomes

$$(9A) \quad K_1 + h_1 + Q = K_2 + h_2 + W,$$

$$(9B) \quad \frac{v_1^2}{2g_\sigma J} + h_1 + Q = \frac{v_2^2}{2g_\sigma J} + h_2 + W \text{ Btu.} \quad [\Delta P = 0]$$

In applications of equation (9), other terms may be zero (ΔK or Q or W or Δh). One of the objectives of this study is for the reader to learn to omit energies which should be omitted and to include the ones which count. There are several other arrangements of the steady flow equation which are advantageous under one circumstance or another, including

$$(9C) \quad Q = h_2 - h_1 + K_2 - K_1 + W = \Delta h + \Delta K + W,$$

$$(9D) \quad Q = \Delta u + \frac{\Delta pv}{J} + \Delta K + W,$$

$$(9E) \quad dQ = du + \frac{d(pv)}{J} + dK + dW = du + \frac{p dv}{J} + \frac{v dp}{J} + dK + dW,$$

$$(9F) \quad dQ = dh + dK + dW.$$

[ANY STEADY FLOW, $\Delta P = 0$]

Equations (9C) and (9F) are important forms. If $\Delta K = 0$, we have $dQ = dh + dW$; compare with the nonflow equation (6B), p. 30. If $W = 0$,

* Other names in use for this property are *total heat* and *heat content*, both of which should be avoided.

we get $dQ - dh + dK$. As for a nonflow process (§ 2a) Q and W are net transfers of energy but as seen $W \neq \int p dV$ here. Practice writing the equation (9) in different arrangements just to get acquainted with it (See § 42.)

29 Applications of the Steady Flow Equation In order to apply any energy equation to a device it is necessary to know what the device does and more or less how it does it. One reason for this is that not all of the energy terms in an energy equation (6) or (7) are applicable to all devices and machines. One or more terms usually drop out because they are irrelevant or inconsequential. Thus the first step in the solution of a thermodynamic problem is to determine by whatever means possible the operation of the thing with which the problem is concerned. With this knowledge you are in a position to decide or tentatively decide which energies are to be considered.

Consider a turbine gas or steam. It receives a stream of fluid more or less steadily at high pressure which then expands to a low pressure doing work. In stationary practice this turbine will be fairly well insulated that is $Q \approx 0$. From a practical point of view the speed at which the fluid approaches the turbine is not very different from the speed at which it departs which is to say that there is not much change in kinetic energy. Thus the change of kinetic energy may be negligible $\Delta K \approx 0$. Now draw an energy diagram Fig. 210 considering all forms of energy, but omitting those which are not apropos or using the full steady flow equation write the energy equation for this particular application. It is best to do both. Thus from the energy equation (9C) we may write

$$(d) \quad W = -\Delta h - \Delta K \quad \text{or} \quad W = -\Delta h \quad h_1 - h_2 \text{ Btu/lb} \\ [Q = 0] \quad [Q = 0 \quad \Delta K = 0]$$

The same results are obtained from energy diagrams.

For another illustration consider a nozzle on a jet engine (see Fig. 219 § 322). A nozzle is a device which receives a fluid guides its expansion to some lower pressure converting some of the entering energy into kinetic energy at exit (Fig. 12). No shafts are turned so that $W = 0$. Also $Q \approx 0$ because the time of passage of a particle of mass through the nozzle is so short (a fraction of a second) that there is not enough time for the heat loss Q to be significant as compared to other energy quantities. From the energy diagram Fig. 12 or from (9C) we get

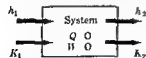


Fig. 12 Energy Diagram for Nozzle. The system is bounded by the internal surface of the nozzle and imaginary planes at its extremities sections 1 and 2.

$$(e) \quad \Delta K = -\Delta h = (h_2 - h_1) = h_1 - h_2 \text{ Btu/lb} \\ [Q = 0 \quad W = 0]$$

which says that the increase in kinetic energy in the nozzle is equal to the decrease of that useful property enthalpy. In many nozzles the entering

kinetic energy is negligible. $K_1 \approx 0$ and $\Delta K \approx K_2$. If this is so, equation (e) becomes

$$(f) \quad K_2 = \frac{v_2^2}{2g_c J} = h_1 - h_2 \text{ Btu/lb.}$$

$$[K_1 = 0, \quad Q = 0, \quad W = 0]$$

The reader probably observes that, even in these two simplest applications, one must have some engineering knowledge in order to judge the appropriateness of the various energies. Such elementary knowledge (and much more) is working knowledge for the engineer. Other applications will be made here and there throughout the book. The student should make an energy diagram for each problem solved.

30. Example. An air compressor takes in air at 15 psia and discharges it at 100 psia: $v_1 = 2$ cu. ft. lb. and $v_2 = 0.5$ cu. ft. lb. The increase of internal energy is 40 Btu lb. and the work is 70 Btu lb.: $\Delta P = 0$ and $\Delta K = 0$. How much heat is transferred?

SOLUTION. The forms of energy to be considered are flow work, internal energy, work, and of course heat. For 1 lb. of substance,

$$W_{f1} = \frac{p_1 v_1}{J} = \frac{(15)(144)(2)}{778} = 5.55 \text{ Btu lb.}$$

$$W_{f2} = \frac{p_2 v_2}{J} = \frac{(100)(144)(0.5)}{778} = 9.25 \text{ Btu lb.}$$

$$\Delta W_f = W_{f2} - W_{f1} = 9.25 - 5.55 = +3.7 \text{ Btu/lb.}$$

Since work must be done on the air to compress it, the work is negative,

$$W = -70 \text{ Btu lb.}$$

and since the internal energy increases, $\Delta u = u_2 - u_1 = +40$ Btu/lb. Using these various values in equation (9D), we solve for Q and find

$$Q = \Delta u + \Delta W_f + W = (+40) + (+3.7) + (-70) = -26.3 \text{ Btu lb.}$$

where the negative sign indicates that heat is rejected by the system. Some air compressors have water jackets for the purpose of cooling the air during compression; hence the negative sign would be expected.

Since it is important to be able to utilize energy diagrams, we shall repeat the solution from this viewpoint. The procedure is to decide upon what the system is, say, the air on its way through the compressor. Figure 13 sets the boundaries and shows what is known. The internal energy may be measured above the datum of the entering air, which gives $u_1 = 0$ and $u_2 = 40$, because $u_2 - u_1 = +40$. At this stage, it may not be known whether heat flows in or out. If we choose to show heat Q as transferred in, as in Fig. 13, a plus sign for the answer would say that

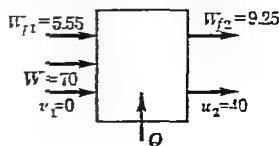


Fig. 13.

The direction of energy flow as shown is correct.

points 1 and 2 for different processes between the states. Therefore, the specific heat c as defined in equation (10B) depends on the process, and there are an infinite number of possible processes. For example, the specific heat of a constant volume process is different from that of a constant pressure process. Moreover, even in a particular process, the specific heat is not constant (§ 44), but luckily it can usually be expressed as a function of temperature so that equation (10B) may be integrated readily. For small variations of temperature, the specific heat may be considered constant. Therefore, if the mean or instantaneous value is used, equation (10B) may be integrated to give

$$(11) \quad Q = wc \int dT = wc(T_2 - T_1) \text{ Btu,}$$

[CONSTANT SPECIFIC HEAT]

between the limits of states 1 to 2 where the value of c is taken according to the process



Fig 14 Constant Volume System

36 Constant Volume Specific Heat There are two processes whose specific heats are especially important and useful: constant volume c_v and constant pressure c_p . Consider first a nonflow constant volume system (Fig 14). Since $\dot{W} = 0$, the heat transferred into the system must go into internal energy, and the nonflow equation for unit mass gives $Q_v = \Delta u$, where the subscript v suggests that the volume remains constant. Thus we have

$$(b) \quad Q_v = \Delta U = wc_v(T_2 - T_1) \text{ Btu} \quad \text{or} \quad Q_v = \Delta u = c_v(T_2 - T_1) \text{ Btu/lb}$$

[CONSTANT c_v]

and a definition of c_v in terms of point functions as

$$(12) \quad c_v = \left(\frac{\Delta u}{\Delta T} \right)_v \quad \text{or} \quad c_v = \left(\frac{du}{dT} \right)_v \text{ Btu/lb-}^\circ\text{R}$$

[AVERAGE VALUE] [INSTANTANEOUS VALUE]

The second form is recognized as a partial differentiation of u with respect to T with v constant, but we shall not adopt the partial notation at this time. For the *ideal gas* to be studied soon, it is not necessary for the volume to remain constant in order for equation (12) to be true. See Table I for some values of c_v .

37 Constant Pressure Specific Heat Let the system be a nonflow system as before, and add heat while the pressure is maintained constant (Fig 15). The pressure is balanced by a constant load F (Fig 15) on a frictionless, movable piston. For an average value of c_p , the heat is

$$(c) \quad Q_p = wc_p \Delta T \text{ Btu} \quad \text{or} \quad Q_p = c_p \Delta T \text{ Btu/lb,}$$



Fig 15 Constant Pressure System

Table I. CHARACTERISTIC CONSTANTS FOR GASES

(a) For the gases marked (a), the values of c_p are instantaneous values at $540^\circ\text{R} = 80^\circ\text{F}$ taken from spectroscopic data. Then c_v was found from $c_v = c_p - R/J$, and k from c_p/c_v . These values are for zero pressure, but they are suitably accurate for ordinary computations at ordinary pressures.

(b) For the gases marked (b), the values of c_p and k were taken from the *International Critical Tables*, Vol. V, for standard atmospheric pressure and 15°C (59°F). Then c_v was computed from $c_v = c_p/k$.

The gas constant R for each gas was computed from $R = p/(\rho T)$, where $\rho = 1/v$ was taken from the *International Critical Tables*, Vol. III, for standard atmospheric pressure and 32°F . M = molecular weight; c_p and c_v are in Btu per lb-°R. To obtain C_p Btu per mol, use $C_p = C_p - 1.986$.

Gas		M	c_p	c_v	k	C_p	R	MR
Air	(a)	28.970	0.24	0.1715	1.4	6.95	53.3	1545
Carbon monoxide (CO)	(a)	28.010	0.2487	0.1779	1.398	6.97	55.1	1543
Hydrogen (H ₂)	(a)	2.016	3.421	2.4354	1.405	6.9	767	1546
Nitric oxide (NO)	(a)	30.008	0.2378	0.1717	1.384	7.14	51.4	1542
Nitrogen (N ₂)	(a)	28.016	0.2484	0.1776	1.4	6.96	55.1	1543
Oxygen (O ₂)	(a)	32	0.2193	0.1573	1.394	7.02	48.25	1544
Argon (A)	(b)	39.95	0.125	0.0749	1.668	4.99	38.7	1544
Helium (He)	(b)	4.003	1.25	0.754	1.659	5.00	386	1544
Carbon dioxide (CO ₂)	(a)	44.01	0.202	0.157	1.29	8.89	34.9	1536
Hydrogen sulfide (H ₂ S)	(a)	34.08	0.328	0.270	1.21	11.18	44.8	1523
Nitrous oxide (N ₂ O)	(a)	44.02	0.211	0.166	1.27	9.29	34.9	1536
Sulfur dioxide (SO ₂)	(a)	64.06	0.154	0.123	1.25	9.87	23.6	1512
Acetylene (C ₂ H ₂)	(a)	26.04	0.409	0.333	1.23	10.65	58.8	1529
Ethane (C ₂ H ₆)	(a)	30.07	0.422	0.357	1.18	12.69	50.8	1524
Ethylene (C ₂ H ₄)	(a)	28.05	0.374	0.304	1.23	10.49	54.7	1532
Isobutane (C ₄ H ₁₀)	(a)	58.12	0.420	0.387	1.09	24.4	25.8	1499
Methane (CH ₄)	(a)	16.04	0.533	0.409	1.30	8.55	96.2	1539
Propane (C ₃ H ₈)	(a)	44.09	0.404	0.360	1.12	17.81	34.1	1504

this heat goes into internal energy and into work being done in raising the weight F and overcoming the pressure of the surroundings. For one pound the work is (§ 18),

$$(d) \quad W = \frac{1}{J} \int p \, dv = \frac{p}{J} \int dv = \frac{p(v_2 - v_1)}{J} = \frac{p_2 v_2}{J} - \frac{p_1 v_1}{J}.$$

[PRESSURE CONSTANT ONLY]

Then the energy equation, $dQ = du + p \, dv$ for 1 lb., applied to a constant pressure process gives

$$(e) \quad Q_p = u_2 - u_1 + \frac{p_2 v_2}{J} - \frac{p_1 v_1}{J} = h_2 - h_1 = \Delta h = c_p(T_2 - T_1) \text{ Btu/lb.},$$

where $u + pv/J$ has been replaced by h , the specific enthalpy. Thus we see that c_p may be defined in terms of point functions as

$$(13) \quad c_p = \left(\frac{\Delta h}{\Delta T} \right)_p, \quad \text{or} \quad c_p = \left(\frac{dh}{dT} \right)_p \text{ Btu/lb-}^\circ\text{R}$$

[AVERAGE VALUE] [INSTANTANEOUS VALUE]

There are advantages in some advanced thermodynamic problems in having the definitions of c_v and c_p in terms of point functions. Moreover, since work entering the systems in Figs 14 and 15 may bring about the same terminal state, equations (12) and (13) free the definitions of these specific heats of dependence on heat. Nevertheless the concept of heat per unit mass $^\circ\text{R}$ is very useful and holds for processes in which there is no fluid friction. See Table I for values of c_p for a few common substances.

Air is so often the working substance that the student should memorize its specific heats at normal atmospheric temperatures

$$\text{AIR} \quad c_p = 0.24, \quad \text{and} \quad c_v = 0.1715 \text{ Btu/lb } ^\circ\text{R},$$

or Btu per lb $^\circ\text{F}$. The ratio of the specific heats c_p and c_v appears so often in thermodynamic equations that it is given a symbol k

$$(14) \quad k = \frac{c_p}{c_v} > 1$$

The value of k is always greater than unity because for a particular temperature difference ΔT , $\Delta h = \Delta u + \Delta pv$ is always greater than Δu that is to say that the heat in the constant pressure process not only increases the internal energy but also does work. From equations (b), (e) and (14) we notice that $\Delta h/\Delta u = k$.

38 Reversibility A process or a cycle which is reversible in every respect is one which is as perfect as the mind can conceive. Inasmuch as thermodynamic analyses treat first of events which are reversible (ideal) in some respect, it behooves us to search for the significant meanings of the word *reversibility*. (There are many metaphors in the language which show that mankind has long realized the inevitable irreversibility of time and actual events)*

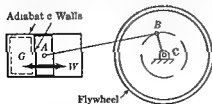


Fig 16

Consider a nonflow system of an expansible fluid G in a cylinder (Fig 16) surrounded by *adiabatic walls*. By definition, adiabatic means that the transferred heat is zero ($Q = 0$), and adiabatic walls are ones which are perfectly insulated. (There is no such thing.) Imagine that the piston is an

infinitesimal movement past head-end dead center position (piston at its

* No use crying over spilled milk. All the king's horses and all the king's men could not put Humpty Dumpty together again.

farthest leftward position) and that the fluid G is at a pressure above atmospheric. If the engine is free to move, G expands and does work at the expense of its internal energy ($Q = \Delta U + W = 0$). If the start is from rest, this work can be used to speed up the flywheel, storing kinetic energy in it. When the expansion of G is completed at the end of the stroke, the kinetic energy in the flywheel is a maximum. Now if this process is reversible, the kinetic energy in the flywheel is just enough to compress G back to its original state (p_1, v_1, T_1), where the engine again comes to rest. This event will not happen because no actual process is reversible.

(a) *External irreversibility* is some irreversibility external to the system, when the system is the working substance; for example, friction at the bearings A, B , and C , between the piston and cylinder walls, and between the atmosphere and the rotating members; all absorb some of the W output of the system. Instinctively you know that you cannot use the "frictional energy" to recompress the fluid to its original state (which is one of the things said by the second law of thermodynamics—see Chapter 7).

Another form of external irreversibility is due to the flow of heat through the containing walls, inasmuch as an adiabatic wall is a fiction of the mind. Since the temperature of the substance changes during the expansion, there is certain to be a temperature difference ΔT with the surroundings. Heat is that energy which is transferred because of a temperature difference, and if a temperature difference exists, heat is necessarily transferred. We say that the flow of heat from a body at high temperature to a body at low temperature is irreversible because all experience suggests the improbability of its flowing in the opposite direction. As Clausius* said in 1850, "It is impossible for a self-acting machine unaided by external agency to move heat from one body to another at a higher temperature." (This is a statement of the second law of thermodynamics, Chapter 7.) We see then that heat can be transferred reversibly only when $\Delta T = 0$ and that the transfer approaches reversibility as ΔT approaches zero.

(b) *Internal irreversibility* is irreversibility caused by fluid friction within the system, or it is an internal event which is irreversible, such as the mixing or diffusion of two or more gases. You can easily imagine that a large and fast movement of the piston would result in a sudden expansion of G (Fig. 16) and momentary differences in the properties, say, pressure and temperature, in various parts of the fluid. That part adjacent to the piston would expand first. Because of the lower pressure in the vicinity of the piston, gas would rush in from the higher pressure regions, there would be

* Rudolf Julius Emmanuel Clausius (1822–1888), born in northern Germany, a professor of physics, was a genius in mathematical investigations of natural phenomena. He elaborated and restated the work of Carnot, deducing the principle of the second law of thermodynamics. His mathematical work in optics, electricity, and electrolysis was significant. James Clerk Maxwell credits him with being the founder of the kinetic theory of gases, on the basis of which Clausius made many important calculations, one of which was the mean free path of a molecule. He is the author of an exhaustive treatise on the steam engine, wherein he emphasized the then new conception of entropy.

whirlpools and eddies, and consequently fluid friction. These effects can be neither ascertained in detail nor completely controlled. While this internal motion exists, the fluid is not in an equilibrium condition. Some of the energy which might have been delivered as work remains stored in the substance. In order for this phase of the operation to be reversible, the piston would have to move very slowly (at an infinitesimal speed in the limit), the substance must pass through a series of equilibrium states, which is to say that there is no fluid turbulence of any degree. At any instant, all parts of the system must be at the same temperature and pressure. While internal reversibility cannot be achieved, actual processes can approach it closely. It is internal reversibility that we most often assume in this study. For example, the *work of a fluid* is $W = \int p \, dV$ when the nonflow process is *internally reversible* (p is the instantaneous pressure in all parts of the system), whether there is or is not external reversibility.

The internally reversible process is a controlled process, as contrasted with an uncontrolled expansion, as in opening a valve and letting the fluid flow out without doing work. On the other hand, a guided expansion in a nozzle may closely approach a reversible process, because the kinetic energy of the jet can be reconverted in such a manner as to bring the substance back close to its original state in the absence of fluid friction and turbulence during the compression process.

To summarize: *If, after a process consisting of a continuous series of equilibrium states is completed, the substance can be made to retrace in the reverse order the various states of the original process, if all energy quantities to or from the surroundings can be returned to their original states (everything as it was before the process), then the process is externally and internally reversible.* The conditions for reversibility are (1) processes controlled through a series of equilibrium states (no fluid friction), (2) no mechanical friction, and (3) no temperature difference during transfer of heat. Perhaps we should add a fourth condition, that there is no diffusion. For example, if gases A and B are separate but at the same temperature and pressure, they will gradually mix (diffuse) when the separating partition is removed. This is an irreversible process which could be construed as being covered by condition (1). In engineering, we usually care for irreversibilities or convert ideal results into actual results by the use of ratios and efficiency numbers which are based on experience.

39 Entropy. The time has come to introduce another property, entropy (en'-tro-py). We shall discuss this property, which happens to be of great utility, in more detail later after you have become better acquainted with it. Entropy (s , S) is zero at absolute zero temperature, but unless tabulated absolute values of entropy are available, we generally compute only changes of entropy and we generally define it in terms of a change. For an internally reversible process, the change of entropy of a substance receiving (or deliver-

ing) heat is defined by

$$(15A) \quad dS = \frac{dQ}{T} \quad \text{or} \quad \Delta S = \int \frac{dQ}{T} \text{ Btu/}^\circ\text{R.}$$

$$(15B) \quad \Delta s = s_2 - s_1 = \int \frac{dQ}{T} \text{ Btu/lb-}^\circ\text{R,}$$

[REVERSIBLE PROCESS]

where dQ is the heat transferred [total amount in equation (15A), for one pound in equation (15B)] at the temperature T ; ΔS is the total change of entropy for w lb.; Δs is the change of specific entropy. If the integrations are made from 1 to 2, the signs of the answers obtained will always accord with the conventions set up in Chapter 2; that is, an increase is positive, a decrease negative.

In processes where the specific heat is a known finite value, we may use $dQ = wc \, dT$ and evaluate the change in entropy as

$$(f) \quad \Delta S = w \int \frac{c \, dT}{T} \text{ Btu/}^\circ\text{R; or}$$

$$(g) \quad \Delta S = wc \int \frac{dT}{T} = wc \ln \frac{T_2}{T_1} \text{ Btu/}^\circ\text{R,}$$

[CONSTANT SPECIFIC HEAT]

between the limits of states 1 and 2, where the specific heat is the proper average value for the kind of process involved; for example, for a reversible constant pressure process, $\Delta S = c_p \ln (T_2/T_1)$ Btu per lb- $^\circ\text{R}$.

Considering equation (15), $\Delta S = \int dQ/T$, it appears that S is not a point function since it is defined in terms of a path function Q ; yet we truly stated that entropy is a property. As will be shown later, dividing dQ by T results in a property.

40. Temperature-Entropy Coordinates. If equation (15) is rewritten as

$$(15C) \quad Q = \int T \, dS \text{ Btu, or}$$

$$Q = \int T \, ds \text{ Btu/lb.}$$

and compared with $W = \int p \, dV$, we observe that T , like p , is an intensity factor and (s, S) , like (v, V) , is an extensive factor. We see that the product of T and dS , during a reversible process, gives heat and that this product represents an area on the TS plane (Fig. 17). The integration of $T \, dS$ from 1 to 2 (Fig. 17) gives the area under the curve down to absolute zero temperature, area m -1-2- n , which represents the amount of heat transferred during the process 1-2. This is a useful attribute of entropy and makes the TS or T 's plane a favorite one for depicting processes and cycles. Since

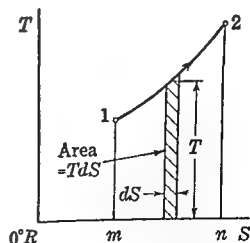


Fig. 17. TS Plane. A differential area is $T \, dS$, recognized as dQ .

$dQ = dU + p dV/J = T dS$, we can write

$$(h) \quad dS = \frac{dU}{T} + \frac{p dV}{J},$$

which can be integrated for certain circumstances (§ 62)

The process must be reversible for either of the areas, $\int p dV$ or $\int T dS$, to have significance. Irreversible processes will be shown dotted as a reminder that the areas under the curves do not represent work or heat.

41 $\int p dV$ for Reversible Steady Flow Processes It might be supposed that the integral $\int p dV$, which gives the work for a *nonflow* process has some significance in steady flow processes. Let us write the energy equation as established for reversible nonflow processes as follows

$$(i) \quad Q - \Delta U - \int p dV \text{ ft-lb,}$$

and the steady flow equation as

$$(j) \quad Q - \Delta U - \Delta W_f + \Delta K + \Delta P + W \text{ ft-lb}$$

We recall that the change of internal energy depends only on the location of the state points 1 and 2 that it is independent of the nature of the process connecting the points and that therefore the change is the same between any two states no matter whether the process is reversible or irreversible non-flow or steady flow. Limiting our discussion to reversible processes we recall that the heat added in an internally reversible process is expressed in general by $wfc dT$ where c has a definite and characteristic value depending upon the kind of process only. That is $wfc dT$ has the same value for say, a constant pressure process no matter whether this process is nonflow or steady flow. Thus the *left hand sides* of the two foregoing equations are identical for a particular reversible process. Since the right hand sides of equations (i) and (j) are equal to the same thing they must be equal to each other,

$$(k) \quad \int p dV = \Delta W_f + \Delta K + \Delta P + W \text{ ft-lb}$$

In other words, the integral $\int p dV$ evaluates the total change in the mechanical energy terms of the steady flow equation for *reversible* processes. Equation (i) is therefore an energy equation applying to either reversible nonflow or steady flow processes. In differential form, (i) becomes

$$(16) \quad dQ - T dS = dU + \frac{p dV}{J} \text{ Btu or } dQ = T ds = du + \frac{p dv}{J} \text{ Btu/lb,}$$

[ANY REVERSIBLE PROCESS]

where $p dV$ is defined by equation (k). Note that equation (k) reduces to the nonflow relation $\int p dV = W$ when the terms associated with motion are taken out.

42. Other Energy Relations. With equation (16) established for general use and with the definition of enthalpy and other energy quantities, a variety of energy equations can be obtained. Some are particularly useful and significant. By way of illustration, we differentiate $H = U + pV/J$, and get

$$(l) \quad dH = dU + \frac{p dV}{J} + \frac{V dp}{J} \text{ Btu.}$$

But $dU + p dV/J = dQ = T dS$ from equation (16); so for reversible processes, equation (l) becomes

$$(m) \quad dH = dQ + \frac{V dp}{J}, \quad \text{or} \quad dQ = dH - \frac{V dp}{J},$$

or

$$(17) \quad T dS = dH - \frac{V dp}{J} \quad \text{and} \quad T ds = dh - \frac{v dp}{J},$$

[ANY REVERSIBLE PROCESS] [FOR ONE POUND]

where $dQ = T dS$ (and $T ds$) has been used and where the integration would be from state 1 to 2. The integral $\int V dp$ represents an area (Fig. 18), the area "behind" the curve. Equation (17) says this area is

$$(n) \quad -\frac{1}{J} \int_1^2 V dp = +\frac{1}{J} \int_2^1 V dp = \int_1^2 (T dS - dH) = Q - \Delta H \text{ Btu.}$$

[REVERSIBLE PROCESS]

If Q should be zero, as it is often assumed in ideal situations, the area represents ΔH . Now in equation (9E), which is

$$(o) \quad dQ = du + \frac{p dv}{J} + \frac{v dp}{J} + dK + dW$$

let $du + p dv/J = dQ$, and get

$$(p) \quad -\int_1^2 v dp = W + \Delta K \text{ ft-lb./lb.}$$

and

$$-\int_1^2 V dp = W + \Delta K \text{ ft-lb.}$$

[REVERSIBLE STEADY FLOW, $\Delta P = 0$]

As mentioned before, the potential energy is seldom, if ever, significant in a heat engine, $\int dP = 0$. Therefore, we may say that the area $-\int V dp$ represents the work of a steady flow process when $\Delta K = 0$, or it represents ΔK when $W = 0$; and in both cases it represents $-\Delta H$ if $Q = 0$. Observe that $-\int V dp$ is a positive number when integrated from 1 to 2 (Fig. 18), the work being done by the substance. In order to make the integration, one must know the

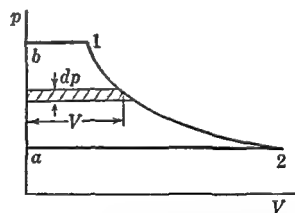


Fig. 18. Area for $\int V dp$. Area 1-2-a-b represents the work of a reversible steady flow process when $\Delta K = 0$ and $\Delta P = 0$.

relation between p and V (as $pV^n = C$). Practice arranging energy equations now

43. Molar Specific Heats. It is often convenient to use specific heats for a mol of substance. A mol is M pounds, where M is the molecular weight of the substance. For example, a mol of oxygen is 32 lb. The corresponding values are called *molar specific heats* C , $C = Mc$ Btu per mol-°R;

$$(q) \quad C_p = Mc_p, \quad C_v = Mc_v$$

44. Variable Specific Heats.* As previously mentioned, specific heats in general are not constant. They not only vary markedly with temperature, but also with pressure. The specific heat values to be given below are for zero pressure, but they are used for finite pressures in the absence of better data and serve very well for most engineering situations. The specific heats of vapors close to temperatures where they will condense vary markedly with pressure. For gases such as air at very high pressures and at temperatures below, say, 1000°F, the variation of specific heat with pressure is large (15). At ordinary atmospheric temperatures, the values for gases at zero pressure are beginning to be questionable if accuracy is important when the pressure exceeds, say, 200 psia. However, specific heats for zero pressure are successfully used in analyzing internal combustion engines where pressures are much higher than 200 psia (but the temperatures are higher also and pressure has less effect at high temperature—above 2000°F).

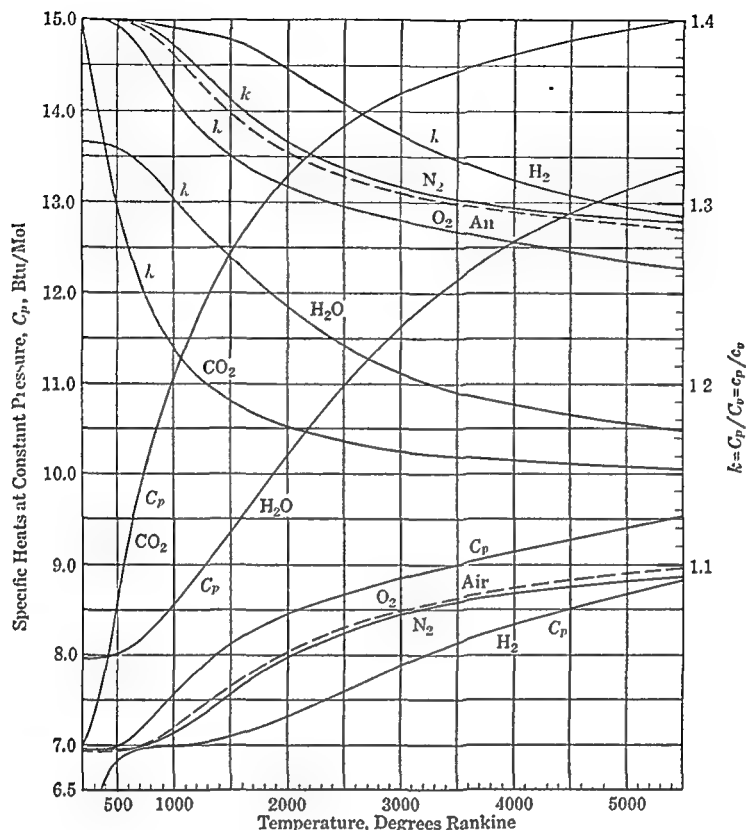
The curves of Fig. 10 give instantaneous values of the molar specific heat at constant pressure. These curves can also be used to estimate approximately the mean values of C_p between any two temperatures. A constant volume specific heat is obtained from $C_v = C_p/k$, or better from

$$C_v = C_p - 1.986 \text{ Btu/mol-°R}$$

As previously explained, the internal energy of a gas is largely due to the translational energy of the molecule, to the energy of rotation, and to the energy of vibration of the atoms within the molecule. At high temperatures, the vibrational energy increases markedly, and since only the translational energy is measured by the temperature, the specific heat must increase to account for the absorption of the energy which goes to increase the vibrations. Observe how abruptly the curve for CO_2 rises, this suggests that the vibrational energy of a polyatomic molecule undergoes a considerable change at ordinary temperatures. From this theory, it follows that the specific heats of monatomic gases, in which the molecule has practically only translation energy, do *not* vary with temperature.

It is interesting to recall that in accordance with the kinetic theory of

* In short courses, it may be desired to omit part of or all the remainder of this chapter, except perhaps § 51.



Plotted from data in Keenan and Kaye, Gas Tables (John Wiley)

Fig. 19. Molar Specific Heats at Constant Pressure. The molar specific heat at constant volume is $C_v = C_p - 1.986$ or $C_v = C_p/k$ Btu per mol, where values of k are on the right-hand ordinate. Divide C_p (or C_v) by the molecular weight M to obtain c_p Btu per lb.

gases, which you studied in physics, the specific heats of gases *in the absence of vibration of the atoms* have the values:

$$\text{Monatomic gases, } c_v = \frac{3R}{2J}, \quad c_p = \frac{5R}{2J}, \quad k = 1.667;$$

$$\text{Diatomic gases, } c_v = \frac{5R}{2J}, \quad c_p = \frac{7R}{2J}, \quad k = 1.4;$$

$$\text{Polyatomic gases, } c_v = \frac{3R}{J}, \quad c_p = \frac{4R}{J}, \quad k = 1.333;$$

where R is called the gas constant (Chapter 4). Compare these values with those in Table I, which come from experiment.

Table II. VARIABLE SPECIFIC HEATS

All equations derived from spectrographic data, $c_p = c_p - R/J$, $C_p = C_p - 1.986$

(a) This value is derived from Spencer and Justice (18), (b) from Spencer and Flannagan (17), (c) from Chipman and Fontana (20), (d) from Sweigert and Beardsley (19), (e) from Spencer (21)

Substance (Temp range)	Mol Wt	Btu per lb °R	Btu per mol-°R
(a) Air (500-2700°R)	29	$c_p = 0.219 + 0.342T/10^4$ $- 0.293T^2/10^6$ $R/J = 0.0681$	$C_p = 6.36 + 9.92T/10^4$ $- 8.52T^2/10^6$
(b) SO ₂ sul diox (510-3400°R)	64.06	$c_p = 0.1875 + 0.0944T/10^4$ $- 1.336 \times 10^{-6}/T^2$ $R/J = 0.0328$	$C_p = 11.89 + 0.05T/10^4$ $- 85.6 \times 10^{-6}/T^2$
(b) NH ₃ , ammonia (540-1800°R)	17.03	$c_p = 0.363 + 2.57T/10^4$ $- 1.319T^2/10^6$ $R/J = 0.116$	$C_p = 6.19 + 43.8T/10^4$ $- 22.47T^2/10^6$
(c) H ₂ , hydrogen (540-1000°R)	2.016	$c_p = 2.857 + 2.867T/10^4$ $+ 9.92/T^{1/2}$ $R/J = 0.986$	$C_p = 5.76 + 5.78T/10^4$ $+ 20/T^{1/2}$
(d) O ₂ oxygen (540-5000°R)	32	$c_p = 0.36 - 5.375/T^{1/2}$ $+ 47.8/T$ $R/J = 0.062$	$C_p = 11.515 - 172/T^{1/2}$ $+ 1530/T$
(d) N ₂ , nitrogen (510-9000°R)	28.016	$c_p = 0.338 - 123.8/T$ $+ 4.14 \times 10^{-6}/T^2$ $R/J = 0.071$	$C_p = 9.47 - 3470/T$ $+ 116 \times 10^{-6}/T^2$
(d) CO carb mon (510-9000°R)	28	$c_p = 0.338 - 117.5/T$ $+ 3.82 \times 10^{-6}/T^2$ $R/J = 0.071$	$C_p = 9.46 - 3290/T$ $+ 107 \times 10^{-6}/T^2$
(d) H ₂ O, steam (540-5400°R)	18.016	$c_p = 1.102 - 33.1/T^{1/2}$ $+ 416/T$ $R/J = 0.110$	$C_p = 19.86 - 597/T^{1/2}$ $+ 7500/T$
(d) CO ₂ carb diox (510-6300°R)	44	$c_p = 0.368 - 148.4/T$ $+ 3.2 \times 10^{-6}/T^2$ $R/J = 0.015$	$C_p = 16.2 - 6530/T$ $+ 141 \times 10^{-6}/T^2$
(e) CH ₄ , methane (540-2700°R)	16.03	$c_p = 0.211 + 0.25T/10^4$ $- 8.28T^2/10^6$ $R/J = 0.1237$	$C_p = 3.38 + 100.2T/10^4$ $- 132.7T^2/10^6$
(b) C ₂ H ₄ , ethylene (540-2700°R)	28.03	$c_p = 0.0965 + 5.78T/10^4$ $- 9.97T^2/10^6$ $R/J = 0.0733$	$C_p = 2.706 + 162T/10^4$ $- 279.6T^2/10^6$
(e) C ₂ H ₆ , ethane (540-2700°R)	30.05	$c_p = 0.0731 + 7.08T/10^4$ $- 11.3T^2/10^6$ $R/J = 0.0653$	$C_p = 2.195 + 212.7T/10^4$ $- 340T^2/10^6$
(e) C ₄ H ₁₀ , n butane (540-2700°R)	58.08	$c_p = 0.075 + 6.94T/10^4$ $- 11.77T^2/10^6$ $R/J = 0.0331$	$C_p = 4.36 + 403T/10^4$ $- 683T^2/10^6$
(e) C ₃ H ₈ , propane (540-2700°R)	44.06	$c_p = 0.0512 + 7.27T/10^4$ $- 12.32T^2/10^6$ $R/J = 0.0439$	$C_p = 2.258 + 320T/10^4$ $- 543T^2/10^6$
(b) C ₂ H ₂ , acetylene (500-2300°R)	26.02	$c_p = 0.459 + 0.937T/10^4$ $- 2.89 \times 10^{-6}/T^2$ $R/J = 0.076$	$C_p = 11.94 + 24.37T/10^4$ $- 75.2 \times 10^{-6}/T^2$

The empirical equations of Table II make it possible to integrate; for example,

$$(r) \quad Q = \int c \, dT \quad \text{and} \quad \Delta s = \int \frac{c \, dT}{T}, \quad [\text{PER POUND}]$$

$$(s) \quad Q = \int C \, dT \quad \text{and} \quad \Delta S = \int \frac{C \, dT}{T}, \quad [\text{PER MOL}]$$

thus allowing for the variation of specific heat with temperature. To find the specific heat at constant volume for each substance in Table II, subtract R/J from the equation for c_p ; that is, the coefficients of the T terms remain the same; $c_v = c_p - R/J$. To find the molar specific heat at constant volume, subtract 1.986 from C_p ; that is, $C_v = C_p - 1.986$.

45. Example. What is the change of entropy of air which is heated at constant pressure from $t_1 = 40^\circ\text{F}$ to $t_2 = 1540^\circ\text{F}$?

SOLUTION. In Table II, we see that the constant pressure specific heat of air is given in terms of absolute temperature ($T_1 = 500^\circ\text{R}$, $T_2 = 2000^\circ\text{R}$) by an equation of the form

$$(t) \quad c_p = \alpha + \beta T + \gamma T^2 \text{ Btu/lb}^\circ\text{R}$$

where $\alpha = 0.219$, $\beta = 0.342 \times 10^{-4}$, and $\gamma = -0.293 \times 10^{-8}$. From $\Delta s = \int dQ/T$, we have

$$\begin{aligned} \Delta s &= \int \frac{c \, dT}{T} = \int_{500}^{2000} \left(\frac{\alpha \, dT}{T} + \beta \, dT + \gamma T \, dT \right) \\ &= \left[\alpha \ln T + \beta T + \gamma \frac{T^2}{2} \right]_{500}^{2000} \\ &= \alpha \ln \frac{2000}{500} + \beta(2000 - 500) + \frac{\gamma}{2} (2000^2 - 500^2), \end{aligned}$$

from which $\Delta s = 0.35 \text{ Btu per lb}^\circ\text{R}$.

46. Mean Specific Heat. The use of equations for specific heats becomes rather tedious at times. To ease the burden of computations, tables and charts of properties, such as entropy and enthalpy, of many common substances are available: air, nitrogen, oxygen (14), ammonia, freon, steam, etc. (See the material at the back of *Problems on Thermodynamics* by Faires, Brewer, and Simmang.) In the absence of tabulated properties, equations of specific heats are useful in many practical applications where accuracy is desired. If a number of repetitive calculations are to be made for approximately a constant temperature range, the mean specific heat \bar{c} and \bar{C} may be used:

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For quick estimations, the mean values as obtained from curves such as Fig. 19 are satisfactory.

47. Steady Flow of More than One Mass Stream We recall the stipulations for steady flow that there be no change in either the total mass or the total energy within the system. In this event, the law of conservation of energy says that energy entering is equal to energy leaving. With this knowledge, it is just as easy to write the energy equation for any number of substances as for one, but it takes more time and the equation looks more formidable. However, by using E to represent the total energy associated

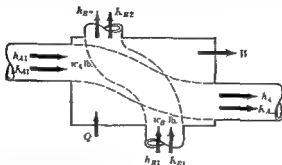


Fig 20 Energy Diagram Steady Flow—Two Fluids

with 1 lb of a particular stream $E = u + p/\rho + h + P = h + K + P$ and by using only two streams for illustration the resulting equation looks harmless enough. Thus for fluid A, Fig 20 we have

Entering $E_{A1} = h_{A1} + K_{A1} + P_{A1}$, Leaving $E_{A2} = h_{A2} + K_{A2} + P_{A2}$,

and for fluid B,

Entering $E_{B1} = h_{B1} + K_{B1} + P_{B1}$, Leaving $E_{B2} = h_{B2} + K_{B2} + P_{B2}$

The potential energy P may be omitted as in Fig 20 or included as appropriate. The energy equation becomes

$$(v) \quad w_A E_{A1} + w_B E_{B1} + Q = w_A E_{A2} + w_B E_{B2} + W \text{ Btu}$$

where Q and W are net values and the convention of signs is as previously explained.

48 Variable Flow Process There are cases in engineering where the quantity of mass and energy within a system do not remain constant. Typical of these would be a fluid flowing into a tank (and not out) or out of a tank (and not in). When the flow is variable one must consider what happens during a particular period of time, or what happens while a certain mass leaves or enters the system, or set up the energy equation in terms of differentials. Let E consist in general of all stored energy quantities in 1 lb of the system at any instant. In terms of the ones we have been considering, $E = u + K + P$ Btu per lb. (Note that this is not the same E as in the previous article.) In the beginning then (Fig 21), there are $w_1 E_1$ Btu of

energy within the system; at the end, there are $w_2 E_2$ Btu. During an infinitesimal time interval, the change of energy stored in the system is $d(wE)$. Now apply the law of the conservation of energy, as stated in equation (5), to Fig. 21, and find

Energy entering = change in stored energy + energy leaving.

$$(w) \quad dw_i(h_i + K_i + P_i) + dQ = d(wE) + dw_f(h_f + K_f + P_f) + dW,$$

where i suggests the initial state at entry and f the state of the working substance at the final (exit) boundary. The mass balance gives

Mass entering - mass leaving = change of mass within.

$$(x) \quad dw_i - dw_f = w_2 - w_1,$$

where $w_2 - w_1$ may be positive (increase) or negative (decrease). The foregoing two equations are good for two unknowns, but in many actual problems, assumptions must be made and special data collected in order to be

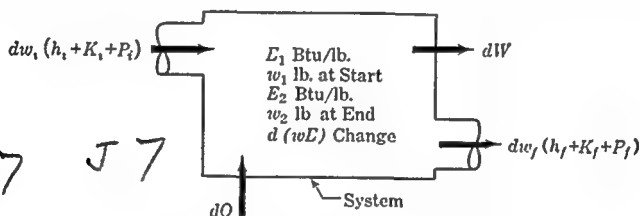


Fig. 21. Energy Diagram—Variable Flow.

able to solve equation (w) for an unknown. If, for example, the system is a tank of gas from which there is an outflow only, the term $dw_i(h_i + K_i + P_i)$, which is the energy accompanying the inflow of dw_i lb. of substance, would be dropped, and $dW = 0$ because W is the kind of work which turns shafts; $P_f = 0$; $d(wE) = d(wu)$; and $dQ = 0$ if the time element is short. Notwithstanding the foregoing simplifications, it is still true that for each dw_f flowing out, there is a different value of h_f and K_f , and hence there would be difficulty in summing the terms $dw_f(h_f + K_f)$ over a finite time. (It could be further assumed that the fluid which remains in the tank undergoes a constant entropy process—a process to be described later.) Observe that if the properties of the system (pressure and temperature, say) are measured before and after the event, and if the amount of outflow (or inflow) is known, the change of internal energy $\Delta wu = w_2 u_2 - w_1 u_1$ can be found, as explained in detail later.

The problem is somewhat simpler if the flow is inward only from a source of substance where h_i and K_i are constant and $P_i = 0$, as from a pipe line in which substance is flowing steadily (more or less). The student should by now be able to set up an energy diagram and obtain the energy equation for this case with little difficulty. The stored energy E becomes internal

energy u because kinetic energy K of a gas in a tank is virtually zero and potential energy P is irrelevant. See references 3, 10, 13, and 22.

49 Frictional Energy. One frequently hears the term "frictional energy" or its equivalent, but it is not a form of energy different from those forms already defined here. To allow for energy losses due to friction, engineers generally use efficiency numbers which have been accumulated through experience. This practice is satisfactory and will be explained as the occasion arises in later pages. However, we may profit now from a brief consideration of the nature of this energy.

If the frictional loss is caused by the rubbing together of two solid parts, with or without lubrication, as in a bearing, what happens is that work is expended to excite the molecules in the vicinity of the surfaces being rubbed, raising their temperatures. This is to say that work is converted into internal energy of the parts (of a bearing say). From the hotter surfaces heat is conducted through the parts, and if the environmental temperature is lower than that of the parts, this energy is radiated and convected away from the parts as Q . That portion of this energy carried off by the oil will also be given off as heat when the oil is cooled. Thus the frictional energy in this situation eventually appears as heat—heat dissipated into the atmosphere, which is such an immense reservoir that it does not notice the difference except locally.

If the frictional loss is that caused by fluid friction, the substance has more internal energy at the end of the process than it would have had in the absence of fluid friction. If the substance also gets hotter than its environment, at least a part of the "frictional energy" eventually leaves as heat to the surroundings. In all events, frictional energy, in effect is eventually spilled into the atmosphere at the expense of work (no form of energy is finer than work).

Suppose in Fig 5, we let E_f represent the frictional losses in system (2) at bearings L , D , C , etc., we see that $W_I = W_s + E_f$. In Fig 22, W_s is the shaft work and E_f is the frictional loss in the bearings, etc. of a steady flow machine. If $Q = 0$, except as E_f is heat, the energy equation is

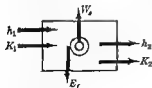


Fig 22 Energy Diagram
—with Friction

$$h_1 + K_1 = h_2 + K_2 + W_s + E_f,$$

where the fluid work W_s , which in the absence of friction could have all been shaft work, is $W = W_s + E_f$.

50 Energy Equation for Flow of Incompressible Fluids The reader has probably derived the Bernoulli equation in his study of fluid mechanics, using a force analysis. Heat and internal energy were not involved for a good historical reason. Bernoulli, a contemporary of Newton, derived the

equation around 1700, whereas the law of conservation of energy was not accepted until about 1850—150 years later. However, the energy law throws additional light on the Bernoulli equation. Consider the steady flow energy equation in the following form,

$$Q = \Delta u + \Delta(pv) + \Delta K + \Delta P + W \text{ ft.-lb./lb.}$$

as applied to Fig. 23, an open system of fluid flowing in a pipe between the boundaries 1 and 2. In this system, $W = 0$ and Q can be negligibly small if there is little temperature difference between the inside and outside, or if the line is well insulated. Use $v = 1/\rho$, where ρ lb. per cu. ft. is the density. An incompressible fluid in frictionless flow with $Q = 0$ will undergo no change of internal energy, $\Delta u = 0$. Between the boundaries 1 and 2, the energy relation then becomes

$$\Delta \left(\frac{p}{\rho} \right) + \Delta K + \Delta P = 0, \quad (y)$$

$$\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} + \frac{v_2^2}{2g_0} - \frac{v_1^2}{2g_0} + z_2 - z_1 = 0 \text{ ft.-lb./lb.,}$$

in which g/g_0 , as it modifies z , is taken as unity [equation (a), § 14]. Bernoulli's equation for the flow of incompressible fluids without friction is sometimes written in the form of equation (y). Observe that if the force pound and the mass pound are canceled (ordinarily not advisable), each term can be interpreted as a *head* in feet, the name usually used in hydraulics. The $v^2/(2g_0)$ term is called the *velocity head*, p/ρ is called the *pressure head*. If the system of Fig. 23 should lead to a hydraulic turbine at 2, the events would be that some of the potential energy Δz would be converted into kinetic energy and then some of the kinetic energy would be converted into work in the turbine. The reason that all the change of potential energy is not converted into kinetic energy is that friction prevents it. If Q is negligible during the flow of an incompressible fluid, the energy loss due to friction appears in the internal energy of the fluid. Thus, using all the energy terms shown in Fig. 23, we have

$$(z) \quad \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} + \frac{v_2^2}{2g_0} - \frac{v_1^2}{2g_0} + z_2 - z_1 = -(u_2 - u_1) \text{ ft.-lb./lb.}$$

This is one form of Bernoulli's equation with friction if we designate $u_2 - u_1$ as the *friction head*, which is the sum of the changes of "all heads," a net loss. Recall also that $Q = 0$ and $W = 0$. If Q is not equal to zero, the equation

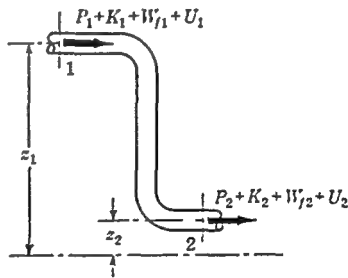


Fig. 23. Energy Diagram—Fluid Flow.

(z) can be made to balance by making the *friction head* on the right hand side arbitrarily equal to the left hand side (rather than equal to $u_2 - u_1$)

51 Closure It would be superb if everything about thermodynamics could be said at once and be done with. Not only is this impossible but it is also often difficult to write about one idea without involving another. Thus we have referred to several notions which are to be explained in detail later. Perhaps these notions are not essential to the purposes of these first three chapters. Nevertheless if the reader will review these chapters a time or two within the next few weeks he will no doubt find meanings which he missed on first study. Specific heats, entropy and the energy relations are basic to further study.

We have arrived at two important mathematical relationships which are applicable to all reversible thermodynamic processes, namely

$$dQ = du + \frac{p dv}{J} \quad \text{and} \quad dQ = dh - \frac{v dp}{J}$$

[EQUATION (16)]

[EQUATION (17)]

In each of these equations $dQ = T ds$ for a reversible process

4

THE IDEAL GAS

52. Introduction. We shall now study the characteristics of the *perfect* or *ideal gas*. Since the mathematics of an ideal gas is relatively simple, it is fitting that this be the first working substance to be examined in detail.

Early experiments on air resulted in laws which we now recognize as being characteristic of an ideal gas (§§ 53 and 54). That is, air is almost an ideal gas in its most common states, as are all the monatomic and diatomic gases; which is one practical reason for studying this hypothetical substance. While no actual gas conforms exactly to the tenets of an ideal gas, many of them conform so closely that ideal-gas calculations give good engineering answers. Hence, this ideal substance is a practical one too.

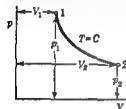
Some substances in a gaseous state deviate substantially from ideal-gas laws. In general, when this is true the temperature of the substance is close to the boiling point of the liquid; that is, the substance is not too far from the point of condensation—at least as compared with nitrogen, for instance, whose boiling point at 14.7 psia is -321°F . A substance which is at or near its condensation point is called a *vapor*. Thus, actual “ideal” gases are either highly *superheated* or are vapors at a very low pressure (low density).

Since ideal-gas laws are usually inaccurate for vapors, which are imperfect gases, we must use either more elaborate mathematical procedures or tabulated values and charts of properties. There is no distinct line of demarcation dividing imperfect gases and ideal gases, but the criterion is simple: If the ideal gas laws yield sufficiently accurate results, the substance is considered as an ideal gas; otherwise, it is a vapor or an imperfect gas. A substance departs from the action characteristic of ideal gases when its molecules are so close together that the forces of attraction between molecules are large enough to affect significantly their behavior (§ 16). In one

condition, a substance may act as an ideal gas, in another, it may not. For instance, steam, as it exists in the atmosphere, is at such a low density that it accords well with ideal-gas laws, steam in a steam power plant does not.

An ideal gas is ideal only in the sense that it conforms to the simple perfect-gas laws. It might lack many characteristics which we would care to assign to it, for instance, a greater heat conductivity.

53 Boyle's Law. Robert Boyle (1627-1691), during the course of experiments with air, observed the following relation between the pressure and volume. *If the temperature of a given quantity of gas is held constant, the volume of gas varies inversely with the absolute pressure during a change of state (Boyle's law).** In mathematical form, if a gas is in a condition represented by state 1 (Fig 24), and undergoes a change of state at constant temperature (*isothermal process*) to state 2, then



$$(18) \quad \frac{P_1}{P_2} = \frac{V_2}{V_1}, \quad \text{or}$$

$$P_1 V_1 = P_2 V_2, \quad \text{or} \quad pV = C$$

(BOYLE'S LAW $T = C$)

The equation of the curve 1-2 (Fig 24) is thus $pV = C$, where C has some particular positive value. This curve is seen to be an equilateral hyperbola ($xy = C$), and since C may have any one of an infinite number of values, it is one of a family of curves.

Observe that we shall use C in a generic sense to mean a constant, but it will be a different constant in different equations, that is, for example, in stating

Boyle's law mathematically, we write $t = C$ and $pV = C$, in which the two C 's are different constants, t is not equal to pV .

The experiments of Boyle were not so precise as modern science can make them. More accurate observations reveal that no actual gas follows this law exactly. Hence, we consider Boyle's law as being a defining characteristic of an ideal gas.

54 Charles' Law. About a hundred years after the discovery of Boyle's law, two Frenchmen, Jacques A. Charles (1746-1823) and Joseph L. Gay-Lussac (1778-1850), each without knowledge of the other's work, discovered the law which we usually call *Charles' law*. This law is in two parts:

1. *If the pressure on a particular quantity of gas is held constant then with any change of state, the volume will vary directly as the absolute temperature.*

* Edme Mariotte, a Frenchman, independently discovered this same principle at approximately the same time that Boyle did. Although Mariotte therefore, is due fully as much credit for the discovery, the law is more commonly called Boyle's in this country.

2. If the volume of a particular quantity of gas is held constant, then, with any change of state, the pressure will vary directly as the absolute temperature.

Equations (19) and (20), which are mathematical expressions of Charles' law, may be considered as other characteristics of the *perfect gas*, usable however for gases which are nearly perfect. As before, consider a change of state for some gas, point 1 to point 2 [Fig. 25(a)], this time using the absolute temperature T and volume V as coordinates, and letting the pressure at 1 equal to the pressure at 2, $p_1 = p_2$. Then we may write

$$(19) \quad \frac{V_1}{V_2} = \frac{T_1}{T_2}, \quad \text{or} \quad \frac{T_1}{V_1} = \frac{T_2}{V_2}, \quad \text{or} \quad \frac{T}{V} = C.$$

[CHARLES' LAW, $p = C$]

The volume V in this law is any volume, including specific volume v , of a constant mass of gas. Next, using the coordinates T and p [Fig. 25 (b)], letting $V_1 = V_2$, we have

$$(20) \quad \frac{p_1}{p_2} = \frac{T_1}{T_2}, \quad \text{or} \quad \frac{T_1}{p_1} = \frac{T_2}{p_2} \quad \text{or} \quad \frac{T}{p} = C.$$

[CHARLES' LAW, $V = C$]

The equations (19) and (20) are seen to be straight lines ($y = Cx$) which pass through the origin with slopes C . However, since all actual gases

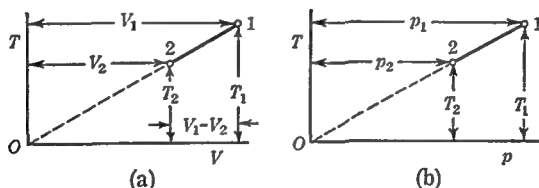


Fig. 25. Charles' Law. In (a), the line 1-2 represents a constant pressure process for an *ideal gas* as it appears on the TV plane. In (b), the line 1-2 represents a constant volume process as it appears on the Tp plane.

undergo changes of phase at low temperatures, this law does not hold down to absolute zero. Charles' law is applicable to gases through only a limited range of temperature.

Each of the parts of Charles' law is used as a basis for measuring temperature. If the most nearly ideal gases are used, accurate temperature indications may be obtained from a constant-volume gas thermometer, in which $T_1/T_2 = p_1/p_2$, or from a constant-pressure gas thermometer, in which $T_1/T_2 = V_1/V_2$. Also, experiments on nearly perfect gases furnish an important clue to absolute zero temperature. For example, the slope of the line in Fig. 25(a) is

$$(a) \quad \frac{T_1 - T_2}{V_1 - V_2} = \frac{\Delta T}{\Delta V} = \frac{\Delta t}{\Delta V} = C.$$

Experimental data for air at $p = 14.696$ psia are: at $t_2 = 32^\circ\text{F}$, the specific

volume is $v_2 = 12.39$ cu ft ; at $t_1 = 100^\circ\text{F}$, $v_1 = 14.1$ cu ft. Then

$$C = \frac{t_1 - t_2}{v_1 - v_2} = \frac{100 - 32}{14.1 - 12.39} \approx 39.76,$$

$$T = Cv = 39.76v$$

This is the equation of the line on Fig. 25(a) for air and the data given. Thus, the absolute temperature at 32°F (where $v = 12.39$) is

$$T = (39.76)(12.39) = 492.6^\circ \text{ abs},$$

which is a rough estimation of the absolute temperature on the Fahrenheit scale corresponding to the freezing point of water, 32°F . If the data for some other gas were used as above, we should find a slightly different absolute zero point, because every gas deviates to some extent from Charles' law. Thus, we may have as many absolute scales as gases. However, using more refined technique than the method given above and using a nearly perfect gas (helium at low pressure), we find that 32°F is 491.69°R . See §§ 8 and 96.

55 Equation of State of an Ideal Gas. An equation of state of a substance is one which relates three properties of the substance for example, pressure as a function of volume and temperature $p = f(v, T)$. Satisfactory equations of state for vapors are complicated (Chapter 14), but the one for an ideal gas is simple. What we wish is a relation between p , v , and T that relates any two states whatsoever. Such a relation may be obtained by combining Boyle's law with either part of Charles' law, or by combining the two parts of Charles' law.

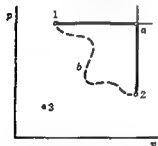


Fig. 26

Let an ideal gas be in state 1 (Fig. 26), and let the state be changed at random 1-b-2, until it is represented by point 2. Let the mass of gas be one pound, for convenience so, the corresponding volume is the specific volume v . Now through point 1, draw a horizontal line of indefinite extent representing a constant pressure process on the p - v plane, through point 2, draw a vertical line of indefinite extent representing a constant volume process. The lines representing these processes intersect at point a . Applying Charles' law and noting that $t_a = v_2$ and $p_a = p_1$, we find from the constant pressure process 1-a,

$$(b) \quad \frac{t_1}{t_a} = \frac{T_1}{T_a}, \quad \text{or} \quad T_a = T_1 \left(\frac{v_2}{v_1} \right) = T_1 \left(\frac{v_2}{v_1} \right),$$

and from the constant volume process 2-a,

$$(c) \quad \frac{p_2}{p_a} = \frac{T_a}{T_2}, \quad \text{or} \quad T_a = T_2 \left(\frac{p_a}{p_2} \right) = T_2 \left(\frac{p_1}{p_2} \right)$$

From equations (b) and (c), we find

$$T_a = T_2 \left(\frac{p_1}{p_2} \right) = T_1 \left(\frac{v_2}{v_1} \right),$$

or, by transposing terms,

$$(d) \quad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = R, \text{ a constant.}$$

Since points 1 and 2 were selected at random, it follows that a like relation could be derived for any other pair of points, say, 1 and 3; thus

$$\frac{p_1 v_1}{T_1} = \frac{p_3 v_3}{T_3}.$$

Consequently, this expression pv/T must be constant for a particular gas. Since the constant R is associated with a unit mass of substance, it is designated as the *specific gas constant*, ordinarily shortened, however, to the *gas constant*. It is expressed in general terms as $pv/T = R$, or

$$(21) \quad \underset{[1 \text{ LB.}]}{pv} = RT \quad \text{and} \quad pV = \underset{[w \text{ LB.}]}{w}RT,$$

where $V = wv$. Equation (21) is the *equation of state* or the *characteristic equation of a perfect gas*. From it may be obtained Boyle's and Charles' laws by letting T , p , and V be successively constant. For the gas constants R given in Table I, always use p in pounds per square foot (psf), V in cubic feet, and T in degrees Rankine in equation (21).

56. The Gas Constant. The value of R for any gas may be determined from accurate experimental observations of simultaneous values of p , v , and T . Thus, at 32°F and standard atmospheric pressure, the specific volume of *air* is 12.39 cu. ft. Hence,

$$(e) \quad R = \frac{pv}{T} = \frac{(14.7)(144)(12.39)}{(460 + 32)} = 53.3 \text{ ft-lb./lb.}^\circ\text{R},$$

the approximate gas constant for air only. Air is so frequently used that the student should memorize this number. Gas constants for other gases are given in Table I.

Consider the units of R . From equation (e) above, we may write

$$R \rightarrow \frac{(\text{pressure unit})(\text{volume unit/unit mass})}{\text{absolute temperature}}.$$

Thus, R could be computed to be in (tons/sq. ft.)(cu. meters)/lb. $^\circ\text{K}$; that is, any pressure unit, volume unit, and temperature scale could be used. From another point of view, the unit of pv is an energy unit,

$$(\text{lb./ft.}^2)(\text{ft.}^3) = \text{ft-lb.}$$

Therefore, R could be expressed in terms of any energy unit,

$$R \rightarrow \frac{\text{energy unit}}{(\text{mass})(\text{absolute temperature})}$$

Common English units for the specific gas constant are

$$R \rightarrow \text{ft lb /lb}^\circ\text{R}, \quad \text{and} \quad \text{Btu/lb}^\circ\text{R}$$

The gas constant for a mol (M lb) of substance is often needed. Multiply both sides of $pv = RT$ by M and get

$$(f) \quad MR = \frac{p(Mv)}{T}.$$

According to Avogadro, an Italian (1776–1856) *All ideal gases at a particular pressure and temperature have the same number of molecules in a given volume (Avogadro's law)*. Since the molecular weight is an index to the weight of the molecule, it follows that the density ρ , say, in pounds per cubic foot, is proportional to the weight of the molecule, or

$$(g) \quad \frac{\rho_x}{\rho_y} = \frac{M_x}{M_y},$$

where ρ_x and ρ_y are the densities, and M_x and M_y are the molecular weights of the gases X and Y , respectively. As the densities are inversely proportional to the specific volumes, $\rho_x/\rho_y = v_y/v_x = M_x/M_y$, or

$$(h) \quad M_x v_x = M_y v_y$$

We have shown that the molecular weight times the specific volume for gas X is equal to the product of these numbers for gas Y , the gases being at the same pressure and temperature. Since gases X and Y may be any two gases, it follows from Avogadro's law that the product Mv must be the same for all ideal gases at any particular temperature and pressure. Therefore, in equation (f), which applies to an ideal gas, the volume Mv varies inversely as the pressure (Boyle) and directly as the absolute temperature (Charles). Since Mv is the same for all gases (Avogadro), it follows that MR is the same constant for all gases. This gas constant is called the *universal gas constant* \bar{R} and its accepted value is (14)

$$\bar{R} = 1545.32 \text{ ft lb /mol }^\circ\text{R},$$

use 1545 for slide-rule work. Now compare this value of \bar{R} with MR values given in Table I, p. 39, and observe that a typical value for the more nearly perfect gases is not too different from 1545. This is a convenient constant to have in mind, because one can often decide from memory the molecular weight of a substance and then find the gas constant in ft-lb. per lb- $^\circ\text{R}$ from $R = 1545/M$.

For our purposes, the unit of the molecular weight number is M lb. per mol. (In the European system of units, used in scientific work here, it might be M gm. per mol, which we call a gram-mol, or M kg. per mol.) From equation (f), the units of $MR = \bar{R}$ are

$$(i) \quad \bar{R} \rightarrow \frac{(\text{pressure unit})(\text{volume unit})}{\text{mol} \cdot ^\circ\text{R}}, \quad \text{or} \quad \rightarrow \frac{\text{energy unit}}{\text{mol} \cdot ^\circ\text{R}}.$$

Values of the universal gas constant \bar{R} in various units are given in the Appendix (23). When the universal gas constant is in terms of any cgs units, as 82.06 atm-cc per (g-mol)- $^\circ\text{K}$, the mol is a gram-mol; that is, for example, 32 grams of O_2 ; not 32 lb.

As you would expect by now, the actual value of Mv , called the *mol volume*, is not constant because gases are not ideal. At 14.696 psia and 32 $^\circ\text{F}$, typical values of Mv are: hydrogen (H), 359 cu. ft.; helium (He), 358.8 cu. ft.; carbon dioxide (CO_2), 356.4 cu. ft.; ammonia (NH_3), 353.5 cu. ft. The standard mol volume (14.696 psia and 32 $^\circ\text{F}$) for the ideal gas may be taken as 359 cu. ft.

57. Joule's Law. Joule arranged two copper containers, as shown diagrammatically in Fig. 27, in a bath of water. The water was in an insulated vessel, so that the whole apparatus could be considered as an isolated system, without thermal contact with outside bodies. One of the containers held air at a pressure of 22 atm [(22)(14.7) psia], and the other container was evacuated as nearly as possible. After all parts had reached thermal equilibrium, the valve was opened. The air rushed into the evacuated container, and the whole mass of air finally came to rest at a pressure of 11 atm. Joule observed that there was no change in the temperature of the water surrounding the containers, a simple observation that leads to an important deduction.

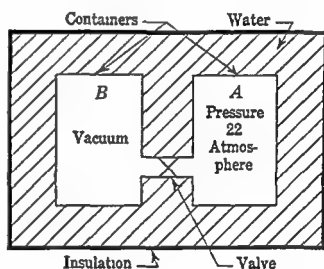


Fig. 27. Joule's Experiment.

There are several facts to be noted. First, although there is a momentary flow of air, giving rise to energy terms characteristic of flow, the net effect due to flow is zero, since the gas is at rest in the initial and final states. Consequently, the experiment is considered in relation to the simple energy equation. Second, inasmuch as there is no connection by which energy may leave the system as work, $W = 0$. Third, because the temperature of the water did not change, there must have been no flow of heat from the air in the containers to the water, or from the water to the air, $Q = 0$. Fourth, the temperature of the air in the containers did not change, since otherwise,

due to the temperature difference there would have been a transfer of heat *

Now from the simple energy equation $\Delta U - Q - W = 0$ The pressure changed the volume changed but the internal energy did not change. Consequently we conclude that the internal energy of this gas is not a function of the pressure or volume. However since the temperature did not change Joule deduced that *the change of internal energy is a function of the temperature change*. The italicized statement is known as *Joule's law*. While the evidence from Joule's experiment leading to this deduction is not conclusive the law is true for an ideal gas and can be proved mathematically for a gas conforming to the characteristic equation $p = RT$. Moreover the law may be explained from the viewpoint of the kinetic theory of gases. As we have learned (§ 16) the molecules in a gas are so far from one another that the forces of attraction between molecules are very small. Even though the distance between the molecules is changed by a considerable amount the change of *potential* internal energy will be negligibly small because the attractive forces are so small. In ordinary expansions or compressions of gases then the change of *potential* internal energy is practically zero. Consequently any internal energy change must be a change of the kinetic internal energy, the internal energy due to the motion of the molecule. Since temperature is a measure of the molecular kinetic energy it follows that the change of internal energy is a function of the temperature change.

58 Internal Energy of an Ideal Gas With respect to the nonflow constant volume system of § 36 we observed that the total effect of the inflow of energy was to increase the internal energy of the system and we found $c_v = (du/dT)_v$ [equation (12)] applicable to any substance. It can be shown that for an ideal gas the limitation of constant volume is not necessary. Therefore we may write $c_v = du/dT$ and

$$(22) \quad \Delta u = \int c_v dT, \quad \Delta U = w \int c_v dT$$

[IDEAL GAS ANY KIND OF PROCESS]

For actual gases ΔU approaches a function of T only as the pressure approaches zero but equation (22) often gives a good estimate of ΔU for any gas. If it is permissible to take the specific heat as constant integration of

* More precise experiments conducted later by Joule and William Thomson (Lord Kelvin) showed a change of temperature of the air. A porous plug was used at the valve and extreme care was taken to prevent heat transfer to or from an external source. Thus real gases do have a change of temperature with a change in pressure. The rate of change of temperature with pressure under the conditions of this experiment is called the *Joule-Thomson coefficient*. Mathematically the Joule-Thomson coefficient $= (dT/dp)_h$ where the subscript h means that the enthalpy is constant. In a perfect gas this coefficient is zero. Thus Joule's law is true only for ideal gases; is nearly true for nearly ideal gases; is decidedly in error for vapors. The magnitude of the Joule-Thomson coefficient measures the degree of departure of the gas from the ideal.

equation (22) gives

$$(j) \quad \Delta U = U_2 - U_1 = wc_v(T_2 - T_1) \text{ Btu,}$$

[CONSTANT SPECIFIC HEAT]

which is the internal energy change of an ideal gas whenever the temperature changes from T_1 to T_2 and c_v is constant.

59. Enthalpy of an Ideal Gas. In § 37, we found $c_p = (dh/dT)_p$, equation (13). As for the case of internal energy, it can be shown that the constant pressure limitation is not necessary for an ideal gas. Therefore, from $c_p = dh/dT$, we get

$$(23) \quad \Delta h = \int c_p dT, \quad \Delta H = w \int c_p dT,$$

[IDEAL GAS, ANY KIND OF PROCESS]

applicable to actual gases in most engineering processes. For constant specific heat,

$$(k) \quad \Delta H = H_2 - H_1 = wc_p(T_2 - T_1) = wc_p \Delta T \text{ Btu,}$$

[CONSTANT SPECIFIC HEAT]

which is the enthalpy change of an ideal gas whenever the temperature changes from T_1 to T_2 and c_p is constant.

60. Example. A gas in the initial state of $p_1 = 75$ psia and $V_1 = 5$ cu. ft. undergoes a process to $p_2 = 25$ psia and $V_2 = 9.68$ cu. ft., during which the enthalpy decreases 62 Btu. The specific heat at constant volume is $c_v = 0.745$ Btu/lb-°R. Determine (a) the change of internal energy, (b) the specific heat at constant pressure, (c) the gas constant R .

SOLUTION. In general, if a method of solution is not immediately apparent, it is a good idea to write down on your work sheet all that you can think of which might have a bearing, including the most fundamental relations. If this is done for this problem, the work sheet should have on it $\Delta U = wc_v(T_2 - T_1)$ and $pV = wRT$; with these two equations there to look at, it might occur to you to use $pV = wRT$ in order to eliminate the temperature in the equation for ΔU . [See part (c).] However, the gas constant R is unknown and this solution cannot be made. If you have written the fundamental definition of enthalpy change, $\Delta H = \Delta U + \Delta(pV)$, a solution for (a) is evident.

(a) Substitute $\Delta H = -62$ Btu because the change is a decrease. Then

$$-62 = \Delta U + \frac{[(25)(9.68) - (75)(5)]144}{778} = \Delta U - 24.6,$$

or $\Delta U = -62 + 24.6 = -37.4$ Btu, a decrease of U as indicated by the negative sign. Be sure to remember to convert psi to psf (144) and to have all energy terms in the same units (778).

(b) We notice from equations (j) and (k) above (constant k) that

$$\frac{\Delta h}{\Delta u} = \frac{c_p}{c_v} = \frac{-62}{-37.4} = 1.655 = k,$$

$$c_p = 1.655c_v = (1.655)(0.745) = 1.25 \text{ Btu/lb-°R}$$

(c) Now we may use $T_1 = (p_1 V_1)/(wR)$ and $T_2 = (p_2 V_2)/(wR)$ in the internal energy equation and find

$$\begin{aligned}\Delta U &= wc_v(T_2 - T_1) = \frac{c_v}{R} (p_2 V_2 - p_1 V_1) \text{ Btu}, \\ -37.4 &= \frac{0.754}{R} [(25)(9.68) - (75)(5)] 144, \\ R &= 386 \text{ ft-lb/lb-}^\circ\text{R}\end{aligned}$$

NOTE If the substance is known, these constants c_p and R can be found in the literature, as in Table I. Comparing the answers found with the values in Table I, we see that the gas in this problem is helium if it is a single gas. However, such a problem as this might be a mixture of gases with constants different from those of any single gas. Note that equation (24) below could be used to find R in this example.

ANOTHER NOTE The way to study examples is with pencil and paper. You solve, or try to solve, the example before or after reading the solution.

61. Relation between c_p and c_v . The relations derived below will be repeatedly useful. Starting with the definition $dH = dU + d(pV)/J$, we substitute $dH = wc_p dT$, $dU = wc_v dT$, and $pV = wRT$ [or $d(pV) = wR dT$] and find

$$\begin{aligned}wc_p dT &= wc_v dT + \frac{wR dT}{J}, \\ (24) \quad c_p &= c_v + \frac{R}{J}, \quad \text{or} \quad c_p - c_v = \frac{R}{J} \text{ Btu/lb-}^\circ\text{R}\end{aligned}$$

This equation says that the difference between the specific heats is equal to the gas constant in Btu units (§ 56), not only true for ideal gases, but also substantially true for real gases. Now, using $k = c_p/c_v$ or $c_p = kc_v$ [see equation (14), p. 40], we get from equation (24) $kc_v - c_v = R/J$, or

$$(25) \quad c_v = \frac{R}{J(k-1)}$$

Since $c_p = kc_v$, we find from equation (25)

$$(26) \quad c_p = \frac{kR}{J(k-1)}$$

Recall that k varies with the temperature (Fig. 19).

To get the relation between molar specific heats, we have $C_p = Mc_p$, $C_v = Mc_v$, and $MR = \bar{R}$. Thus, equation (24) yields

$$(27) \quad Mc_p - Mc_v = \frac{MR}{J}, \quad \text{or} \quad C_p - C_v = \frac{\bar{R}}{J} = 1.986,$$

that is, the difference between these molar specific heats is equal to the universal gas constant in Btu units, the same constant for all gases,

$$1545/778 = 1.986 \text{ Btu per mol-}^\circ\text{R}$$

(See § 56.) From equations (25) and (26), we have

$$(m) \quad C_v = \frac{\bar{R}}{J(k-1)} = \frac{1.986}{k-1}, \quad \text{and} \quad C_p = \frac{k\bar{R}}{J(k-1)} = \frac{1.986k}{k-1}.$$

62. Entropy Change of an Ideal Gas. In § 41, the energy equation was set up in the form $dQ = dU + p dV/J$, where for

$$\begin{aligned} \text{Steady flow,} \quad & \int p dV = \Delta W_f + \Delta K + \Delta P + W, \\ \text{Nonflow,} \quad & \int p dV = W \text{ ft-lb.} \end{aligned}$$

If we let $dQ = T dS$ and $dU = w c_v dT$, we get an important form of the energy equation;

$$(28) \quad dQ = T dS = w c_v dT + \frac{p dV}{J}. \quad [\text{IDEAL GAS}]$$

Also using the enthalpy change of equation (23), § 59, and equation (17), p. 45, we write

$$(29) \quad dQ = T dS = w c_p dT - \frac{V dp}{J}. \quad [\text{IDEAL GAS}]$$

It will be worth while for the reader to practice deriving these two equations from the various fundamental definitions which have been given. By dividing through equations (28) and (29) by T , we have two values of dS in terms of point functions only. Since entropy is a point function, $S_2 - S_1$ in Fig. 28 is a certain definite number no matter whether the path followed is 1- d -2, or 1- b -2, or any other path, reversible or irreversible. Equations to relate the entropies at any two points are particularly desirable inasmuch as entropy has been defined only for a reversible process, $\Delta S = \int dQ/T$.

First, divide equation (28) by T ; use $p/T = wR/V$ from $pV = wRT$ (note that the results are restricted to ideal gases), and get

$$\int dS = w \int \frac{c_v dT}{T} + \frac{wR}{J} \int \frac{dV}{V}.$$

If the specific heat is constant, we integrate from 1 to 2 and get

$$(n) \quad \Delta S = S_2 - S_1 = w c_v \ln \frac{T_2}{T_1} + \frac{wR}{J} \ln \frac{V_2}{V_1} \text{ Btu/}^\circ\text{R.}$$

[IDEAL GAS, CONSTANT c_v]

While it is true that $Q = \int T dS$ and $W = \int p dV$ for reversible processes only, the foregoing integration [equation (n)] in terms of point functions is no longer dependent upon the path, as are Q and W . However, with points 1 and 2 as

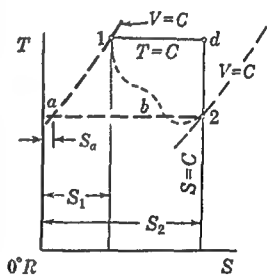


Fig. 28. Entropy Change.

shown in Fig 28, the first term of equation (n) is the change of entropy along a reversible constant volume line 1-a ($S_a - S_1 = \int dQ/T = w c_v \int dT/T$), and the second term is the change along a reversible constant temperature line a-2 (§ 69), $S_a - S_1 + S_2 - S_a = S_2 - S_1$ (Fig 28). In other words, equation (n) and those that follow actually give the entropy change along two reversible paths joining the points.

Similarly from equation (29), after dividing by T and using $V/T = wR/p$, we have

$$\int dS = w \int \frac{c_p dT}{T} - \frac{wR}{J} \int \frac{dp}{p}$$

[IDEAL GAS]

If the specific heat is constant, integration from 1 to 2 yields

$$(o) \quad \Delta S = S_2 - S_1 = w c_p \ln \frac{T_2}{T_1} - \frac{wR}{J} \ln \frac{p_2}{p_1} \text{ Btu/}^\circ\text{R}$$

[IDEAL GAS CONSTANT c_p]

It may be useful to recall that

$$-\log \frac{A}{B} = +\log \frac{B}{A}, \quad \text{and} \quad \ln V = \log_e V = 2.3 \log_{10} V$$

Using various known relations ($pV = wRT$, $c_p - c_v = R/J$ etc.) many different forms of equations for ΔS can be obtained. These variations will be left for the reader to find as needed. For example ΔS in terms of p and V (no T) may be found from equation (n) as

$$(p) \quad \Delta S = w c_v \ln \frac{p_2}{p_1} + w c_p \ln \frac{V_2}{V_1} \text{ Btu } ^\circ\text{R}$$

[IDEAL GAS CONSTANT c_v AND c_p]

The equations of this article (§ 62) are primarily for an ideal gas as explained. In most engineering situations, allowing for the variation of specific heats will result in working answers for any gas.

63 Closure. The various relations evolved in this chapter apply to ideal gases only. When you think of Boyle, think $T = C$ and $pV = C$, when you think of Charles, think $p = C$ and $V/T = C$ and also $V = C$ and $p/T = C$. Apply $pV = wRT$ only to monatomic (He, A) or diatomic gases. Other gases are much too imperfect, except at low pressures. Joule's law ($\Delta U = w c_v \Delta T$, and also $\Delta H = w c_p \Delta T$) applies only to an ideal gas (or for all practical purposes a nearly ideal gas).

5

PROCESSES OF IDEAL GASES

64. Introduction. Knowing the characteristics of one working substance, the ideal gas, we are in a position to study processes of such a substance, after which we shall be able to investigate cycles in detail. Since graphical representations are of inestimable value in the solution of problems, the reader should always sketch the processes on the pV and TS planes.

Area "under" curve on pV plane, $\int p dV$, represents the work of a reversible nonflow process,

Area "behind" curve on pV plane, $-\int V dp$, represents $\Delta K + W$ for a reversible steady flow process,

Area "under" curve on TS plane, $\int T dS$, represents heat of a reversible process.

It is worth while to know the energy equation in the forms

$$wc dT = wc_v dT + \frac{p dV}{J} \quad \text{and} \quad wc dT = wc_p dT - \frac{V dp}{J},$$

[AFTER EQUATION (28)] [AFTER EQUATION (29)]

where the specific heat c is chosen in accord with the kind of process and the substance.

For all ideal gas processes (constant specific heats):

$$\Delta U = wc_v(T_2 - T_1), \quad \text{and} \quad \Delta H = wc_p(T_2 - T_1).$$

For all ideal gas reversible processes (constant specific heats):

$$\Delta S = wc \ln \frac{T_2}{T_1} \quad \left\{ \begin{array}{l} \text{except } T = C. \\ c \text{ accords with process} \end{array} \right\},$$

$$Q = wc(T_2 - T_1) \quad \left\{ \begin{array}{l} \text{except } T = C. \\ c \text{ accords with process} \end{array} \right\}.$$

For all reversible nonflow processes

$$W = \int p dV$$

Positive values of ΔU , ΔH , and ΔS mean increases, negative values, decreases. Positive Q is heat added, negative, heat rejected. Positive W is work done by, negative, work done on the system. With some of the material of this chapter, you are already familiar from preceding chapters, but it is necessary to organize the knowledge with respect to processes.

65 Constant Volume Process On the pV plane, this process, which is also called an *isometric process*, is represented by a vertical line 1-2 [Fig 29(a)]. On the TS plane, a constant volume line slopes upward toward the right, the equation of the curve being obtained from $\int dS = u c_v \int dT/T$ for constant c_v , or

$$S = u c_v \ln T + C,$$

where C is a constant of integration. As shown in Fig 29, the gas is being heated up, at least, energy is flowing into the system.

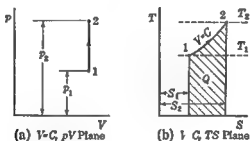


Fig 29 Reversible Isometric Process On the pV plane, there is no area under 1-2, $\int p dV = 0$

In the solution of problems, it is frequently necessary to recall the p , V , T relations for the process at hand. For $V = C$, we think immediately of Charles' law and have

$$(a) \quad \frac{p_2}{p_1} = \frac{T_2}{T_1} \quad [V = C, \text{ CHARLES' LAW}]$$

An internally reversible constant volume process is one in which the rise in temperature is due to heat only and

$$(b) \quad Q = w c_v (T_2 - T_1)$$

Since $dV = 0$, the work $W = \int p dV = 0$

The change of entropy is

$$(c) \quad \Delta S = u c_v \ln \frac{T_2}{T_1}$$

From the energy equation, we observe that

$$(d) \quad Q = \Delta U + \frac{p dV}{J}, \quad \text{or} \quad Q = \Delta U.$$

For a reversible steady flow isometric process (it could happen to an incompressible liquid), we have the energy equation from equation (7A), p. 31, in the form

$$Q = \Delta U + \Delta W_f + \Delta K + \Delta P + W.$$

Since $Q = \Delta U$, this equation yields

$$W = -(\Delta W_f + \Delta K + \Delta P).$$

66. Example: Constant Volume. A 10-cu. ft. vessel of hydrogen at a pressure of 305 psia is stirred by paddles (Fig. 30) until the pressure becomes 400 psia. Determine (a) the change of internal energy, (b) the work, (c) the transferred heat, and (d) the change of entropy of 1 lb. of the system. The system is the hydrogen in the tank, which is perfectly insulated (adiabatic walls).

SOLUTION. (a) We know that $\Delta U = w c_v \Delta T$ for any kind of process of an ideal gas, but we have no temperatures. Therefore, eliminate temperatures with $pV = wRT$ and find

$$\Delta U = \frac{c_v V}{R} (p_2 - p_1) = \frac{(2.4354)(10)}{767} (400 - 305)(144),$$

or $\Delta U = 434$ Btu, where c_v and R have been taken from Table I and where we remembered (144) that p must be in pounds per square foot.

(b) Although $\int p dV = 0$ because $dV = 0$, the work is not zero. We have repeatedly warned that $W = \int p dV$ only for a reversible process. The kind of constant volume process that has been discussed up to this point is one which is internally reversible; that is, the process was brought about by the addition or abstraction of heat. While the heating process is irreversible externally, it is reversible internally in that heat which is added can be made to flow out again (in both directions as heat). However, the work flow into the system cannot be reversed. We shall learn later how part of this energy can be got out as work—but the flow of energy as work, dissipated as fluid friction, is irreversible.

Since the walls do not permit flow of heat, the nonflow energy equation becomes

$$Q = \Delta U + W = 0, \quad \text{or} \quad W = -\Delta U,$$

or $W = -434$ Btu, where the minus sign says that the work is done on the system.

NOTE. If electricity is to be used in a motor to cause work to flow into the system, Fig. 30 shows the hard way—driving a paddle wheel. The easy way would be to install an electric resistance coil inside of the system and feed electricity directly to it. Since electricity is 100% convertible into work, it can be classified as work W

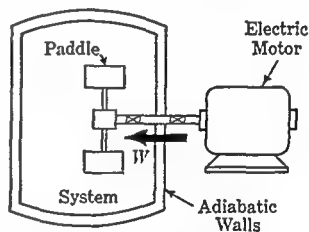


Fig. 30. Irreversible Constant Volume Process. $V = C$; $Q = 0$.

insofar as all our energy equations are concerned, but what leaves the resistance coil is heat. Think about this until it is clear. Evidently, the specified state change in this problem can be brought about entirely by heat entirely by work or by part heat and part work in any proportions.

(c) The heat $Q = 0$ by statement of the problem.

(d) We have defined specific heat as the amount of heat to be added to one pound of substance to change its temperature 1°F . But no heat is added in this example. However, since the entropy change of the system is a function of state properties only, we may refer to equation (n), p. 65 and its antecedents and get

$$\Delta s = c_v \ln \frac{T_2}{T_1} + \frac{R}{J} \ln \frac{v_2}{v_1} = c_v \ln \frac{T_2}{T_1}$$

The term $(R/J) \ln (v_2/v_1)$ drops out because $v_2/v_1 = 1$ and $\ln 1 = 0$. Using Charles' law, $T_2/T_1 = p_2/p_1$ in this expression, we find

$$\Delta s = c_v \ln \frac{p_2}{p_1} = 2.4354 \ln \frac{400}{305} = 0.659 \text{ Btu lb}^{-1}\text{R},$$

an increase within the system. There are not enough data given to determine the total mass and total entropy involved.

67. Constant Pressure Process A *constant pressure process*, also called an *isobaric process*, is a change of state during which the pressure remains constant. It may be reversible or irreversible nonflow or flow. In any case, for an ideal gas,

$$(e) \quad \frac{V_2}{V_1} = \frac{v_2}{v_1} = \frac{T_2}{T_1} \quad (p = C \text{ CHARLES' LAW})$$

On the pV plane (Fig. 31), the process is represented by a horizontal line, and on the TS plane the reversible process is represented by the curve obtained from the indefinite integral of $dS = dQ/T$, or

$$S = w c_p \ln T + C,$$

where C is the constant of integration. In the reversible process 1-2 (Fig. 31), heat is being added to the system and work is being done by it. If the state point should have a leftward movement, as from 2 to 1 (Fig. 31), heat is *rejected* and work done *on* the system would be indicated. (See § 64.)

Notice the similarity of appearance of the constant volume and constant pressure lines on the TS plane (Fig. 31). The area under the constant pressure line must be larger than the area under the constant volume line for two particular temperatures T_1 and T_2 , because c_p is larger than c_v ($dQ = c dT$). Thus, to accord with this fact, it is essential for the constant pressure line to be *less steep* than the constant volume line within a particular temperature range.

(a) $\int p dV$. For constant pressure,

$$(f) \quad \int p dV = p \int dV = p(V_2 - V_1) = wR(T_2 - T_1) \text{ ft-lb},$$

[ANY FLUID]
[IDEAL GAS]

where the last term is obtained by use of $pV = wRT$ and applies to an ideal gas only. Thus, for a nonflow process,

$$W = \int p dV = p(V_2 - V_1) \text{ ft-lb.}$$

For a steady flow process, equation (k), § 41, shows that (with $\Delta P = 0$)

$$\begin{aligned} \int p dV &= p(V_2 - V_1) = \Delta W_f + \Delta K + W \\ &= p(V_2 - V_1) + \Delta K + W. \end{aligned}$$

Since the terms $p(V_2 - V_1)$ cancel each other, the shaft work, if any, is $W = -\Delta K$ for any fluid in steady flow at constant pressure. The same conclusion is arrived at for $dp = 0$ in $\int v dp$ in equation (p), § 42; to wit, $W = -\Delta K$.

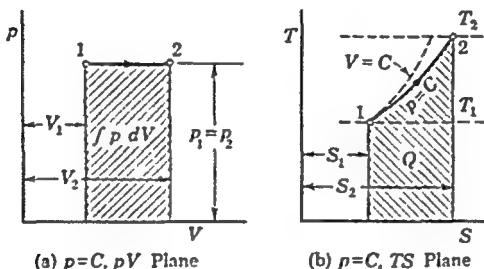


Fig. 31. *Reversible Isobaric Process.* Observe that the shaded area of figure (a) is equal to $W = p(V_2 - V_1)$, the work for a nonflow process, which is the same as the result obtained from $\int p dV$.

(b) *Heat.* Using a constant c_p in $dQ = wc_p dT$, we find for an internally reversible process,

$$(g) \quad Q = wc_p \int dT = wc_p(T_2 - T_1) \text{ Btu.}$$

The energy for a change of state from T_1 to T_2 could be supplied as work (see paddle, § 66), in which case the process is irreversible. In any reversible process, flow or nonflow, $Q = \Delta U + \int (p dV)/J$; and for $p = C$, this is

$$(h) \quad Q = U_2 - U_1 + p_2V_2 - p_1V_1 = H_2 - H_1, \\ \text{[ANY FLUID, FLOW OR NONFLOW, } p = C\text{]}$$

which shows that the heat transferred in any reversible constant pressure process is ΔH , which for an ideal gas is $\Delta H = wc_p \Delta T = Q$ as in equation (g). If the process is steady flow (with $\Delta P = 0$),

$$Q = \Delta H + \Delta K + W. \\ \text{[ANY FLUID, ANY PROCESS]}$$

If this flow occurs with little or no change in kinetic energy ($\Delta K = 0$) and with no shaft work ($W = 0$), as it does in all kinds of heat exchangers, then,

even with the pressure varying,

$$(i) \quad \begin{array}{ll} Q = \Delta H & Q = w c_p (T_2 - T_1) \\ \text{[ANY FLUID, STEADY FLOW PROCESS,]} & \text{[IDEAL GAS, STEADY FLOW,]} \\ \Delta K = \Delta P = W = 0] & \Delta K = \Delta P = W = 0] \end{array}$$

This equation holds whether the process is reversible or irreversible when the stated conditions are met. See equation (h) again. The energy equation for $p = C$ is ($Q = \Delta U + \int p dV/J$)

$$w c_p \Delta T = w c_v \Delta T + \frac{p \Delta V}{J} \text{ Btu}$$

[c_p AND c_v CONSTANT, IDEAL GAS]

68 Example: Constant Pressure. An ideal gas, for which $R = 386$ and $k = 1.659$, undergoes a reversible isobaric process during which 500 Btu are added to 5 lb of gas. The initial temperature is 100°F . Determine (a) t_2 , (b) ΔH , (c) ΔS , (d) ΔU , and (e) W for a nonflow process.

SOLUTION The specific heats which will be needed, are found from equations (25) and (26) (The gas is not stated)

$$c_p = \frac{R}{J(k-1)} = \frac{386}{778(1.659-1)} = 0.754 \text{ Btu/lb-}^\circ\text{R}$$

$$c_v = \frac{kR}{J(k-1)} = \frac{(1.659)(386)}{778(1.659-1)} = 1.25 \text{ Btu/lb-}^\circ\text{R}$$

(a) The heat is $500 = w c_p (T_2 - T_1)$, or

$$T_2 = \frac{500}{w c_p} + T_1 = \frac{500}{(5)(1.25)} + 560 = 640^\circ\text{R}$$

or $t_2 = 180^\circ\text{F}$

(b) Since $\Delta H = w c_p \Delta T$ then $\Delta H = 500$ Btu [This is also the heat in any steady flow process, under certain conditions. See equation (i)]

(c) The change of entropy is

$$\Delta S = w c_p \ln \frac{T_2}{T_1} = (5)(1.25) \ln \frac{640}{560} = 0.83 \text{ Btu/}^\circ\text{R}$$

(d) The change in internal energy is

$$\Delta U = w c_v (T_2 - T_1) = (5)(0.754)(640 - 560) = 301.6 \text{ Btu}$$

(e) The work may be found from $W = p(v_2 - v_1) = wR(T_2 - T_1)$ ft-lb, but it is easily found from the nonflow energy equation

$$W = Q - \Delta U = 500 - 301.6 = 198.4 \text{ Btu}$$

69 Isothermal Process. An *isothermal process* is one carried out at constant temperature, $T = C$. Unless stated otherwise, we shall mean a *reversible* isothermal process whenever the term is used. Since $T = C$, we think immediately of Boyle's law,

$$pV = C, \quad \text{or} \quad p_1 V_1 = p_2 V_2 = C \quad [T = C, \text{BOYLE}]$$

[ANY REVERSIBLE OR IRREVERSIBLE PROCESS]

The pV and TS curves of this process are shown in Fig. 32, where, on both planes, the

State point has a component motion toward the right;
Heat is added, work is done *by* the system.

If the process is run in the opposite direction, the

State point has a component motion toward the left;
Heat is rejected, work is done *on* the system.

One can conceive of a process for which the state point has a component motion on one plane in the opposite direction to the motion on the other plane—but not for the isothermal process.

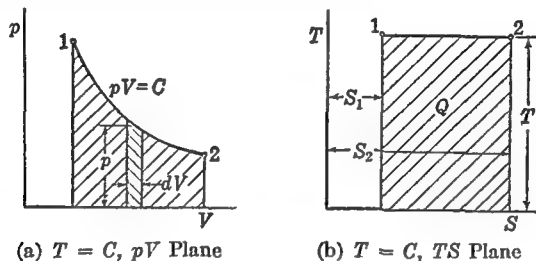


Fig. 32. *Reversible Isothermal Process.* Observe that the differential area in (a), $p dV$, is a differential work quantity dW in a nonflow system. When the volume increases, as 1 to 2, the process is called an *expansion*. When the volume decreases, it is said to be a *compression*.

Since U and H are functions of temperature only for an ideal gas, and since $T = C$, it follows that $U = C$ and $H = C$; or $\Delta U = 0$ and $\Delta H = 0$. Thus, for an ideal gas, we could (but we will not) call this process a constant-internal-energy process or a constant-enthalpy process.

(a) $\int p dV$. This integration has been made (§ 18) using $p = C/V$.

$$\int p dV = C \int \frac{dV}{V} = p_1 V_1 \ln \frac{V_2}{V_1} = wRT \ln \frac{p_1}{p_2} \text{ ft-lb.},$$

where, in the last form, we have used $p_1 V_1 = wRT$ and $V_2/V_1 = p_1/p_2$.

For the nonflow process, then

$$(j) \quad W = p_1 V_1 \ln \frac{V_2}{V_1} \text{ ft-lb.} = \frac{p_1 V_1}{J} \ln \frac{V_2}{V_1} \text{ Btu.}$$

[REVERSIBLE NONFLOW, IDEAL GAS]

For the steady flow process at constant temperature ($\Delta P = 0$) we may use equation (p), § 42, and obtain

$$- \int V dp = p_1 V_1 \ln \frac{V_2}{V_1} = \Delta K + W \text{ ft-lb.},$$

from which one may solve for W if the other terms are known. The states 1 and 2 are at the boundaries of the system where the substance enters and leaves.

(b) *Heat* Since heat is transferred but the temperature does not change, the specific heat of the process is infinite, $c = \infty$. With $dT = 0$,

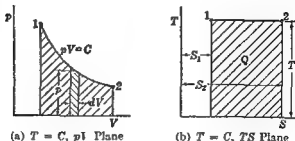


Fig 32 Repeated

we have $Q = c dT = (\infty)(0)$, an indeterminate. Therefore, to find Q , we must use the energy equation

$$Q = \Delta U + \int p dV \quad \text{or} \quad Q = \Delta H - \int V dp$$

$$(k) \quad Q = \int p dV = \frac{p_1 V_1}{J} \ln \frac{V_2}{V_1} = \frac{wRT}{J} \ln \frac{p_1}{p_2} \text{ Btu}$$

[ANY REVERSIBLE ISOTHERMAL, FLOW OR NONFLOW IDEAL GAS]

This equation applies as well to a steady flow isothermal as to a nonflow, as you realize when you recall that $\int p dV$ is equal to the sum of all the mechanical energy terms in the steady flow equation given in § 41. Equation (k) says that the work of a nonflow isothermal process is exactly compensated by heat—without increase or decrease of internal energy.

The change of entropy is

$$(l) \quad \Delta S = \int \frac{dQ}{T} = \frac{1}{T} \int dQ = \frac{Q}{T} = \frac{wR}{J} \ln \frac{V_2}{V_1} \text{ Btu}/^\circ\text{R}$$

Compare with the rectangular area under 1-2 on the TS plane (Fig 32)

70 Adiabatic Process—Isentropic Process An *adiabatic process* is one in which no heat is transferred, $Q = 0$. An *isentropic process* is defined as one of constant entropy. In a reversible adiabatic $Q = 0$ as in any adiabatic, but also $T dS = 0$, since $Q = T dS$ in a reversible process. We know that T is not zero; hence, $dS = 0$. In this book, the word *isentropic* shall mean a reversible adiabatic.*

In any *nonflow adiabatic*, reversible or irreversible any substance, $Q = \Delta U + W = 0$, or

$$(30) \quad W = -\Delta U = U_1 - U_2 \quad \text{or} \quad W = w c_v (T_1 - T_2) \text{ Btu}$$

[ANY FLUID]
[IDEAL GAS]

* Occasionally, *isentropic* is taken to mean only *constant entropy* and an irreversible constant entropy process which is not adiabatic is easily conceived though it is most unlikely to be of any use.

In any *steady flow adiabatic*, reversible or irreversible, any fluid (with $Q = 0$ and $\Delta P = 0$).

$$(31) \quad W = -\Delta H - \Delta K,$$

and if ΔK happens to be negligible, as it often is,

$$(32) \quad W = -\Delta H = H_1 - H_2 \quad \text{or} \quad W = w c_p (T_1 - T_2) \text{ Btu.}$$

[ANY FLUID, $\Delta K = 0$] [IDEAL GAS]

If $W = 0$, as in a nozzle, $\Delta K = -\Delta H$, or

$$(33) \quad \frac{w u_2^2}{2gJ} - \frac{w u_1^2}{2gJ} = H_1 - H_2 = w c_p (T_1 - T_2) \text{ Btu.}$$

[ANY FLUID, $W = 0$] [IDEAL GAS]

We see that the work done in any nonflow adiabatic [equation (30)] is at the expense of the internal energy. In any adiabatic steady flow [equation (32)], the work is usually largely at the expense of a decrease in enthalpy.

(c) *Relation between p , V , and T .* If we wish to obtain the relation between p and V , we use the energy equation and $pV = wRT$, eliminating the temperature. For a reversible process,

$$(34) \quad T dS = 0 = w c_v dT + \frac{p dV}{J}.$$

Differentiate $pV = RT$ and get

$$p dV + V dp = wR dT,$$

or

$$dT = \frac{p dV + V dp}{wR}.$$

Put this value of dT into equation (34) and find

$$-\frac{p dV}{J} = \frac{c_v p dV + c_v V dp}{R}.$$

Multiplying both sides of this equation by R and dividing by pV , we have

$$-\frac{R}{J} \frac{dV}{V} = c_v \frac{dV}{V} + c_v \frac{dp}{p} \quad \text{or} \quad -\left(\frac{R}{J} + c_v\right) \frac{dV}{V} = c_v \frac{dp}{p}.$$

Since $R/J + c_v = c_p$ (§ 61), and since $c_p/c_v = k$, we have

$$-k \frac{dV}{V} = \frac{dp}{p}.$$

If k is taken as a constant (say, the mean value), we have now arrived at an integrable form.* Thus, between two states 1 and 2,

$$-k \int_{V_1}^{V_2} \frac{dV}{V} = \int_{p_1}^{p_2} \frac{dp}{p}.$$

* See § 77 for a treatment of the isentropic process with variable specific heats.

or

$$-k \ln \frac{V_2}{V_1} = \ln \left(\frac{V_1}{V_2} \right)^k = \ln \frac{p_2}{p_1}$$

Taking the antilogarithm of the last two expressions, we find

$$(n) \quad \left(\frac{V_1}{V_2} \right)^k = \frac{p_2}{p_1} \quad \text{or} \quad p_1 V_1^k = p_2 V_2^k$$

or, since states 1 and 2 were chosen at random along any isentropic line,

$$(34) \quad pV^k = C,$$

the equation of an isentropic process for an ideal gas in pV coordinates. The corresponding curve is similar in general appearance to the equilateral

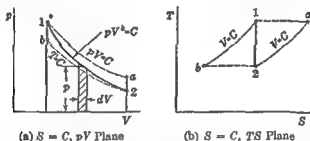


Fig 33 Isentropic Process For a comparison of the isothermal and the isentropic curves, isothermals 1-a and b-2 are shown dotted. Point a is related to 2 in being on the constant volume line through 2. Points 1 and b are also on a constant volume line. Observe that the isentropic line on the pV plane is steeper than the isothermal, also that there is no area under the curve 1-2 on the TS plane, which is in accordance with the definition that $Q = 0$ and $S = C$.

hyperbola [Fig 33(a)]. With equation (n), we can derive the relation between T and V , and the relation between T and p . From the equation of state, $p_1 V_1 / T_1 = p_2 V_2 / T_2$ we get

$$\frac{p_2}{p_1} = \frac{V_1 T_2}{V_2 T_1}$$

which, substituted into equation (n), gives

$$\left(\frac{V_1}{V_2} \right)^k = \frac{V_1 T_2}{V_2 T_1}$$

or

$$(35) \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1} = \left(\frac{V_2}{V_1} \right)^{1-k}$$

Now, from equation (n),

$$\frac{V_1}{V_2} = \left(\frac{p_2}{p_1} \right)^{1/k}.$$

Substituting this value of V_1/V_2 in equation (35), we find

$$(36) \quad \frac{T_2}{T_1} = \left[\left(\frac{p_2}{p_1} \right)^{1/k} \right]^{k-1} = \left(\frac{p_2}{p_1} \right)^{(k-1)/k}.$$

Equations (35) and (36) will be useful in problem work and in deriving special forms of other equations. All the foregoing p , V , T relations apply to an ideal gas.

(b) *Work.* We have found work from the energy equations (30), (31), and (32), all of which are very useful, but we must also learn how to obtain the work from the pV plane—area under the curve is *nonflow work*. Since $pV^k = C$ [Fig. 33(a)], we use $p = C/V^k$ in $p dV$ and get

$$(c) \quad \int p dV = C \int_{V_1}^{V_2} \frac{dV}{V^k} = \left[\frac{CV^{-k+1}}{-k+1} \right]_{V_1}^{V_2} = \frac{CV_2^{1-k} - CV_1^{1-k}}{1-k}.$$

Setting the first $C = p_2V_2^k$ and the second $C = p_1V_1^k$, we find

$$(37) \quad \int p dV = \frac{p_2V_2 - p_1V_1}{1-k} \text{ ft-lb.} = \frac{p_2V_2 - p_1V_1}{J(1-k)} \text{ Btu,}$$

[WHENEVER $pV^k = C$]

which is, then, the work W of a nonflow isentropic process. This integral may be converted to temperatures by using $pV = wRT$ and it can be shown to be identical to equation (30) when both are in the same units.

In a steady-flow isentropic ($\Delta P = 0$) with $pV^k = C$, we get from equation (p), § 42,

$$(p) \quad - \int V dp = -C \int_1^2 \frac{dp}{p^{1/k}} \\ = \frac{k(p_2V_2 - p_1V_1)}{1-k} = \Delta K + W.$$

The elementary area $V dp$ is shown in Fig. 34; let $C = p^{1/k}V$ or $V = Cp^{1/k}$. If the change of kinetic energy is negligible ($\Delta K = 0$), equation (p) gives

$$(38) \quad W = - \int V dp = \frac{k(p_2V_2 - p_1V_1)}{1-k} \text{ ft-lb.}$$

[REVERSIBLE STEADY FLOW, $\Delta K = 0$]

If $W = 0$, we get $\Delta K = - \int V dp$. Summarizing, we have

$$(q) \quad W = - \int V dp \quad \text{and} \quad \Delta K = - \int V dp.$$

[STEADY FLOW, $\Delta K = 0$] [STEADY FLOW, $W = 0$]

The negative sign for the $\int V dp$ is there because in integrating with respect to pressure from 1 to 2, the direction of integration is downward (negative) for an expansion and the $\int V dp$ turns out to be a negative number. But W done by the system, as in Fig. 34, is here taken positive; hence, we use

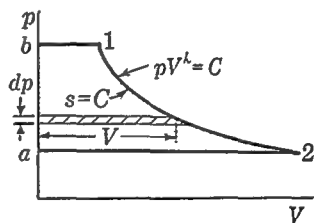


Fig. 34. Isentropic Process—Steady Flow.

$-\int V dp$ to accord with the convention of signs already defined. The student should work through the details of all integrations made in this text.

71. Irreversible Adiabatic with Work.* If, during an adiabatic process in a machine such as a turbine, there is fluid friction and turbulence, the work done is less than that during a corresponding isentropic expansion. The process is internally irreversible because of frictional effects. The end

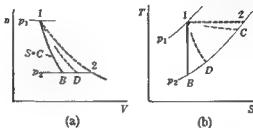


Fig 35

states of the adiabatic with work are represented by 1 D (Fig 35) where D may be anywhere between B and 2. The less the loss from irreversibility, the closer 1- D approaches the reversible 1- B .

In a general way, we shall use prime marks to indicate actual values and actual points, for example W' = actual work done (the work of the fluid unless otherwise indicated—that work which can be converted into shaft work in the absence of mechanical friction) $\Delta H'$ = actual change of enthalpy, 2' = the actual end point when the corresponding ideal end point

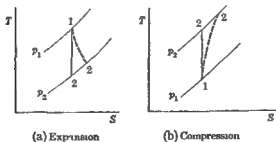


Fig 36 Irreversible Adiabatic Processes

is 2'. Since the energy equations in general form apply to any kind of process, we already have useful forms in § 70. However, rewriting equations (30), (32), and (33) in the prime notation, which indicates that some irreversibility effects are involved and referring to Fig 36, we get

* If it is desired to shorten the course one may touch lightly on irreversible processes and omit § 71, also omit §§ 77 and 78 and other articles on irreversible processes.

$$(30A) \quad \begin{array}{ll} W' = U_1 - U_2' & W' = wc_v(T_1 - T_2'); \\ \text{[ANY NONFLOW SYSTEM, } Q = 0] & \text{[IDEAL GAS]} \end{array}$$

$$(32A) \quad \begin{array}{ll} W' = H_1 - H_2' & W' = wc_p(T_1 - T_2'); \\ \text{[STEADY FLOW, ANY FLUID,} & \text{[IDEAL GAS]} \\ Q = 0, \Delta K = 0] & \end{array}$$

$$(33A) \quad \begin{array}{ll} \Delta K' = \frac{wv_2'^2}{2g_{\phi J}} - \frac{wv_1^2}{2g_{\phi J}} = H_1 - H_2' & \Delta K' = wc_p(T_1 - T_2'). \\ \text{[STEADY FLOW, ANY FLUID,} & \text{[IDEAL GAS]} \\ Q = 0, W = 0] & \end{array}$$

Irreversible *adiabatic* processes always occur with increasing entropy. In Fig. 36(a), an ideal adiabatic expansion from p_1 to p_2 ends at 2, but the actual expansion ends to the right at 2'. Likewise, the isentropic compression [Fig. 36(b)] ends at 2; the actual expansion at 2' at greater entropy than 2.

(a) *Relations between p , V , and T .* Actual expansions occur which accord reasonably with an equation of the form $pV^m = C$. If an actual exponent m can be determined and if the fluid is an *ideal gas* so that we may use $pV/T = C$, we get

$$p_1 V_1^m = p_2 V_2^m, \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{m-1}, \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(m-1)/m}$$

[IDEAL GAS]

after equations (35) and (36). These relations hold for any irreversible change of state to which $pV^m = C$ applies, whether it is adiabatic or not, but the fluid must be an ideal gas, $pv = RT$.

(b) *Change of Entropy.* The change of entropy may be evaluated along any reversible path connecting the points. In Fig. 35, ΔS from 1 to 2 is equal to ΔS from B to 2, in this case along a constant pressure line. The equations of § 62 may be used to compute the change of entropy, as convenient; for example, equation (n);

$$\Delta S = wc_v \ln \frac{T_2}{T_1} + \frac{wR}{J} \ln \frac{V_2}{V_1}.$$

Using $V_2/V_1 = (T_2/T_1)^{1/(1-m)}$ in this equation, we get

$$\begin{aligned} \Delta S &= wc_v \ln \frac{T_2}{T_1} + \frac{wR}{J} \ln \left(\frac{T_2}{T_1} \right)^{1/(1-m)} \\ &= wc_v \ln \frac{T_2}{T_1} + \frac{wc_v(k-1)}{1-m} \ln \frac{T_2}{T_1}, \\ (r) \quad \Delta S &= wc_v \left(\frac{k-m}{1-m} \right) \ln \frac{T_2}{T_1}, \end{aligned}$$

which gives the change of entropy in terms of temperatures, often a con-

venience * Remember that in irreversible processes, $\int p dV$ is not work and $\int T ds$ is not heat. Irreversible adiabatics are closely approached in some centrifugal pumps and blowers, in steam and gas turbines, and in reciprocating compressors and engines

72 Example. There are compressed 3500 cfm of air from $p_1 = 14.5$ psia and $t_1 = 75^\circ\text{F}$ to $p_2 = 29$ psia. Determine the work for each of the following processes (a) nonflow isentropic (b) nonflow irreversible adiabatic to a temperature of 190°F (650°R), (c) steady flow isentropic for which $v_1 = 40$ fps and $v_2 = 120$ fps (d) steady flow adiabatic to a temperature of 190°F , velocities as in (c) (e) If the irreversible processes accord with the relation $pT^m = C$, what is the value of m ? (f) What is the change of entropy?

SOLUTION The temperature after isentropic compression is Fig 36(b),

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}, \quad \text{or} \quad T_2 = 535 \left(\frac{29}{14.5}\right)^{(1.4-1)/1.4} = 625^\circ\text{R}$$

The amount of air involved is

$$w = \frac{p_1 V_1}{RT_1} = \frac{(14.5)(144)(3500)}{(53.3)(535)} = 256 \text{ lb/min}$$

(a) For a nonflow isentropic

$$W = -\Delta U = u c_v (T_1 - T_2) = (256)(0.1715)(535 - 625) = -3950 \text{ Btu/min}$$

This work is for the case where all the air is in a cylinder and the piston compresses it. It does not include work that is necessary to get the air into and out of the cylinder

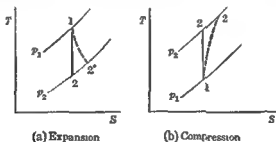


Fig 36 Repeated

(b) For the nonflow irreversible adiabatic, Fig 36(b)

$$W = u c_v (T_1 - T_2) = (256)(0.1715)(535 - 650) = -5050 \text{ Btu/min}$$

(c) For a steady flow process $Q = 0$ and $s = C$,

$$\begin{aligned} W &= -\Delta K - \Delta H = w \left(\frac{v_1^2}{2g_c J} - \frac{v_2^2}{2g_c J} + h_1 - h_2 \right) \\ &= 256 \left[\frac{40^2}{2g_c J} - \frac{120^2}{2g_c J} + 0.24(535 - 625) \right] = -5600 \text{ Btu/min} \end{aligned}$$

This work includes that necessary to get the air into and out of the cylinder

* We might note the similarity of $c_v[(k-m)/(1-m)]$ in equation (r), which is not a specific heat, to the polytropic specific heat equation $c_v[(k-n)/(1-n)]$, § 73(b). The polytropic is defined as a reversible process and $Q \neq 0$

(d) For the irreversible steady flow adiabatic, $-\Delta H' = wc_p(T_1 - T_2')$ and

$$W = 256 \left[\frac{40^2}{2g_o J} - \frac{120^2}{2g_o J} + 0.24(535 - 650) \right] = -7130 \text{ Btu/min.}$$

(e) To find m , we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(m-1)/m} = \frac{650}{535} = \left(\frac{25}{14.5} \right)^{(m-1)/m}$$

Using logarithms (or log log slide rule),

$$\frac{m-1}{m} = \frac{\log(650/535)}{\log(25/14.5)}, \quad \text{or} \quad m = 1.553.$$

(f) To find the entropy change of the system, use one of the basic equations of § 62 or use the derived form, equation (r), p. 79, which is

$$\Delta S = wc_v \left(\frac{k-m}{1-m} \right) \ln \frac{T_2}{T_1} = (256)(0.1715) \left(\frac{1.4-1.553}{1-1.553} \right) \ln \frac{650}{535} = 2.35 \text{ Btu/}^\circ\text{R.}$$

73. Polytropic Process. A polytropic process, Fig. 37, is an internally reversible process, which conforms to the relation

$$(39) \quad pV^n = C \quad \text{or} \quad p_1V_1^n = p_2V_2^n, \quad [\text{ANY FLUID}]$$

where n is any constant. Since many actual processes follow closely the relation in (39), this discussion is especially important. Because $pV^n = C$

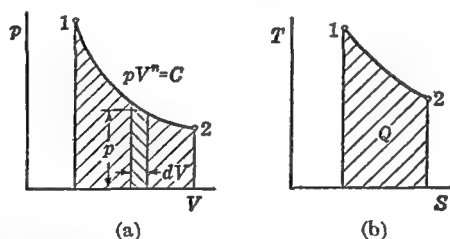


Fig. 37. Reversible Polytropic Process. These curves show the general appearance of a polytropic process where the value of n is between 1 and k . Since, in theory, the polytropic exponent may have any value from $n = -\infty$ to $n = +\infty$, curves sloping in any general direction may be obtained (see Fig. 39). In practice, the value of n does not vary greatly from k .

is mathematically the same as $pV^k = C$, all the mathematic consequences of § 70 apply here when $k = n$. Thus, the relation between T and V is

$$(40) \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_1} \right)^{1-n} = \left(\frac{V_1}{V_2} \right)^{n-1}. \quad [\text{IDEAL GAS}]$$

Between T and p , we have

$$(41) \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(n-1)/n} \quad [\text{IDEAL GAS}]$$

A negative specific heat means that heat is rejected by the substance even though the temperature increases, or that heat is added even though the temperature decreases. A process of heat rejection with simultaneous temperature increase actually occurs in the usual air compressor. The work done on the air raises the temperature of the air (stores some internal energy), yet at the same time, in order to obtain a lower final temperature at the end of compression, cooling water is circulated around the cylinder to carry some heat away. The area *under* the curve on the TS plane [Fig. 37(b) or 38(b)] represents Q in Btu units.

The change of entropy of a reversible polytropic process is

$$(u) \quad \Delta S = \int \frac{dQ}{T} = w \int \frac{c_n dT}{T} = wc_n \ln \frac{T_2}{T_1}.$$

[c_n CONSTANT]

Notice, for example, that by using the values of T_2/T_1 given by equations (40) and (41) in equation (u), ΔS may be expressed in terms of other properties.

74. Example: Polytropic Process. Five pounds of an ideal gas are compressed polytropically from a pressure of 15 psia and 40°F to 85.3 psig. The gas constant is $R = 50$, $c_p = 0.25$, and the change of state is according to $pV^{1.3} = C$. Find (a) the initial volume, (b) the final temperature, (c) the work for nonflow and steady flow processes, (d) the change of internal energy, (e) the transferred heat, (f) the change of entropy.

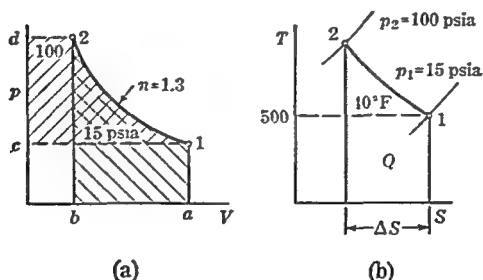


Fig. 38.

SOLUTION. (a) See Fig. 38. The student should cultivate the habit of making sketches for every problem. From the characteristic equation

$$V_1 = \frac{wRT_1}{p_1} = \frac{(5)(50)(40 + 460)}{(144)(15)} = 57.9 \text{ cu. ft.}$$

(b) From equation (41), $T_2 = T_1(p_2/p_1)^{(n-1)/n}$. Assume standard atmospheric pressure of 14.7 psia. Then $p_2 = 85.3 + 14.7 = 100$ psia, and

$$T_2 = (500) \left(\frac{100}{15} \right)^{(1.3-1)/1.3} = (500)(1.549) = 774^\circ\text{R} = 314^\circ\text{F}.$$

(c) The nonflow work is $W = (p_2V_2 - p_1V_1)/(1 - n)$. The volume V_2 must be calculated or eliminated. Using $pV = nRT$ we have

$$W_{\text{non}} = \frac{wR(T_2 - T_1)}{1 - n} = \frac{(5)(50)(774 - 500)}{1 - 1.3} = -228\,000 \text{ ft-lb} = -293 \text{ Btu}$$

represented by the area $a12b$ (Fig. 38) in foot-pound units. The negative sign indicates work done *on* the substance. The work of a steady flow process in which

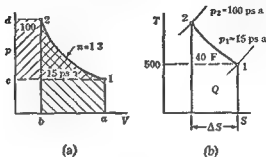


Fig 38 Repeated

$\Delta K = 0$ and $\Delta P = 0$ is n times the nonflow work. Compare equations (42) and (t). Therefore

$$W \text{ (steady flow)} = (1.3)(-293) = -381 \text{ Btu}$$

represented in foot-pound units by area $c12d$ (Fig. 38)

(d) To find c_v use $c_p - c_v = R/J$ or

$$c_v = 0.25 - \frac{50}{778} = 0.25 - 0.0043 = 0.1857 \text{ Btu/lb-}^\circ\text{R}$$

Then

$$\Delta U = u c_v (T_2 - T_1) = (5)(0.1857)(774 - 500) = +254 \text{ Btu}$$

The positive sign indicates an increase in internal energy.

(e) The value of $k = c_p/c_v = 0.25/0.1857 = 1.346$. The polytropic specific heat is

$$c_n = c_v \left(\frac{k - n}{1 - n} \right) = 0.1857 \left(\frac{1.346 - 1.3}{1 - 1.3} \right) = -0.02845$$

$$Q = u c_n (T_2 - T_1) = (5)(-0.02845)(774 - 500) = -39 \text{ Btu}$$

The negative sign indicates heat rejected by the substance. As a check on this calculation use the nonflow energy equation

$$Q = \Delta U + W = +254 - 293 = -39 \text{ Btu (check)}$$

The area under the curve 1-2 on the TS plane (Fig. 38) represents Q . Figure 38 is *not* drawn to scale.

(f) The change of entropy is

$$\Delta S = w c_n \ln \frac{T_2}{T_1} = (5)(-0.02845) \ln \frac{774}{500} = -0.0624 \text{ Btu/}^\circ\text{R}$$

The negative sign indicates a decrease of entropy.

75. Effect of Varying n in Polytrropic Equations. Polytrropic processes are all inclusive in that the equations for the change of state of all the foregoing reversible processes can be obtained from $pV^n = C$ by choosing proper values of n .

$p = C$; $n = 0$; then $pV^0 = C$ or $p = C$, a constant pressure process. This value of $n = 0$ substituted in polytrropic equations (39), (40), (42), (43), and (44) will give the correct forms for the constant pressure process.

$V = C$; $n = \infty$. Then from $pV^n = C$, we have $p^{1/n} V = C'$, or

$$p^{1/\infty} V = p^0 V = C',$$

or $V = C'$, a constant volume process. This value of $n = \infty$ substituted in the polytrropic equations (41), (42), (43), and (44) will yield the correct forms for the constant volume process.

Let $n = k$, then $pV^k = C$, which is recognized as the equation for an isentropic process. This value of $n = k$ substituted wherever n appears yields the correct forms for the isentropic process.

Let $n = 1$, then $pV = C$, which is recognized as Boyle's law and the equation of an isothermal process. This value of $n = 1$ yields the correct result in equations (39) and (41). However, when $n = 1$ is substituted in equations (42), (43), or (44), the resulting expressions are indeterminate.

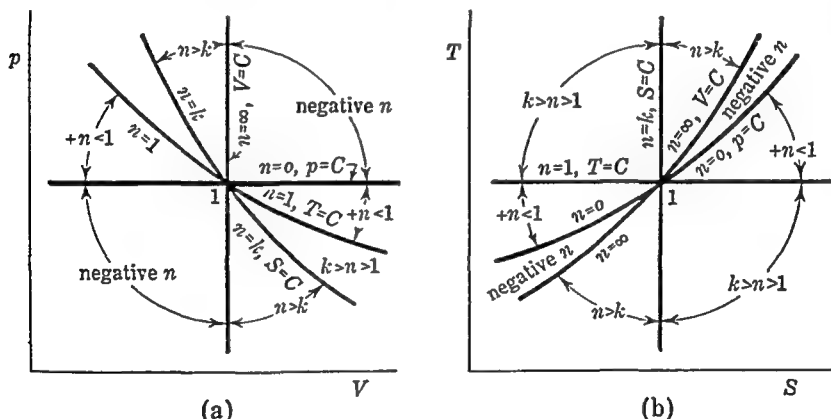


Fig. 39. Effect of Varying n . Expansions or compressions are imagined to take place from some common point 1. Notice that all positive values of n give curves in the second and fourth quadrants on the pV plane (a); that positive values of n may produce curves in all four quadrants on the TS plane (b). Notice too that curves with values of n between 1 and k will fall in the second and fourth quadrants on the TS plane and within a very narrow region on the pV plane.

It is important to be able to sketch various curves of thermodynamic processes on the pV and TS planes, and sketch them about as they would appear if they were actually plotted to scale. Figure 39 will be of assistance in learning to make these sketches and it shows also the effect of varying n . Remember that, although they appear much alike, the isentropic curve on

the pV plane is *steeper* than the isothermal curve, and that, on the TS plane, the constant volume curve is *steeper* than the constant pressure curve when both are drawn between the same temperature limits

76. Throttling Process. A throttling process is taken as an adiabatic flow process ($Q = 0$) from one region to another at lower pressure, during which no work is done. It is a free expansion, as in opening the valve on a compressed air tank or the household faucet. It is usually analyzed for a period during which the flow is steady. If $\Delta P = 0$, as well as $W = 0$ and $Q = 0$, the steady flow energy equation becomes

$$H_1 + K_1 = H_2 + K_2$$

Possibly somewhere in the stage of a throttling expansion, the kinetic energy K might be relatively large. In practical application however, the end conditions for the substance are virtually stationary or $K_1 \approx K_2$. In either event,

$$(45) \quad \begin{array}{ccc} H_1 = H_2 & \text{or} & u c_p (T_2 - T_1) = 0, \\ \text{[ANY FLUID]} & & \text{[IDEAL GAS]} \end{array}$$

represent defining conditions for a throttling process. If the substance is an ideal gas, $T_1 = T_2$ as well as $H_1 = H_2$, but not otherwise. (See footnote, p. 62.) Since $T = C$, $p_1 V_1 = p_2 V_2$ for an ideal gas (Fig. 35). Imperfect gases generally undergo a temperature decrease as a result of throttling, corresponding to some end conditions such as 1-C [Fig. 35(b)]

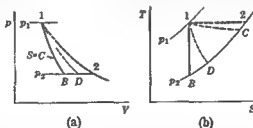


Fig. 35 Repeated (1-2 = Throttling Process)

In Fig. 35, the curve 1-B represents a reversible adiabat ($S = C$), during which the maximum work is obtained from the energy which the substance had in state 1. The throttling adiabat 1-2, which produces no work, is shown dotted to remind us that the areas under the curves no longer represent work and heat, that is, the points along the curve are not a sequence of equilibrium points and the dotted curve 1-2 does not necessarily represent the path of the state point. The expansion is irreversible, and unorganized for the production of work and the conservation of available energy.

The change of entropy, which will always be an increase, can be computed

from any one of the equations of § 62 (be sure to refer back in order to become oriented), say, equation (n) which reduces to

$$(v) \quad \Delta S = \frac{wR}{J} \ln \frac{V_2}{V_1} \text{ Btu/}^\circ\text{R},$$

because $T_1 = T_2$ and $\ln 1 = 0$. Compare equation (v) with the entropy change of a reversible isothermal process [equation (l), § 69].

77. Isentropic Process with Variable Specific Heats.* For the case of constant specific heat, we found the relation between p and V of an isentropic process (ideal gas) to be $pV^k = C$ [equation (34)]. If the specific heats vary, k also varies; so, to solve the isentropic process under this condition, we must start again with fundamental relations. If $pv = RT$, we have $dU = wc_v dT$; and the energy equation $dQ = dU + p dV/J$ becomes $wc_v dT = -p dV/J$. Substituting $p = wRT/V$ into this last equation and separating the variables, we find for $w = 1$ lb.

$$(w) \quad \int c_v \frac{dT}{T} = -\frac{R}{J} \int \frac{dV}{V} = -\frac{R}{J} \ln \frac{V_2}{V_1} = \frac{R}{J} \ln \frac{V_1}{V_2}.$$

To integrate the left-hand side of this equation, substitute the proper value of c_v as obtained from Table II or elsewhere. There is often a convenience in using mol values, $C_v = Mc_v$ and $\bar{R} = MR$, for which equation (w) becomes, in Btu per mol,

$$(x) \quad \int C_v \frac{dT}{T} = -\frac{\bar{R}}{J} \int \frac{dV}{V} = -1.986 \ln \frac{V_2}{V_1} = 1.986 \ln \frac{V_1}{V_2},$$

where the universal gas constant in Btu units is $\bar{R}/J = 1545/778 = 1.986$ (p. 60). After the left-hand side of either equation (w) or (x) has been integrated, one may solve for the temperature T_2 , the usual unknown, by trial and error, as illustrated in the following example.

78. Example. The molar specific heat of a mixture of gases in an internal combustion engine has been found to be given by (see Chapter 12, § 202)

$$C_v = 7.536 + \frac{3.16T}{10^4} - \frac{3.28T^2}{10^8} - \frac{2027.5}{T} + \frac{(80.2)(10^4)}{T^2} - \frac{37.1}{T^{3/2}} \quad \text{Btu/mol-}^\circ\text{R}.$$

The compression ratio of a nonflow isentropic compression in this engine is 6; that is, $V_2 = V_1/6$ and the initial temperature is $t_1 = 140^\circ\text{F}$. Compute the temperature t_2 after isentropic compression.

SOLUTION. The value of the right-hand side of equation (x) is

$$(y) \quad 1.986 \ln \frac{V_1}{V_2} = 1.986 \ln 6 = 3.559.$$

* In short courses, it may be desired to omit part or all of the remainder of this chapter, except perhaps § 79.

What is required is the value of T_2 such that

$$\int_{600}^{T_2} C_v \frac{dT}{T} = 3.559$$

Integrating the left side with C_v as given, we get

$$\begin{aligned} \int_{600}^{T_2} C_v \frac{dT}{T} = & 7.536 \ln T_2 - 7.536 \ln 600 + \frac{3.16 T_2}{(10)^4} - \frac{(3.16)(600)}{(10)^4} - \frac{3.28 T_2^2}{(10)^4(2)} \\ & + \frac{(3.28)(600)^2}{(10)^4(2)} + \frac{2027.5}{T_2} - \frac{2027.5}{600} - \frac{(80.2)(10)^4}{2 T_2^2} + \frac{(80.2)(10)^4}{(2)(600)^2} \\ & + \frac{(37.1)(2)}{T^{1/2}} - \frac{(37.1)(2)}{(600)^{1/2}} \end{aligned}$$

The only practical way to find T_2 is to substitute different assumed values of it into the foregoing expression until one is found which gives 3.559, the required value according to equation (y). Getting help for a start as we can we may compute T_2 for the case of constant specific heats thus

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{\gamma-1}} \quad \text{or} \quad T_2 = (600)(6^{\frac{1}{1.4}}) = 1228^\circ\text{R}$$

Since k decreases with temperature this value is necessarily greater than the answer. One might also compute T_2 for some value of k which is certain to be below its true

Table III IDEAL GAS FORMULAS, CONSTANT SPECIFIC HEATS

Process →	Constant volume $V = C$	Constant pressure $p = C$	Isothermal $T = C$	Isentropic $S = C$	Polytropic $pV^n = C$
p, V, T relations	$\frac{T_2}{T_1} = \frac{p_2}{p_1}$	$\frac{T_2}{T_1} = \frac{V_2}{V_1}$	$p_1 V_1 = p_2 V_2$	$p_1 V_1^\gamma = p_2 V_2^\gamma$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $= \left(\frac{p_2}{p_1}\right)^{(1-\gamma)/\gamma}$	$p_1 V_1^n = p_2 V_2^n$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $= \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$
$\int_1^2 p dV$	0	$p(V_2 - V_1)$	$p_1 V_1 \ln \frac{V_2}{V_1}$	$\frac{p_2 V_2 - p_1 V_1}{1 - \gamma}$	$\frac{p_2 V_2 - p_1 V_1}{1 - n}$
$-\int_1^2 V dp$	$V(p_1 - p_2)$	0	$p_1 V_1 \ln \frac{V_2}{V_1}$	$\frac{k(p_2 V_2 - p_1 V_1)}{1 - k}$	$\frac{n(p_2 V_2 - p_1 V_1)}{1 - n}$
$U_2 - U_1$	$u c_v (T_2 - T_1)$	$u c_p (T_2 - T_1)$	0	$u c_v (T_2 - T_1)$	$u c_v (T_2 - T_1)$
Q	$u c_v (T_2 - T_1)$	$u c_p (T_2 - T_1)$	$\frac{p_1 V_1}{J} \ln \frac{V_2}{V_1}$	0	$u c_v (T_2 - T_1)$
n	∞	0	1	k	$-\infty$ to $+\infty$
Specific heat, c	c_v	c_p	∞	0	$c_n = c_v \left(\frac{k - n}{1 - n}\right)$
$H_2 - H_1$	$u c_p (T_2 - T_1)$	$u c_p (T_2 - T_1)$	0	$u c_p (T_2 - T_1)$	$u c_p (T_2 - T_1)$
$S_2 - S_1$	$u c_v \ln \frac{T_2}{T_1}$	$u c_p \ln \frac{T_2}{T_1}$	$\frac{u R}{J} \ln \frac{V_2}{V_1}$	0	$u c_v \ln \frac{T_2}{T_1}$

value. Then, a range is defined between which the answer should be found. After several guesses, we find for $T_2 \approx 1180^\circ\text{R}$ that

$$\int_{600}^{1180} C_v \frac{dT}{T} = 3.559,$$

which is the desired value; therefore $t_2 = 1180 - 480 = 720^\circ\text{F}$.

79. Closure. Thermodynamics is exacting but fascinating. The beginner is prone to depend unduly on memory for details with which to solve problems, whereas it is easier and more profitable to classify the knowledge in such a way as to see the repetitive utility of the more basic laws. For example, in any nonflow process, the law of conservation of energy is expressed by

$$dW = dQ - dU,$$

where $dU = wc_v dT$ when $pv = RT$; if the process is reversible, $dW = p dV/J$ and $dQ = wc dT$, where c depends upon the kind of process.

In the steady flow process ($\Delta P = 0$),

$$dW = dQ - dH - \Delta K,$$

where $dH = wc_p dT$ when $pv = RT$; dQ is as stated for the nonflow process, and $dW = -\int V dp/J = dQ - dH$ when $dK = 0$. The tabulation of equations in Table III is for the purpose only of highlighting many similarities and emphasizing differences.

Recall that energy is a scalar quantity, so that should there be two or more successive processes, the total work will be the algebraic sum of the works for the individual processes, and the heat transferred will be the algebraic sum of the heats of the processes. This brings us to cycle analysis, Chapter 6.

that is, without subscript, W applied to a cycle is understood to mean the net work. If it is negative, it is the net input work.

Equation (46) embodies the classical definition of the first law (§ 24), namely that work and heat are mutually convertible. Arriving at equation (46) as we have, we see the necessary conditions in order for the first law, as stated in this brief form, to hold. *If there is no change in stored energy during cyclic operation of a system, which may be flow or nonflow, the net amount of energy crossing the boundary as heat is equal to the net amount of energy crossing the boundary as work.*

Note in passing that when the working substance is taken as the system, the net work does not involve the losses from mechanical friction in bearings, etc. It is the fluid work. For a power generating machine, the actual shaft work is less than W , and for a power-consuming machine, the actual shaft work is greater than W , the difference is a loss due to mechanical and fluid friction.

We may and do say

$$W = \Sigma Q \quad \text{and} \quad W = Q_A - Q_R,$$

[ALGEBRAIC SUM] [ARITHMETIC DIFFERENCE]

meaning the same thing in each case. If you use the various equations for process heat, as derived in this book, the result will automatically be positive when heat is added and negative when heat is rejected. When we write $W = \Sigma Q$, we mean an algebraic sum where rejected heats have negative signs. If we write the rejected heat as a positive number then it must be subtracted from the heats added, $W = Q_A - Q_R$. There is no difference, simply different ways of saying the same thing. Since both viewpoints are handy the reader should master both.

✓ 83 **Thermal Efficiency** Efficiency may well be thought of in its simplest form, *output divided by input*. The output of a power cycle, which is a thermodynamic cycle for the production of power is the net work, the input is the heat added to the working substance from an external source of

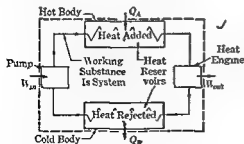


Fig 41 Power Cycle $Q_A > Q_R$

heat Q_A in Fig 41. The net work $W_{\text{net}} = W_{\text{out}} - W_{\text{in}}$ is taken as the output of the power cycle because some of the gross work W_{out} is used to supply the energy W_{in} to drive the pump. Therefore, the thermal efficiency of a power cycle is

$$(47) \quad e = \frac{W}{Q_A} = \frac{Q_A - Q_R}{Q_A} = \frac{\Sigma Q}{Q_A}$$

Observe that equation (47) applies only to a cycle. However, if the cycle is performed wholly within the engine, as we shall imagine for ideal internal combustion engines, then equation (47) gives the efficiency of the engine as well as that of the cycle.

of equation (48) that for the particular temperature limits of operation, it is the highest conceivable thermal efficiency. Equation (48) is the efficiency of the Carnot and other reversible cycles only. Most of the cycles which we shall study are not externally reversible ones.

85. Carnot Cycle with Ideal Gas as the Working Substance. While the rectangle represents the Carnot cycle on the TS plane, the cycle cannot be shown on the pV plane without a knowledge of the properties and the equation of state of the substance. We are familiar with the ideal gas whose equation of state is $pV = wRT$. The pV equation of the isothermal process is $pV = C$ (§ 69); for the isentropic process, it is $pV^k = C$ (§ 70). With known constants, these equations may be plotted to get the cycle. (See Fig. 44.) It will be instructive to analyze the Carnot cycle for an ideal gas, because the method of attack is the same for all cycles.

(a) *Work from the TS Plane.* By this expression, we mean to find an equation for work as ΣQ , that is in terms of heats. For the isentropics, $Q = 0$; for the isothermals, $Q = (wRT/J) \ln (V_2/V_1)$, § 69. Applying this basic form to the process ab (Figs. 42 and 44) and to the process cd , we have

$$Q_A = \frac{wRT_1}{J} \ln \frac{V_b}{V_a}, \quad Q_R = \frac{wRT_2}{J} \ln \frac{V_d}{V_c},$$

$$W = \Sigma Q = \frac{wRT_1}{J} \ln \frac{V_b}{V_a} + \frac{wRT_2}{J} \ln \frac{V_d}{V_c},$$

$$(e) \quad W = \frac{wRT_1}{J} \ln \frac{V_b}{V_a} - \frac{wRT_2}{J} \ln \frac{V_c}{V_d}.$$

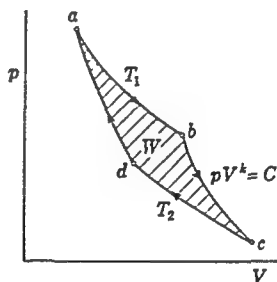


Fig. 44. Carnot Cycle, Ideal Gas. Not to scale. See Fig. 46.

From equation (35) for an isentropic change of state,

$$\frac{V_c}{V_b} = \left(\frac{T_b}{T_c} \right)^{1/(k-1)} = \left(\frac{T_1}{T_2} \right)^{1/(k-1)} \quad \text{and} \quad \frac{V_d}{V_a} = \left(\frac{T_a}{T_d} \right)^{1/(k-1)} = \left(\frac{T_1}{T_2} \right)^{1/(k-1)}$$

whence

$$(f) \quad \frac{V_c}{V_d} = \frac{V_b}{V_a}.$$

Substituting this value of V_c/V_d in equation (e), we find

$$(g) \quad W = (T_1 - T_2) \frac{wR}{J} \ln \frac{V_b}{V_a} \text{ Btu.}$$

which is represented by the enclosed area on the TS plane. The thermal efficiency is

$$e = \frac{W}{Q_A} = \frac{(T_1 - T_2)(wR/J) \ln (V_b/V_a)}{(wR/J) T_1 \ln (V_b/V_a)} = \frac{T_1 - T_2}{T_1},$$

which, as it should be, is the same result as that given by equation (48).

(b) *Work from the pV Plane* We recall that areas under the curve on the pV plane represent the work of the reversible nonflow process ($\int p \, dV$). The net work of a series of processes is the algebraic sum of the works of the individual processes. These work quantities are positive when the state point has a component motion toward the right, as abc (Fig. 45), negative when there is a component motion of the state point toward the left. The result is that the sum, the net work, is represented by the enclosed area on the pV plane in foot-pound units ($abcd$, Fig. 45). Applying the equations for work found in the preceding chapter,

$$W_T = p_1 V_1 \ln \frac{V_2}{V_1} \quad \text{and} \quad W_S = \frac{p_2 V_2 - p_1 V_1}{1 - k},$$

[ISOTHERMAL] [ISENTROPIC]

to this particular cycle (Fig. 44 or 45) we get $W = \Sigma W$,

$$(h) \quad W = p_a V_a \ln \frac{V_b}{V_a} + \frac{p_c V_c - p_b V_b}{1 - k} + p_c V_c \ln \frac{V_d}{V_c} + \frac{p_a V_a - p_d V_d}{1 - k}$$

Equations (d), (g), and (h) should reduce to identical forms (except for units), because each represents the work of the Carnot cycle. For example, in (h), note that $p_b V_b = p_a V_a$ (Boyle's law) and $p_c V_c = p_d V_d$. The work terms of the isentropics are therefore equal but of opposite sign and cancel each other. Also as already proved $V_b/V_a = V_c/V_d$. Therefore, with $pV = wRT$, equation (h) becomes

$$(i) \quad W = wRT_1 \ln \frac{V_b}{V_a} - wRT_2 \ln \frac{V_b}{V_a} = (T_1 - T_2)wR \ln \frac{V_b}{V_a} \text{ ft lb,}$$

the same as equation (g) except for units. The student should show that for an ideal gas, equations (d) and (g) are the same.

Even though there is no practical significance to the Carnot engine, the cycle itself is most important as the measure of the *limit* of efficiency for other ideal or actual cycles. It also serves to establish an absolute zero temperature that is *absolute in the sense that no lower temperature can be conceived* (§ 96).

86 Mean Effective Pressure Nonflow cycles of which the Carnot is an illustration take place within a cylinder, work being obtained through the action of a variable pressure on a movable piston. The work done during the cycle is represented by the enclosed area on the pV plane $abcd$ (Fig. 45). This area may be reproduced in the form of a rectangle, area $efgh$ = area $abcd$. The length of the rectangle is a volume V_D , called the *displacement volume*, which is seen to be the volume swept by the piston in one stroke (Fig. 43), $V_D = V_c - V_a$. The height of the rectangle is a pressure p_m , called the *mean effective pressure*, and known in abbreviated form as the *mep*. Since the rectangle is the same area as the cycle area, it too represents

W to some scale; hence, the mep is that average pressure which, acting through *one stroke*, will produce on the piston the net work of a *single cycle*. In equation form, the mep is

$$(49) \quad p_m = \frac{W}{V_D}$$

$$= \frac{\text{work/cycle (say, ft.-lb.)}}{\text{displacement/stroke (ft.}^3\text{)}} \rightarrow \frac{\text{lb}}{\text{ft.}^2},$$

or $W = p_m V_D.$

This equation shows that the higher the mean effective pressure p_m , the greater is the work for a particular size of engine (a given V_D). Inasmuch as cost is some function of the amount of material used, and because mere bulk is usually a disadvantage, a high mep is desired. Consequently, it is doubtful that a Carnot engine which operates on a gas would be useful in practice even if it could be made operable, because the mep corresponding to reasonable expansions and compressions is so low that the size of the engine as compared with modern engines would be excessively large. [See § 89, part (h).] Furthermore, a comparison of the mean effective pressure with the maximum pressure of the cycle is revealing. The various mechanical parts of an engine must be designed for the maximum forces coming upon them, forces which will be related to the maximum pressure. So, even if the mep of a cycle, any cycle, is relatively high, the cycle may be impracticable if the maximum pressure is too great as compared with the average net pressure (mep), because the large maximum pressure necessitates massive parts for the engine. Thus, the mean effective pressure is a significant characteristic of cycles from a practical point of view.

While the units of equation (49) must be consistent, they need not be on the basis of one cycle. More often, it is convenient to base a computation on the number of cycles completed per minute N , in which case the work is in, say, foot-pounds per minute. In order to match foot-pound per minute, the displacement volume must be in cubic feet per minute (cfm), obtained from

$$V_D \text{ cfm} = \left(\begin{array}{l} \text{volume in cu. ft. swept} \\ \text{by piston in one stroke} \end{array} \right) \times \left(\begin{array}{l} \text{number of cycles} \\ \text{completed per min.} \end{array} \right).$$

Also, the displacement is obtained from the dimensions of the engine,

$$(j) \quad V_D = \frac{\pi D^2}{4} L \text{ cu. ft.,} \quad \text{or} \quad V_D = \frac{\pi D^2}{4} L N \text{ cfm,}$$

[ONE CYCLE] [N CYCLES/MIN.]

where D is the diameter of the cylinder and L the length of stroke, both in feet, and N is the number of cycles completed in one minute. To be safe

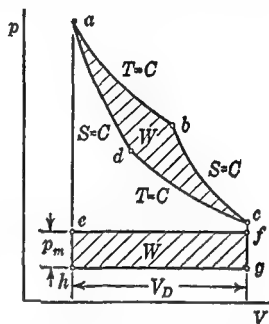


Fig. 45. Mean Effective Pressure.

in regard to units, always use the work in foot pounds and the volume in cubic feet, then convert the resulting pounds per square foot to pounds per square inch, the unit engineers usually use. The mep of the Carnot cycle is the value of W from equation (1) in foot pound units divided by V_D , or

$$p_m = \frac{(T_1 - T_2)wR \ln (V_b/V_a)}{V_b - V_a} \quad [\text{CARNOT}]$$

87 Ratio of Expansion, Ratio of Compression, Pressure Ratio The *ratio of expansion* during any particular process is the ratio of the volume V_2 at the end of the expansion divided by the volume V_1 at the beginning of expansion. For instance, the ratio of isothermal expansion (or the isothermal expansion ratio) in Fig 46 is V_2/V_1 , the ratio of isentropic expansion is $r_e = V_3/V_2$, the over-all ratio of expansion is V_3/V_1 .

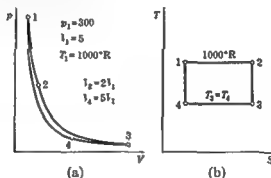


Fig 46 The enclosed areas on each plane represent work. The unit of pV areas is ft lb, of TS areas, Btu. Since the Btu is 778 times larger than the foot pound it would be inconvenient to show these areas to relative size. This pV diagram is drawn approximately to scale for the data in § 89. Notice that the enclosed area is relatively long and slender.

The *ratio of compression* is the ratio of the initial volume V_1 divided by the final volume V_2 . For instance the isentropic compression ratio in Fig 46 is $r_c = V_4/V_1$. Observe that each of these ratios is greater than one.

The *pressure ratio* is also usually written as a number greater than one, and is therefore the highest pressure divided by the lowest pressure of a particular process, p_2/p_1 for the isentropic process 2-3 (Fig 46).

88 Analysis of Cycles In the analysis of gas cycles we shall be particularly interested in (1) the heat supplied to the cycle, (2) the heat rejected (3) the net work, (4) the efficiency, and (5) the mean effective pressure. Other items will command our attention in special cases. All ideal cycles are composed of internally reversible processes, so that $\int p dV$ is nonflow work, $dQ = c dT$, and $\Delta s = \int dQ/T$.

89 Example The working substance of a Carnot cycle is air, considered as an ideal gas. At the beginning of isothermal expansion, we find $p_1 = 300$ psia, $V_1 = 5$ cu ft, and $t_1 = 540^\circ\text{F}$. The ratio of isothermal expansion is $r_1 = 2$ ($= V_2/V_1 =$

V_3/V_4) and the isentropic compression ratio $r_k = 5$ ($= V_4/V_1 = V_3/V_2$). Find (a) the sink temperature and the pressure at each corner of the cycle, (b) the change of entropy during an isothermal process, (c) the heat supplied to the cycle, (d) the heat rejected, (e) the horsepower developed if the volume V_1 is 5 cfm, (f) the efficiency, (g) the volume V_3 and the over-all ratio of expansion, (h) the mep.

SOLUTION. (a) See Fig. 46. For $T_1 = 540 + 460 = 1000^\circ\text{R}$, $k = 1.4$, and for $r_k = V_4/V_1 = 5$, we get

$$T_4 = T_1 \left(\frac{V_1}{V_4} \right)^{k-1} = \frac{1000}{5^{0.4}} = 525^\circ\text{R},$$

or $t_4 = t_2 = 65^\circ\text{F}$. The pressures are (Fig. 46),

$$p_4 = p_1 \left(\frac{V_1}{V_4} \right)^k = \frac{300}{5^{1.4}} = 31.6 \text{ psia};$$

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^k = \frac{300}{2} = 150 \text{ psia};$$

$$p_3 = p_2 \left(\frac{V_2}{V_3} \right)^k = \frac{150}{5^{1.4}} = 15.8 \text{ psia}.$$

(b) The change of entropy is

$$\Delta S = \frac{wR}{J} \ln \frac{V_2}{V_1} = \frac{p_1 V_1}{J T_1} \ln \frac{V_2}{V_1} = \frac{(300)(144)(5)}{(778)(1000)} \ln 2 = 0.1925 \text{ Btu}/^\circ\text{R},$$

an increase for 1-2, a decrease for 3-4.

(c) The heat supplied during 1-2 is

$$Q_{1-2} = \frac{wRT_1}{J} \ln \frac{V_2}{V_1} = (S_2 - S_1)T_1 = (0.1925)(1000) = 192.5 \text{ Btu}.$$

(d) The heat rejected during 3-4 is

$$Q_{3-4} = \frac{wRT_3}{J} \ln \frac{V_4}{V_3} = -(S_2 - S_1)T_2 = -(0.1925)(525) = -101 \text{ Btu}.$$

(e) The work is ΣQ , or

$$W = 192.5 - 101 = 91.5 \text{ Btu}.$$

for 5 cu. ft. If we consider, for example, that one cycle is completed each minute with $V_1 = 5$ cu. ft., the work is 91.5 Btu/min. and $\text{hp} = 91.5/42.4 = 2.16$. See Appendix for conversion constants.

(f) The efficiency is W/Q_A or $(T_1 - T_2)/T_1$, or

$$e = \frac{91.5}{192.5} = 47.5\%, \quad \text{or} \quad e = \frac{1000 - 525}{1000} = 47.5\%.$$

(g) For the volume V_3 , we have

$$V_2 = 2V_1 = 10 \text{ cu. ft.}, \quad \text{and} \quad V_3 = 5V_2 = 50 \text{ cu. ft.}$$

The over-all ratio of expansion is

$$r = \frac{V_3}{V_1} = \left(\frac{V_3}{V_2} \right) \left(\frac{V_2}{V_1} \right) = (5)(2) = 10.$$

(h) The mean effective pressure is

$$p_m = \frac{W}{V_D} = \frac{(778)(91.5)}{(50 - 5)(144)} = 11 \text{ psi}$$

As practical engines go this is a low mep characteristic of Carnot engines. In general a Carnot cycle encloses an area on the pV plane that is small for its pressure range (Fig. 46). Refer to § 86 for the significance of this low pressure.

90 The Ericsson Cycle * There are several other reversible cycles (§ 91) (conceivably reversible that is) but we shall describe only one other the Ericsson cycle. The reason for doing this is that the Ericsson cycle incorporates a feature of particular importance in large modern, steam power plants and in other cycles a *regenerative effect*.

Knowing that a reversible cycle is the most efficient Ericsson set out to design an air engine to operate on such a cycle. As in the Carnot cycle heat is added and rejected at constant temperatures so that the transfer of heat can theoretically occur with a temperature difference ΔT which approaches zero and reversibility. In Fig. 47 heat is received from an external source at T_1 along ab . The hot air then leaves the cylinder and passes through a *regenerator* while the pressure remains constant bc . The regenerator is a chamber in which energy can be transferred from the gas and stored. In practice a regenerator of the kind intended here may contain a labyrinth of brickwork or it may be filled with mesh wire. One end of the regenerator is at a temperature T_1 and the other end at T_2 and there is a gradual temperature gradient from one end to the other.

To get the energy into storage reversibly we imagine the gas entering the regenerator at a temperature $T_1 + \Delta T$ [Fig. 47(c)] so that heat begins to be transferred to the contents of the regenerator at A . As the gas passes through the regenerator its temperature drops gradually to $T_2 + \Delta T$ at the exit B and at all points in the regenerator the gas temperature is higher than the adjacent content temperature by an amount ΔT which is zero in the ideal reversible case.

* John Ericsson (1803-1889) born a Swede methodical stubborn irascible designer and inventor extraordinary had a passion for machinery. In England he built a locomotive to compete with Stephenson's Rocket. Although the Rocket won the prize on tractive effort and dependability Ericsson's locomotive established the amazing speed record of 30 mph. The *London Times* said: "It seemed indeed to fly presenting one of the most sublime spectacles of human ingenuity and human daring the world ever beheld." Because he was unable to interest anyone in England in his screw propeller—a substitute at the time for paddle wheels—he came to the United States to promote it. He is the designer of the *Monitor* which defeated the Confederate iron-clad the *Merrimac*. He designed and built a hot air engine for a boat launched in 1853 sunk shortly afterwards in a squall. The weight of the engine for the power developed was tremendous and the boat was not capable of carrying it in rough weather. He was such a prolific producer that space does not permit even a list of his inventions and designs. During the last few years of his life he was a recluse in New York City irate at the elevated and disbeliever in the telephone.

After this storage operation, the gas rejects heat at constant temperature to an external receiver, the sink; that is, there is an isothermal compression of the gas cd . Now this gas reenters the regenerator at B [Fig. 47(c)] at a temperature $T_2 - \Delta T$, and being slightly cooler than the contents, it receives heat and continues to receive heat until the gas leaves at a temperature $T_1 - \Delta T$ at A . As in the previous passage in the opposite direction, there is always a temperature difference of ΔT between the gas and adjacent contents. In the limit, as ΔT becomes zero, the whole operation becomes reversible. It is essential to observe that the heat picked up by the gas in moving from B to A [Fig. 47(c)] is the same as that stored when the gas passed through from A to B . This heat does not involve an external source;

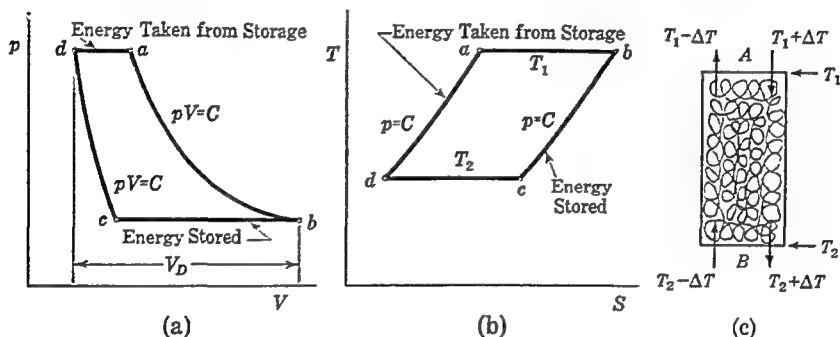


Fig. 47. *Ericsson Cycle.* This cycle consists of two isothermal processes ab and cd and two constant pressure processes bc and da .

it is an interchange of heat *within* the system. Now if the source of heat and the sink are at temperatures T_1 and T_2 , respectively, all processes are reversible *externally and internally*.

Thus, with a system consisting of the gas and the regenerator, no matter crosses the boundaries, and the energy crossing consists of Q_A , Q_R , and W .

$$Q_A = \frac{wRT_1}{J} \ln \frac{V_b}{V_a} \text{ Btu, along } ab.$$

$$Q_R = \frac{wRT_2}{J} \ln \frac{V_d}{V_c} = -\frac{wRT_2}{J} \ln \frac{V_c}{V_d} \text{ Btu, along } cd.$$

From Charles' law, we have

$$\frac{V_c}{V_b} = \frac{T_2}{T_1} \quad \text{and} \quad \frac{V_d}{V_a} = \frac{T_2}{T_1} \quad \text{or} \quad \frac{V_c}{V_d} = \frac{V_b}{V_a}.$$

Using this relation, we find the work to be

$$(k) \quad W = \Sigma Q = (T_1 - T_2) \frac{wR}{J} \ln \frac{V_b}{V_a} \text{ Btu,}$$

and the thermal efficiency is

$$e = \frac{W}{Q_A} = \frac{T_1 - T_2}{T_1},$$

which is the same as that of the Carnot cycle, the highest possible efficiency for any working substance operating between temperatures T_1 and T_2 .

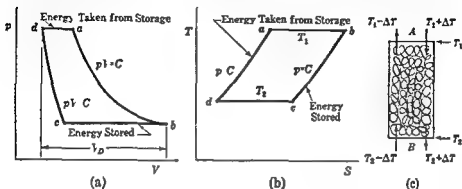


Fig 47 Repeated

The work may also be taken from the $p\bar{v}$ plane by algebraically summing the work quantities for the processes of the cycle

$$(l) \quad W = p_a V_a \ln \frac{V_b}{V_a} + p_b (V_c - V_b) + p_c V_c \ln \frac{V_d}{V_c} + p_d (V_a - V_d),$$

in foot pounds. The student should check this equation with Fig 47 and show that it is the same as equation (k), except for units. The mean effective pressure is

$$(m) \quad p_m = \frac{W}{V_D} = \frac{(T_1 - T_2) w R \ln (V_b/V_a)}{V_b - V_a} \text{ psf,}$$

where $V_D = V_b - V_a$. Ericsson engines have been used to a limited extent but the mep is too low for it to be a competitor of modern reciprocating engines.

The *Stirling cycle* is a regenerative cycle similar to the Ericsson cycle, except that it is composed of two isothermal and two constant volume processes, the regenerative action occurring during the constant volume processes.

91 Reversible Cycle In a reversed cycle, the net work is an energy input, the net heat is rejected $Q_R > Q_A$ and the cycle is a *heat pump*. We refrained from defining a reversible cycle until the reader had a chance to learn something about cycles. Now we may say that a *reversible cycle* is one composed of processes which are reversible internally and externally; there is no friction and heat is transferred with an infinitesimal temperature drop.

The consequences of this definition are that heat is *received* while the working substance is at the same constant temperature as the source; heat is *rejected* while the working substance is at the same constant temperature as the sink (ΔT nearly zero), except as *reversible* heat interchanges occur within the system in a regenerator. A process which receives heat from an external source while its temperature varies can be made externally reversible with the use of reversible engines, but if such a process is part of a cycle, the cycle will not qualify as a reversible cycle as we have defined it. Any cycle with such a process (exclusive of reversible engines, which may be conceivably included) will have a lower thermal efficiency than $(T_1 - T_2)/T_1$. The following discussion will explain further.

92. The Reversed Carnot Cycle. If a cycle is a reversible cycle, the energy effects all take place in the opposite sense, but the paths traversed

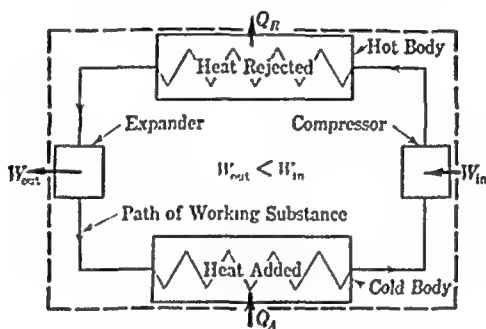


Fig. 48. Reversed Cycle—Heat Pump. This figure, with certain devices not shown, represents a reversed cycle. Note that the work is supplied to the system, that heat is rejected at the higher temperature, and that heat is added at the lower temperature. The net work is $W = W_{out} - W_{in} = \Sigma Q = Q_A - Q_R$, a negative number which indicates that work is done on the working substance.

by the state point are identical with the paths before the cycle was reversed. Customarily, in a *power cycle*, the state point is pictured as moving in a clockwise sense on the pV and TS planes. In a reversed cycle, the state point conventionally moves in a counterclockwise sense. We can reverse cycles, make them go counterclockwise, which are not reversible in the defined sense. For instance, cycles which receive and reject heat while the temperature of the working substance varies can be made to "run" in either direction. But since the external irreversibility is inevitable in a natural heat transfer with varying temperature, such a cycle, though reversed, is not a reversible cycle, which is ideal in *all* respects.

Reversed cycles are used for two purposes: (1) to provide a cooling effect (a refrigerating machine) and (2) to provide a heating effect (a heat pump, Chapter 14). If the system receives work from the outside, heat can be made to flow into the system from the cold reservoir and flow from the

that the assumption that engine *I* is more efficient than engine *R* is absurd and impossible

Engine *I* could be taken as another reversible engine as easily as not, and the same sort of reasoning as above would lead to the conclusion that *one reversible engine cannot be more efficient than another when both engines operate between the same temperature limits all reversible engines operating between the same temperature limits have the same thermal efficiency* namely $(T_1 - T_2)/T_1$

94 Example Suppose a cycle is composed of the following processes from $p_1 = 20$ psia and $t_1 = 200^\circ\text{F}$ 1 lb/sec of air is compressed polytropically with $pV^{1.2} = C$ until $p_2 = 100$ psia it is then cooled at constant pressure until $t_3 = t_1$ an isothermal process completes the cycle Determine (a) the heat added (b) the heat rejected (c) the work and horsepower and (d) the mep (e) Write the equation for the work from the pV plane in terms of the state names in Fig 51

SOLUTION This example illustrates the kind of knowledge you should have about analyzing cycles The cycle defined is probably like none which has ever been used but it is easy to analyze it The first thing to do is to construct the cycle (freehand) on the pV and TS planes This you should do from the problem

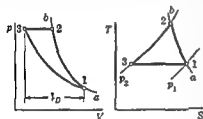


Fig 51

statement alone before studying Fig 51 A polytropic compression curve slopes upward toward the left on the pV plane and also on the TS plane when $1 < n < \lambda$ (Fig 39 p 85) Therefore such a curve ab can be drawn of indefinite extent on both planes (Fig 51) Now draw a constant pressure line of indefinite extent intersecting the polytropic This locates point 2 Now the problem states that cooling occurs from 2

therefore the volume and entropy decrease and point 3 is located somewhere as shown relative to point 2 The final process is one in which $T = C$ so this curve is drawn through 3 Where it intersects the polytropic ab is located point 1 It usually does not matter that the various points in the cycle are not located to scale We now see that positive Q (heat added) occurs along 3-1 negative Q along 1-2 and 2-3—see the TS plane and the component motion of the state point right and left Likewise there is positive work during 3-1 and negative work during 1-2 and 2-3 The cycle goes counterclockwise (W should be negative) and is therefore a reversed cycle (but not a reversible one)

To find the items asked for we shall need (cfs = cu ft/sec)

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = (660) \left(\frac{100}{20} \right)^{0.2/1.2} = 863^\circ\text{R}$$

$$V_2 = \frac{wRT_1}{p_2} = \frac{(53.3)(660)}{(20)(144)} = 12.2 \text{ cfs} \quad V_1 = V_2 \left(\frac{p_2}{p_1} \right) = 12.2 \left(\frac{20}{100} \right) = 2.44 \text{ cfs}$$

(a) The heat added along 3-1 is

$$Q_A = p_1 V_1 \ln \frac{V_2}{V_1} = \frac{(20)(144)(12.2)}{778} \ln \frac{12.2}{2.44} = 72.6 \text{ Btu/sec}$$

(b) The heat rejected is the sum of ($w = 1 \text{ lb./sec.}$)

$$\begin{aligned} Q_{1-2} &= wc_p \left(\frac{k-r}{1-r} \right) (T_2 - T_1) \\ &= (0.1715) \left(\frac{1.4-1.2}{1-1.2} \right) (863 - 660) = -34.8 \text{ Btu sec.} \\ Q_{2-3} &= wc_p (T_3 - T_2) = 0.24(660 - 863) = -48.7 \text{ Btu/sec.} \\ Q_2 &= -34.8 - 48.7 = -83.5 \text{ Btu sec.} \end{aligned}$$

(c) The work is ΣQ , or

$$\begin{aligned} W &= 72.6 - 83.5 = -10.9 \text{ Btu/sec.} \\ hp &= \frac{10.9 \times 60}{42.4} = 15.4 \text{ hp.} \end{aligned}$$

The negative sign for W indicates that this much work must be supplied to the system to keep it running.

(d) The map is

$$p_r = \frac{W}{V_D} = \frac{W}{V_1 - V_2} = \frac{10.9 \times 778}{(12.2 - 2.44)(144)} = 6.03 \text{ psi.}$$

Observe how low this pressure is.

(e) The work equation in terms of the $\Sigma p dV$ around the cycle is often useful. We should be able to write it easily whenever the cycle is defined, and although it serves no particular purpose in this problem, we shall write it for the practice. For Fig. 51, we have

$$W = \frac{p_2 V_2 - p_1 V_1}{1-k} + p_1 (V_2 - V_1) + p_2 V_2 \ln \frac{V_1}{V_2} \text{ ft.-lb.,}$$

which is the algebraic sum of the work quantities. The signs are automatically negative where they should be when numbers are inserted. The value of W obtained from the foregoing equation should check with that in part (c), when expressed in the same units. In important calculations, all available checks should be made.

95. External Irreversibility. You have probably observed that the analyses of cycles is on the basis of internal reversibility. Suppose a Carnot engine receives heat from a source at T_F while the temperature in the engine is T_1 , where $T_F > T_1$ (Fig. 52). Similarly, the system at T_2 discharges heat to a sink at T_c , where $T_2 > T_c$. The efficiency of the engine is $(T_1 - T_2)/T_1$, as usual. But if the heat were transferred without temperature drop from T_F to T_1 and from T_2 to T_c , the efficiency would be $(T_F - T_c)/T_F$, which, even without numbers inserted, is seen to be greater than before.

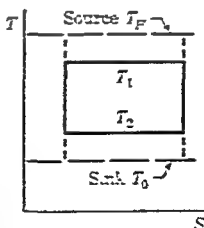


Fig. 52. External Irreversibility.

96. Thermodynamic Temperature. The absolute temperature that we have been using is a thermodynamic temperature, which is independent

of the thermometric substance. It is an energy scale of temperature deduced by Lord Kelvin*. Suppose a quantity of heat Q_1 is supplied to a Carnot engine A (Fig 53) at a temperature level θ_1 on some unknown scale. Heat Q_2 is rejected at temperature θ_2 . Since the Carnot cycle is a rectangle on the θ S plane, we see from Fig 53 that

$$(o) \quad \frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}$$

Let the heat Q_2 be rejected by engine A to another Carnot engine B (Fig 53), which then discharges heat Q_3 at θ_3 . As before,

$$\frac{Q_2}{Q_3} = \frac{\theta_2}{\theta_3},$$

and so on for any number of engines. If each engine A, B, C , etc. does the same work, then $\theta_1 - \theta_2 = \theta_2 - \theta_3$, etc. Thus we may define a degree of temperature as the temperature change of the working substance in a Carnot engine under specified conditions as illustrated by Fig 53. These conditions might be adjusted so that any desired number of reversible engines are interposed between temperature level θ_1 and that level at

which $\theta_n = 0$.

Fig 53 Kelvin Temperature

Such a scale is based on energy quantities and is independent of the properties of the substance. The efficiency of a Carnot engine on this scale is

$$(p) \quad e = \frac{Q_A - Q_R}{Q_A} = \frac{\theta_1 - \theta_2}{\theta_1} = 1 - \frac{\theta_2}{\theta_1}$$

[ANY CYCLE] [CARNOT]

* William Thomson (Lord Kelvin) (1824-1907) who was a professor of physics at Glasgow University is credited by some as being the greatest English physicist. Certainly he possessed a rare combination of talents. His early education was received from his father, who also was a professor at Glasgow University. As a youth he was robust and an active participant in athletics and student affairs at Cambridge; yet he was most distinguished in his studies and before his graduation at age 21 from Cambridge he had established an enviable reputation in scientific circles by his original contributions. An excellent mathematician, a genius at inventing and designing laboratory apparatus and models, he claimed that he could not understand his own ideas until he saw them at work in models. He contributed most to the science of thermodynamics, having established a thermometric scale of absolute temperatures which is independent of the properties of any gas, having aided in establishing the first law of thermodynamics on a firm foundation, and having stated significantly the second law. He was the inventor of some fifty-six instruments and machines and in addition to all this he was interested in the arts and was himself a musician. He was knighted for his indispensable services in laying the first successful transatlantic cable and later was made a peer, Baron Kelvin of Largs. He vigorously denounced the absurd, ridiculous time-wasting brain-destroying British system of weights and measures, favoring the metric system. He received honorary degrees from nearly every important university in Europe, was elected a member of every foreign academy of science and art. He probably received more honors for and recognition of his achievements during his lifetime than has any other scientist.

as compared to equation (48) already found,

$$(48) \quad e = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}. \quad [\text{CARNOT}]$$

Since each of these equations refers to the efficiency of the same reversible engine,

$$(a) \quad \frac{\theta_2}{\theta_1} = \frac{T_2}{T_1},$$

which is to say that in the previous work with the Carnot cycle, a *thermodynamic scale* of temperature, one where $\theta_1/\theta_2 = Q_1/Q_2$ in a reversible engine, was implicit, although it had not been defined. The temperature degree on the θ scale is not necessarily the same as on the Rankine (or Kelvin) scale, but can be so chosen by assigning the corresponding number of degrees between the freezing and boiling points of water. (Read § 8 again.)

Suppose that a Carnot engine operates between the limits of θ_1 and $\theta_n = 0$ (Fig. 53). Its efficiency would be

$$e = \frac{\theta_1 - \theta_n}{\theta_1} = \frac{\theta_1}{\theta_1} = 100\%.$$

If a Carnot engine discharges heat at some negative value of θ , say, $-\theta'$, then its efficiency would be

$$e = \frac{\theta_1 - (-\theta')}{\theta_1} = \frac{\theta_1 + \theta'}{\theta_1} > 100\%,$$

or some number greater than 100%. If this were true, the engine would be discarding more energy than it received, doing work at the same time; that is, it would be *creating energy*, a violation of the first law of thermodynamics. Thus the energy scale of temperatures is an absolute scale in a special sense—zero on the energy scale is the lowest temperature conceivable, because a lower temperature violates the first law.

Inasmuch as the Carnot engine cannot be used for actual temperature measurements, the question arises as to how thermodynamic temperatures can be determined. Also, considering that temperature is measured through the change of some other property, we might conclude (with Kelvin) that the trick can be accomplished if we use a substance for which p_v is proportional to T . Then, for example, if v is held constant, Δp is proportional to ΔT . Such a substance is an ideal gas, $p_v = RT$. That is, the *ideal gas* temperature scale (Charles' law) and the energy scale are the same. (See § 85.) Although there is no ideal gas, both hydrogen and helium approach ideal, getting closer as the pressure is reduced. First choice for calibration purposes is a constant-volume helium thermometer, with corrections made to give thermodynamic temperatures. Current working limit for gas thermometers is about 2000°F.

In order to calibrate actual temperature-measuring devices, there needs to be agreement on some reference temperatures, guide points along the way. By way of illustration, the following primary temperatures, at standard atmospheric pressure, are rather accurately known (24) boiling point of oxygen, -182.970°C , boiling point of sulfur, 444.60°C , melting point of silver, 960.8°C , melting point of gold, 1063.0°C . A number (some 20 or more) of other secondary temperatures are accepted internationally for example, melting point of mercury, -38.87°C , melting point of tin 231.9°C , melting point of platinum, 1769°C , melting point of tungsten, 3380°C . It may be of interest to know that the lowest temperature so far reached in the laboratory is about 0.01°K .

97 Various Statements of the Second Law of Thermodynamics The second law, like the first, is a consequence of experience and logic. It is founded on the work of Carnot. A macroscopic phenomenon which violates the second law has never been observed hence we say that it is most improbable that a violation will ever be observed. There are so many facets to this law that it is being stated in different ways below (recall Clausius, too, § 93)

Kelvin It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects

Kelvin-Planck (18) It is impossible to construct an engine which while operating in a cycle produces no effects except to do work and exchange heat with a single reservoir [See Fig. 50(c)]

All spontaneous processes result in a more probable state

The net result of any actual process is an increase in entropy of the system and its surroundings considered together (not necessarily an increase in each) This statement and the preceding one are the most general in their implications, but they are not so obviously based on experience.

No actual or ideal heat engine operating in cycles can convert into work all the heat supplied to the working substance. It must discharge some heat into a naturally accessible sink. Because of this aspect, the second law is often aptly referred to as the *law of degradation of energy*.

Certain of the foregoing statements overlap to the extent of saying the same thing in different ways, some apply especially to the production of power from heat, some are more rigorous and general than others, but each one contains a significant idea that the reader should make a part of his store of knowledge. Those who enjoy exercises in logic may prove that if one statement is true, another is necessarily true (1). Several of these aspects of the second law will be discussed further.

98 Available and Unavailable Energy Since one of the more important endeavors of an engineer is to obtain the maximum amount of work from a process or cycle and to use the minimum work in reversed operations, it

behooves us to inquire into conditions which ideally produce the maximum work. We already know what this is for a reversible engine, but inasmuch as most engines do not propose to be reversible, even ideally, we need to set up standards for other situations. From the two expressions of the Carnot efficiency,

$$e = 1 - \frac{T_2}{T_1} \quad \text{and} \quad e = 1 - \frac{Q_R}{Q_A},$$

we write

$$(r) \quad \frac{Q_R}{Q_A} = \frac{T_2}{T_1}, \quad \text{or} \quad Q_R = T_2 \left(\frac{Q_A}{T_1} \right).$$

[CARNOT, OTHER REVERSIBLE CYCLES]

This value of Q_R is the heat that must be discharged by a reversible engine, the most perfect conceivable, operating between T_1 and T_2 . Since no imaginable engine could convert a larger portion of heat into work, we say that this Q_R is **unavailable energy**. None of the unavailable energy can be converted into work—unless a colder sink becomes available. See § 97.

Equation (r) is useful only when the heat Q_A is added at constant temperature. To get a more general expression, let the heat be added in accordance with some internally reversible path ab (Fig. 54) and consider an infinitesimal Carnot cycle 1-2-3-4. The working substance is a closed system. Since the width of this cycle dS is very small, heat is effectively added at constant temperature along 4-1; call this temperature T . The sink at temperature T_2 is some natural heat reservoir, such as the atmosphere, a river, or a lake. In practice, over a short period of time, the sink temperature is constant; that is, the temperature of such a vast reservoir as the atmosphere, the ultimate repository, is not noticeably affected by all the heat discharged from all engines. Let the natural sink temperature be designated by T_0 . Now, in equation (r), let $Q_A = dQ$, $Q_R = dQ_R$, $T_1 = T$, and $T_2 = T_0$, a constant, and get

$$(s) \quad \frac{dQ_R}{T_0} = \frac{dQ}{T} \quad \text{or} \quad Q_R = T_0 \int \frac{dQ}{T}.$$

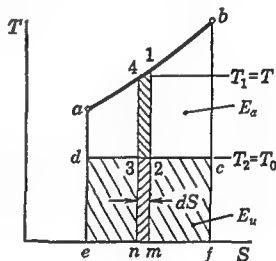


Fig. 54.

In words, this equation says that when heat Q is transferred to a system whose temperature T is variable (or constant), the portion Q_R as defined by equation (s) is the minimum heat that *must* be rejected as heat in case we try to obtain work from the heat Q . For this reason, we call this particular rejected heat the **unavailable energy**. Thus, the unavailable energy E_u may be defined as that portion of any transferred heat which cannot be converted

into work when the heat is used in a frictionless engine for which all processes are ideal. Since the Q_R of equation (s) has this special significance we shall designate it by E_a . Moreover recalling that $\int dQ/T = \Delta S$ for a reversible process (§ 39), we write

$$(50) \quad E_a = T_s \int \frac{dQ}{T} = T_s \Delta S,$$

where ΔS is the system's change of entropy during the transfer of heat Q . Available energy E_a may now be found as the transferred heat minus the unavailable energy *

$$(51) \quad E_a = Q - E_u = Q - T_s \Delta S$$

Available energy is energy which is convertible 100% into work in the absence of irreversibilities, and in this case it is that portion of the heat transferred to a system during an internally reversible process which is available for conversion into work in a closed cycle and its amount is given by equation (51) whether or not the system remains at constant temperature while receiving the heat. After heat has been added ab (Fig 54), the available energy area $abcd$ can be realized as work by completing the cycle with two isentropics bc and da and an isothermal cd . The area $cdef$ represents the unavailable energy. The actual work obtained from any heat Q will always be less than the available energy.

99 Example One pound of air is heated at constant pressure in a closed system from $t_a = 140^\circ\text{F}$ to $t_b = 540^\circ\text{F}$. Determine (a) the heat supplied (b) the change of entropy (c) the unavailable energy for sink temperatures of 40°F and -60°F (d) the value of Q_R/T_s for $t_s = 40^\circ\text{F}$ and for $t_s = -60^\circ\text{F}$ (e) the maximum possible work which might be done by the heat found in (a) when the sink temperature is $t_s = 40^\circ\text{F}$.

SOLUTION (a) The heat added is represented by area $abmn$ (Fig 55). $T_s = 600^\circ\text{R}$ and $T_b \approx 1000^\circ\text{R}$.

$$Q_A = c_p(T_b - T_s) = (0.24)(1000 - 600) = 96 \text{ Btu/lb}$$

$$(b) \quad s_b - s_a = \Delta s = c_p \ln \frac{T_b}{T_s} \\ = 0.24 \ln \frac{1000}{600} = 0.123 \text{ Btu/lb } ^\circ\text{R}$$

* The term *available energy* is often used synonymously with *ideal work of a cycle* but if *available energy* is applied indiscriminately to cycles which are not externally reversible it has a number of meanings. As defined by equation (51) it has one precise and unique meaning with respect to heat Q for a particular sink temperature T_s .

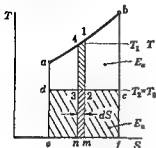


Fig 54 Repeated

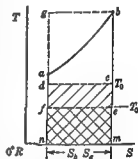


Fig 55 Notice that when the sink temperature is lowered from T_s to T the unavailable energy decreases, the available part increases, conclusion discharge heat at the lowest practicable temperature of the system. Also if all the heat were transferred at the highest temperature, along gb instead of ab , all the extra heat area agb , is available energy conclusion transfer heat while the system is at the highest practicable temperature.

(c) For $t_o = 40^\circ\text{F} = 500^\circ\text{R}$, the unavailable energy is (area $cdnm$)

$$E_u = T_o \Delta s = (500)(0.123) = 61.5 \text{ Btu/lb.};$$

and for $t_o = -60^\circ\text{F} = 400^\circ\text{R}$, (E_u represented by area $efnm$)

$$E_u = T'_o \Delta s = (400)(0.123) = 49.2 \text{ Btu/lb.}$$

(d) For each sink temperature,

$$\frac{Q_R}{T_o} = \frac{61.5}{500} = \frac{49.2}{400} = 0.123 = \Delta s \text{ Btu/lb-}^\circ\text{R}.$$

Note that the heat rejected to the sink divided by the constant sink temperature is always the increase of entropy of the sink.

(e) The maximum possible work which can be obtained from this $Q_A = 96$ Btu is the available energy E_a , which, at $t_o = 40^\circ\text{F}$, is (area $abcd$, Fig. 55)

$$E_a = Q_A - E_u = 96 - 61.5 = 34.5 \text{ Btu/lb.}$$

NOTE. If this air had been heated at constant pressure in an irreversible manner by an electric coil (or by paddle-wheel work), the entropy change of the system would be as computed; because entropy change is dependent on the end states and can be reckoned by an internally reversible path—at constant pressure in this example. However, if the computed entropy change is brought about by heat from a reservoir in the surroundings, the reservoir undergoes a decrease in entropy that offsets to some extent the gain in entropy of the system. If the heating is done with electricity, which is ideally 100% convertible into work, the decrease of entropy of the surroundings is zero, and the gain of entropy of the system is the net gain. The greater the specific entropy gain, the more irreversible the process.

100. Entropy from the Carnot Cycle. It is seen from equations (s) and (50) that by considering the Carnot cycle in terms of differentials, we end up with the expression $\int dQ/T$, and it is this integral for an internally reversible process which was used to define entropy ΔS . We may now demonstrate that entropy is a point function. Write equation (s) in this form,

$$(t) \quad \frac{dQ}{T} - \frac{dQ_R}{T_o} = 0.$$

The integration of (t) may be written (permitting T_o to vary in the general case)

$$(u) \quad \int \frac{dQ}{T} - \int \frac{dQ_R}{T_o} = \sum \int \frac{dQ}{T} = 0, \\ \text{[REVERSIBLE CYCLE]}$$

where $\sum \int dQ/T$ represents the algebraic sum Σ of the integrals around the cycle, which instead of one, may be composed of several processes during which heat is added, and several during which heat is rejected. When the cycle is composed of internally reversible processes, the sum of these integrals is equal to zero, as shown. Since a property is a characteristic which is always the same in a particular state 1, (u) may be accepted as proof that

where ΔS_p is the entropy production during process bc' , and the entire *internal* production of entropy during this particular cycle (Fig. 56), because the other processes are internally reversible.

For a cyclic operation, the entropy of the system returns to its initial value S_a every time a cycle is completed. That is, the *internal* change of entropy is zero for irreversible as well as reversible cycles. The entropy production during a cycle is the growth of entropy of the surroundings. We may say in accordance with the second law that

$$(x) \quad \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \geq 0,$$

where the equals sign applies only when the processes are internally and externally reversible. For all actual processes, therefore, we can write

$$(y) \quad \begin{aligned} \Delta S_p &= \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \\ \Delta S_p &= \Delta S_{\text{system}} + \Delta S_{\text{source}} + \Delta S_{\text{sink}} \end{aligned}$$

where ΔS_p is the net increase in entropy and $\Delta S_{\text{system}} = 0$ for a cycle and the other changes are algebraic (ΔS_{source} is negative when it supplies heat, positive when it receives heat in a reversed cycle). Since for any *process*, the actual change of entropy of the system is $\Delta S' \geq \int dQ/T$, where the equals sign applies for internal reversibility, we may write

$$(z) \quad \Delta S' = \int \frac{dQ}{T} + \Delta S_p,$$

the actual change of entropy, applicable to a process or a series of processes.

If a system's entropy change is a loss, the gain of entropy of the surroundings will be greater than the loss, resulting in a net gain. Thus, entropy, like time, is unidirectional and always increasing, and moreover, *the change of entropy is a measure of the change of unavailable energy* ($T_0 \Delta S$); the greater the entropy change because of irreversibility, the greater the increase of unavailable energy and the greater the loss of available energy. Energy in the form of work is available energy; kinetic energy, potential energy, and electricity are available energies, because in a perfect engine, these forms can be converted 100% into work. Thus, using electricity for heating water, for heating metal for heat treatment, etc., is using the highest grade of energy (100% available) for a job that can be done by low-grade energy. Nevertheless, such use of high-grade energy is often *economic* in industry or is so convenient that the purchaser is willing to pay the extra cost. As implied by the second law, energy may be graded. The energy from a source at high temperature is of higher grade (larger percentage of available energy if used directly in a heat engine) than energy from a source at a low temperature. See example, § 103.

Ultimately, the available energy which is evolved becomes unavailable energy. All actual work produced is eventually dissipated by friction, or the equivalent, and becomes energy in the sink. For example, all the

(e) At 800°R, the available energy, area $efmh$, is

$$E_a = 3200 - T_o \Delta S_{ef} = 3200 - (600)(4) = 800 \text{ Btu/}^\circ\text{R-min.},$$

which is only 25% of $Q = 3200$ as compared with 62.5% before transfer. Another and telling point of view is that the loss of *available energy* is

$$\frac{2000 - 800}{2000} = 60\%$$

of the original available energy, a horrifying percentage. Let's repeat that idea: the process of heat transfer is 100% efficient as far as the first law is concerned (all the heat leaving the source was received by the system), but only 60% efficient with respect to the second law!

These results emphasize the desirability of using energy in an engine at as high a temperature as possible and of transferring heat with as small a temperature difference as practicable. The rapidity with which heat is transferred is proportional to the temperature difference, other things being equal. Hence, the difference must not be too small, because it would then take too long to transfer the required heat (or a very large heat transfer surface would become necessary). Also, it should be easy to visualize from Fig. 57 that the lower the temperature at which real engines discharge heat, the more efficient the engines are likely to be. Many steam power plants are more efficient in the winter than in the summer because the sink temperature T_o is lower in the winter. The lowest economic temperature T_o is one which is naturally at hand. If the discharge temperature T_o were lowered by refrigeration, there would be a net loss because the work to run the refrigeration cycle would be more than the work gained from the power cycle—as the second law says.

104. Example: Heat Exchanger. In a heat exchanger (Fig. 58) 100 lb./min. of water ($c = 1$) are to be heated from 140°F (600°R) to 240°F (700°R) by hot gases ($c_p = 0.25$) which enter this exchanger at 440°F (900°R) and flow at the rate of 200 lb./min. Compute the change of unavailable energy of each fluid for a sink temperature of 40°F (500°R), the net increase in entropy, and the loss of available energy. Consider this heat exchanger as an isolated system with no exchange of energy with the surroundings while the materials flow through.

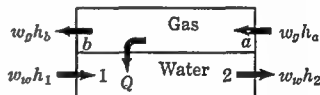


Fig. 58. Energy Diagram, Heat Exchanger.

SOLUTION. The changes in kinetic energies are likely to be negligible and no work is done. Therefore, the energies entering and leaving the system are the enthalpies of the flowing fluid, as shown in Fig. 58. Now we may think of two subsystems, the gas and the water. In accordance with the first law, the heat flow from the gas is numerically equal to the heat flow into the water, and the algebraic sum of these heats is equal to zero. For the water, $\Delta H_w = wc\Delta T$, very nearly—close enough for low-temperature heat exchangers (accurate means of getting Δh for water are given in Chapter 10). For the gas, $\Delta H_g = wc_p\Delta T$. Therefore, we have (Fig. 59)

$$\begin{aligned} Q_{\text{water}} + Q_{\text{gas}} &= 0, \quad \text{or} \quad Q_{\text{gas}} = -Q_{\text{water}} \\ \Delta H_w = Q_{\text{water}} &= wc(T_2 - T_1) = (100)(240 - 140) = 10,000 \text{ Btu/min.} \\ \Delta H_g = Q_{\text{gas}} &= wc_p(T_b - T_a) = (200)(0.25)(T_b - 900) = -10,000, \end{aligned}$$

these equilibrium conditions exist, the system is said to be in *thermodynamic equilibrium* (1) or in the *dead state* (3). A system is in the dead state if it will not spontaneously undergo a process; and its entropy is a maximum.

There are various meanings attached to the word *equilibrium*. We speak of *equilibrium states* of a substance, meaning something which we might call *internal equilibrium*. That is, the substance in a system may not be in thermodynamic equilibrium with the surroundings, yet it may be at an equilibrium state. That is to say the pressure and temperature are the same at all points in the substance; there is no acceleration of the whole or any part of the substance, and the system is chemically inert. Typically, the substance is at rest. This internal equilibrium is an implicit requirement for any reversible process. So, we have been continually dealing with substances at various equilibrium states. Other meanings of *equilibrium* will be taken up as needed.

106. Third Law of Thermodynamics. We have so far dealt only with difference of entropy. Indefinite integrations were made in §§ 65 and 67, but no means of determining the constants of integration were at hand. In most thermodynamic problems, change in entropy from one state to another is sufficient. However, when a chemical reaction occurs, the final substance is different from the initial substance, and in this case, it is often a convenience to have absolute values of entropy; for example, in computing Gibbs function $h - Ts$ before and after the reaction.

Important generalizations come gradually and contributions toward the third law have been made by many: Einstein, Nernst (heat theorem), Boltzmann, Planck (28). The third law may be stated: *At absolute zero temperature, the entropy of a substance in some crystalline form becomes zero.* Since such a statement may be a naïve expectation, there must be experimental verification. Since there are now many experiments which justify the law, it is being used with greater confidence.

107. Perpetual Motion of the Second Kind. A proposed machine which violates the second law of thermodynamics is called a perpetual motion machine of the second kind. If it can be shown that such a machine violates the second law by producing work while taking heat from just one reservoir or if it can be shown that it would produce more work from a given supply of energy than the available energy, the machine will not operate as proposed.

108. Closure. Neither the Carnot nor Ericsson cycles are seriously considered today as a means of manufacturing power. The Carnot cycle is a monumental conception which, as we have seen, has led to important deductions regarding the limitations of any power machine. The Ericsson cycle (and the Stirling) involves an idea, regenerative heating, universally used in large, modern, steam-power plants, although in a form somewhat different from that in Ericsson's engine. Also, in this chapter, we have learned

of the essential elements of any cycle, and we have learned how to analyze cycles. Before proceeding the student should satisfy himself, by working problems concerning other cycles, that this lesson in analysis is mastered. Notwithstanding the visionary or impractical aspect of these cycles in themselves, it should be evident that the study of them should yield results of great practical value.

We write $W = Q_A - Q_R$ or $W = \Sigma Q = Q_A + Q_R$ as may be convenient for a particular purpose, where, in the first instance, Q_R is a positive number, and in the second instance, Q_R is a negative number. Note that both expressions mean the same thing and you will not be confused.

It is one thing to sense the irreversibility of events in general and another to measure the degree of irreversibility, as we are able to do for some events with the use of the concept of entropy. We have noted that whenever a process or cycle deviates from a reversible process or cycle, the amount of work that is delivered decreases below that for reversibility (or the amount of work that must be put into the system for an actual reversed cycle is greater than for the corresponding ideal).

In closing this chapter, we might point out that the thermal efficiency of a machine might be low because of a small temperature range, but good because there is little loss of available energy or it might be high because of a large temperature range, and yet not be so good because of a large loss of available energy.



Courtesy Johnson Publishing Co., Cleveland

Archimedes and His Spiral Pump. The source of power 2000 years ago was man.

COMPRESSION AND EXPANSION OF GASES

109 Introduction Now we move to some applications. Gases at pressures above atmospheric are a common requirement in industry. The compression process is an integral part of refrigeration cycles and gas turbine cycles. Most widely useful is compressed air in operating air engines and pneumatic tools such as hammers and drills, spraying paint, cleaning by air blast, operating air hoists, pumping water by air lift, and in a host of other jobs. Although this discussion is concerned specifically with the compression of a gas, which is the only substance we have so far learned to handle, the basic energy equations and some of the derived ones under specified conditions apply as well to any compressible substance.

110. The Indicator Card. A widely useful means of studying the performance of reciprocating engines (internal combustion engines, compressors, etc.) is the *indicator card* or *indicator diagram*. This "card" is a pictured record of the variation of pressure and volume of the working substance in a cylinder as the piston reciprocates. The record is obtained with an *indicator*, a sample of which is shown in Fig. 61. Other types of indicators are used for high-speed engines. A typical indicator card for an air compressor is shown in Fig. 60. The area of the card is found by using the planimeter, Fig. 62.

Although the indicator card is a closed pressure-volume diagram, it is not a thermodynamic cycle. A compressor diagram shows the details of what is necessary for *one process* in a cycle, picturing the movement of gas into (4-1) and out of (2-3) the cylinder and the re-expansion process (3-4) (Fig. 60). However, the indicator card and the thermodynamic cycle have

one thing in common: *areas represent work*. The work obtained from an indicator diagram is called *indicated work* W_I , which is the work done on the substance in getting it into the cylinder, compressing it, and discharging it from the cylinder.

The valves (Fig. 63) in an air compressor typically operate on a difference in pressure, which explains the waviness in the lines 2-3 and 4-1, Fig. 60. A relatively large pressure difference is necessary to *start* the opening action, because of friction and inertia, so that there is generally an abrupt opening followed by a fluttering. The intake valve does not open until some pressure a little below atmospheric is reached. Then it often flutters, as after 4, Fig. 60, producing the wavy portion of the suction line 4-1. Notice that the intake pressure is slightly below atmospheric. The compression 1-2, which is often close to an adiabatic process, continues until a pressure greater than the delivery pressure is reached, at which point the discharge valve opens. Here again, valve fluttering occurs and the discharge line is wavy.

For most miscellaneous uses, compressed air, though hot on delivery, is cool when used. There is time enough before use for heat to be transferred to the atmosphere, until the compressed air is almost at atmospheric temperature. Since higher-pressure air cannot "hold" as much water vapor as lower-pressure air (temperature the same, Chapter 15), moisture condenses as the air cools. Therefore, in a small system, delivery of air to a *receiver* (Fig. 64), equipped with a drain device is likely to be the situation. In large systems, it will be desirable to let the discharged air flow through an *aftercooler*, which is simply a heat exchanger wherein water is used to cool the air and rid it of excess moisture before it gets into the distribution system. An aftercooler and an intercooler, Fig. 72, are much the same. See § 119 for cases where aftercooling is not desired.

111. Mean Effective Pressure and Ihp. The actual mep is found from the indicator card. The indicator, Fig. 61, is fitted with a spring of a certain scale. If the scale is 100 lb. *in the case of an indicator*, we mean that a vertical movement of 1 in. of the stylus indicates a pressure of 100 psi.* Therefore the actual heights on the indicator card multiplied by the scale of the indicator spring will be the actual pressures in pounds per square inch (neglecting errors inherent in the indicator). Thus, to find the actual

* This statement is true when a standard piston is used in the indicator. If an undersized piston is used, a common practice, the scale stamped on the spring must, of course be modified.

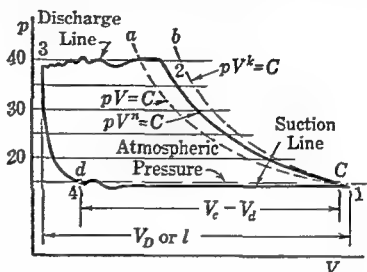


Fig. 60. Actual Indicator Card, Compressor.

mep, find the area of the indicator card in square inches, divide the area by the length of the card l in inches, Fig 60, and obtain the *average height* of card in inches. This average height multiplied by the scale of the indicator spring is the average pressure, called the indicated mep (p_{mi})

$$(a) \quad p_{mi} = \frac{(\text{area of indicator card})(\text{scale of indicator spring})}{\text{length of indicator card}}$$

This is the pressure acting on the face of the piston during one stroke, which results in the same work as produced by the actual pressures throughout all events of the indicator card. Now reread § 86 again

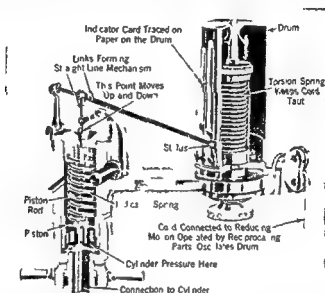


Fig 61 Indicator Steam enters through the connection to the cylinder and acts on a small piston, whose motion is opposed by the indicator spring. The greater the pressure from the cylinder, the more the piston moves. The piston rod actuates the stylus, through a straight line linkage, and the indicator card is traced. The cord is attached to the drum, which moves in phase (the motion is reduced in magnitude) with the engine piston (and crosshead). No indicator paper is shown on the drum.

To get horsepower from the mep, divide the work in foot pounds per minute by 33,000. The force corresponding to a pressure of p_{mi} psi is $p_{mi}A$ lb, where A sq in is the piston area. This force acts through a distance equal to the stroke of the piston, L ft, so that the work per stroke is $p_{mi}AL$ ft-lb, which is also the work for one complete card. If the number of cards per minute (or the number of cycles per minute) is N , the work per minute is $p_{mi}ALN = p_{mi}LAN$ ft-lb per min. The indicated horsepower is therefore

$$(52) \quad \text{ihp} = \frac{p_{mi}LAN}{33,000}$$

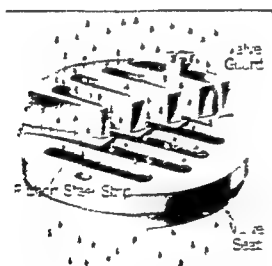
where p_m is obtained from the indicator card as explained above. This equation is often called the mep equation and also the p - L - A - N equation. The beginner frequently makes an error in applying equation (52) because, as derived, A is in square inches (to match pressure in psi) but the stroke L is in feet. However, it is quite correct to use square feet and pounds per square foot together. Observe carefully that, in general, N is *not* the number of revolutions per minute but the number of cards (or cycles) completed per minute. These happen to be the same for a single-cylinder, single-acting compressor. If the single cylinder is double-acting, $N = 2n$, where n is rpm. In a *single-acting engine*, the working substance acts on one side of the piston; in a *double-acting engine*, the working substance acts on both sides of the piston.

We have already defined the symbol p_m as the mep of an ideal cycle (or indicator diagram) and hp as ideal horsepower; hence, the mep equation as



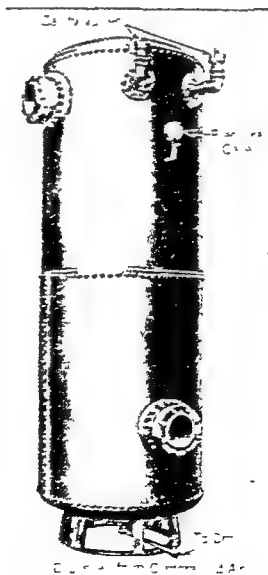
Courtesy Crosby Steam Gage and Valve Co., Boston

Fig. 62. Planimeter. Point F is fixed. Point P is moved around the boundaries of the area to be measured. Wheel D turns with the movement of P , and after the complete outline has been traced, the magnitude of the area may be read from the dial G and the vernier E .



Courtesy Worthington Corp., Harrison, N. J.

Fig. 63. Feather Valve for Gas and Vapor Compressors. The strips of ribbon steel are shown pressing against the valve guard in the open position. When the pressure on the valve-seat side of the valve is less than that on the guard side, the strips lie flat on the ground seats, closing the valve. Intake and discharge valves of this type are identical.



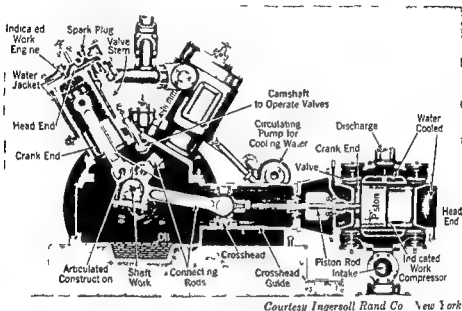
Courtesy Ingersoll-Rand Co., New York

Fig. 64. Receiver.

applied to ideal engines is

$$(53) \quad hp = \frac{p_m LAN}{33,000}$$

When the mep equation is applied to the crank end of a double-acting cylinder, Fig 65, the A in equation (53) should be the area of the piston minus the area of the piston rod, which is the net area over which the mep acts. In run-of-the-mill compressor tests, engineers often neglect to correct for the area of the piston rod.



Courtesy Ingersoll Rand Co New York

Fig 65 Air Compressor, Driven by Twin Cylinder Gas Engine Study the illustration long enough that you will remember the significant names *head end, crank end, packing, piston, piston rod, crosshead, crosshead guide, connecting rod, water jackets on heads and cylinders, indicated work, shaft work, camshaft for ICE*. Observe the articulated construction for the connecting rods, a type of construction also used in radial aircraft engines. Notice that the ICE is *single acting* (work done on one face of the piston), that the compressor is *double acting* (work done on two faces of the piston).

The shaft horsepower, also called the *brake horsepower* (bhp), is another important power term. This bhp is the horsepower which must be delivered to the compressor shaft to drive the compressor, and it is larger than the ihp of the compressor by the amount of frictional losses in bearings, packing, etc., Fig 65.

112 Work of a Compressor The reciprocating as well as the rotary types of compressors can be considered as steady flow machines on an overall basis, in which case the work from equation (9) is

$$W = h_1 - h_2 + K_1 - K_2 + Q.$$

There is generally little difference between the entrance and exit velocities, so that for w' lb.,

$$(54) \quad W = w'(h_1 - h_2) + Q = -\Delta H + Q.$$

[ANY SUBSTANCE, ANY STEADY FLOW PROCESS, $\Delta K = 0$]

If the substance is an ideal gas, $h_1 - h_2 = c_p(T_1 - T_2)$. If the process is internally reversible, $Q = 0$ or $Q = \int c \, dT$. In compressor problems, other forms of the work equation are more convenient.

(a) *Work for Adiabatic and Isentropic Compressions.* Review § 70. If the process is adiabatic, $Q = 0$ and $W = -\Delta h$ Btu per lb.; or for a flow through the compressor of w' lb., we have for constant specific heat

$$(b) \quad W = -w'c_p(T_2 - T_1) = -w'c_pT_1\left(\frac{T_2}{T_1} - 1\right) \text{ Btu.}$$

[ANY ADIABATIC, IDEAL GAS, $\Delta K = 0$]

From equations (26) and (36), we have

$$(c) \quad c_p = \frac{kR}{J(k-1)} \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}.$$

These values and $p_1V'_1 = w'RT_1$ substituted into the work equation give

$$(55) \quad W = \frac{kw'RT_1}{J(1-k)} \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right] = \frac{kp_1V'_1}{J(1-k)} \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right] \text{ Btu,}$$

[ISENTROPIC ONLY, IDEAL GAS, $\Delta K = 0$]

where V'_1 is the volume measured at p_1 and T_1 corresponding to the mass w' .

(b) *Work for Polytropic Compression.* Review § 73. We recall that the polytropic is defined as a reversible process, that $Q = w'c_n \Delta T$ and

$$c_n = \frac{c_v(k-n)}{(1-n)}$$

for an ideal gas; that is,

$$Q = w'c_n(T_2 - T_1) = \frac{w'c_v(k-n)T_1}{1-n} \left(\frac{T_2}{T_1} - 1\right) \text{ Btu.}$$

During polytropic compression from atmospheric temperatures, this value of Q is normally negative. Using known values in equation (54), we get

$$\begin{aligned} W &= -w'c_pT_1\left(\frac{T_2}{T_1} - 1\right) + \frac{w'c_v(k-n)T_1}{1-n}\left(\frac{T_2}{T_1} - 1\right) \\ &= w'T_1 \left[-c_p + \frac{c_v(k-n)}{1-n} \right] \left(\frac{T_2}{T_1} - 1\right) \text{ Btu.} \end{aligned}$$

Putting the term in the brackets over a common denominator $(1-n)$, using $c_p = kc_v$ and $c_p - c_v = R/J$, this equation becomes

$$(56) \quad W = \frac{nw'RT_1}{J(1-n)} \left(\frac{T_2}{T_1} - 1\right) = \frac{np_1V'_1}{J(1-n)} \left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right], \quad [\Delta K = 0]$$

the latter form being obtained by using $T_2/T_1 = (p_2/p_1)^{(n-1)/n}$ and

$$w'RT_1 = p_1V'_1$$

This equation is seen to be the same as equation (55) except for n in place of k , good for an ideal gas when $\Delta K = 0$

(c) *Work for Isothermal Compression* Review § 69 If the temperature is constant for an ideal gas, $\Delta h = 0$ and $W = Q$ For an isothermal process, $Q = p_1V_1 \ln (V_2/V_1)$ ft-lb, and $p_1V_1 = p_2V_2$, or

$$(57) \quad W = \frac{p_1V'_1}{J} \ln \frac{p_1}{p_2} = \frac{w'RT_1}{J} \ln \frac{p_1}{p_2} \text{ Btu,} \quad [\Delta K = 0]$$

where, as before, V'_1 is the volume of w' lb at p_1 and T_1

(d) *Work for Irreversible Adiabatic Compression* Review § 71 Actual rotary types of compressors consume work approximating the irreversible adiabatic in steady flow In equation (54) let the actual final state be represented by 2, $W = w'(h_1 - h_2)$ and find

$$(d) \quad W = -w'c_p(T_2 - T_1) = \frac{-w'kRT_1}{J(k-1)} \left(\frac{T_2}{T_1} - 1 \right),$$

where we have used the ideal gas relation $c_p = kR/[J(k-1)]$ Usually the temperature T_2 is measured in the actual compressor unless from experience with a particular type of compressor, we have some approximate pT relation such as (see § 71)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(m-1)/m}$$

Substituting this value for T_2/T_1 in (d) we get

$$(e) \quad W = \frac{k w' R T_1}{J(1-k)} \left[\left(\frac{p_2}{p_1} \right)^{(m-1)/m} - 1 \right] \quad [\Delta K = 0]$$

Since this equation is for an irreversible compression it represents an actual work quantity Thus, we may say of m that it is that number used as shown which results in equation (e) giving the value of the actual fluid work W —but only when $\Delta K = 0$

113 Work from Conventional Diagram A *conventional diagram* is an idealized indicator card or pV diagram which can be used to determine the work of a compressor The analysis from the diagram is not so revealing thermodynamically, but it is helpful in other ways

Consider first a conventional diagram which suitably portrays the work of a reciprocating compressor without clearance, Fig 66 The area under 4-1 represents the work done on the piston during the intake stroke and the area under 2-3 represents the work done on the substance in pushing (delivering) it from the cylinder For purposes of illustration let the com-

pression curve be represented by the equation $pV^n = C$ (which could be a reversible polytropic but is not necessarily so; just $pV^n = C$). Since the work is represented by the enclosed area 1-2-3-4, the algebraic sum of the areas under each line of the card is the work to some scale of the conventional compressor.

$$\begin{aligned} W &= \frac{p_2 V_2 - p_1 V_1}{1 - n} + p_2(V_3 - V_2) + p_1(V_1 - V_4) \\ &= \frac{p_2 V_2 - p_1 V_1}{1 - n} - p_2 V_2 + p_1 V_1, \end{aligned}$$

since V_3 and V_4 are equal to zero. Reduced to a common denominator, the foregoing expression becomes

$$W = \frac{n(p_2 V_2 - p_1 V_1)}{1 - n},$$

which is also $-\int V dp$ for $pV^n = C$. Since $V_2/V_1 = (p_1/p_2)^{1/n}$, this equation becomes

$$(f) \quad W = \frac{n p_1 V_1}{1 - n} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] \text{ ft.-lb.}$$

[ENCLOSED AREA FOR $pV^n = C$]

If $n = k$, we see by comparison with equation (55) that equation (f) gives the work when there is isentropic compression. Similarly, comparing equation (f) with equation (56), we see that if n is for a reversible polytropic, we obtain the work for a polytropic compressor—except of course that the units in equation (f) are foot-pounds instead of Btu as in the other equations. The V_1 in equation (f) is the same as V'_1 and means the volume *passing through* the compressor as measured at p_1 and T_1 . If the substance is a gas, one may use wRT_1 for $p_1 V_1$ in equation (f).

Comparing Fig. 66 with Fig. 18, p. 45, you see immediately that the enclosed area 1-2-3-4 is equal to $-\int V dp$. This integration for $pV^k = C$ is on p. 77. As a purely mathematical relationship, $-\int V dp$ for $pV^n = C$ is identical with equation (f) above.

It is worth noting that equation (f) represents the area of any diagram bounded by the zero volume line (p axis), two constant pressure lines, and a curve of the form $pV^n = C$. All the work equations for compressors so far given in this chapter should give a negative number, inasmuch as they are on an algebraic basis and the work is done *on* the substance (enters the system). For those who must make repeated computations of compressor work, tables are available which give the value of the pressure ratio to its exponent, $(p_2/p_1)^{(1-1)/k}$ of equation (55), or which give the entire term in the bracket (14, 32).

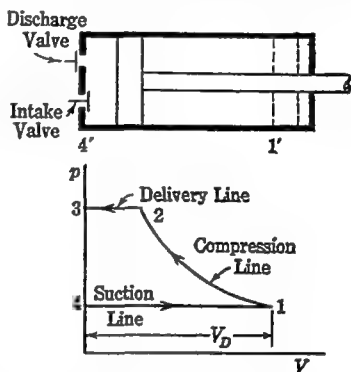


Fig. 66. Conventional Diagram without Clearance.

114. Clearance and Clearance Volume. Displacement volume is defined (§ 86) as the volume swept by the face of the piston in one stroke. In order to be sure that the piston does not strike the cylinder head at the end of the stroke and that there is room for valves, a clearance volume is essential in reciprocating compressors. There is another consideration for clearance volume in internal combustion engines, but in compressors, the smallest practical clearance volumes are desirable (§ 118). Since, as we shall see in the next article, the power consumption is theoretically independent of the amount of clearance, there would be no point in increasing manufacturing costs significantly just to obtain a smaller clearance. The ratio

$$(g) \quad c = \frac{\text{clearance volume}}{\text{displacement volume}}$$

is called the *clearance ratio*, the *percentage clearance*, or just the *clearance*. Values of this ratio in practice vary from about 3% in some large reciprocating compressors to more than 12% in others, with most values falling between 8% and 12%.

115 Work of the Conventional Card with Clearance. The events of the diagram with clearance are the same as those for the case of no clearance,

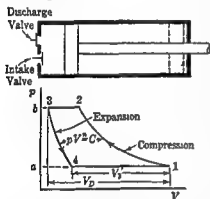


Fig 67 Conventional Diagram with Clearance. The greater the clearance ($= V_1$), the less air drawn in ($= V_1$)

of air drawn into the cylinder is $V_1 - V_4 = V'_1$, and is less than the displacement volume V_D .

To find the work of the diagram with clearance, we imagine it to be made up of two diagrams, $a-1-2-b$ and $a-4-3-b$. Each of these diagrams is similar in all respects to the diagram of the compressor without clearance, Fig 66, hence, equation (f) may be applied to each of them. The work of the diagram 1-2-3-4 will be equal to the work $a-1-2-b$ minus work $a-4-3-b$. Thus for 1-2-3-4 we find

$$\begin{aligned}
 W &= \frac{np_1V_1}{1-n} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] - \frac{np_4V_4}{1-n} \left[\left(\frac{p_3}{p_4} \right)^{(n-1)/n} - 1 \right] \\
 &= \frac{np_1(V_1 - V_4)}{1-n} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right],
 \end{aligned}$$

since $p_4 = p_1$ and $p_3 = p_2$. For $V'_1 = V_1 - V_4$, the work equation becomes

$$(h) \quad W = \frac{np_1V'_1}{1-n} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] = \frac{nw'RT_1}{1-n} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] \text{ ft-lb.,}$$

which again is the same as equation (56) except for units; V'_1 is the volume of air drawn in; w' is the mass of air *passing through* the compressor, corresponding to volume V'_1 . The conclusion is that the amount of work necessary to compress a particular mass of air under given conditions is independent of the clearance, which is perfectly true for the conventional diagrams. In the real compressor, however, there are additional frictional effects. The displacement must be greater with clearance than without, for a particular capacity; this means a larger machine that will cost more and have greater mechanical friction.

116. Free Air. *Free air* is air at normal atmospheric conditions in a particular geographical location. A particular volume of free air at an altitude of 5000 ft. does not have the same mass as the same volume at sea level. At the higher altitude, the barometric pressure is lower, Fig. 68,

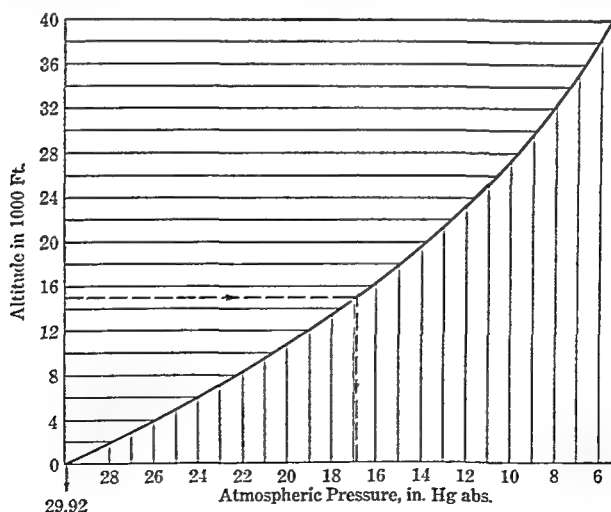


Fig. 68. Variation of Atmospheric Pressure with Altitude. Useful in estimating the mass of air a compressor will deliver at various altitudes. Enter chart at the ordinate, which represents altitude. Move rightward to the curve, as along the dotted line, then downward to the abscissa and read the normal atmospheric pressure at the corresponding altitude.

and a given mass of air occupies a greater volume. It is to be inferred too that the volume of free air varies with the temperature. In the absence of a specified temperature, 68°F may be used.

117. Capacity and Volumetric Efficiency. The *capacity of a compressor* is the actual quantity of gas delivered, as measured by an orifice at intake pressure and temperature, expressed in cubic feet per minute (cfm). The actual *volumetric efficiency* of a reciprocating compressor is the ratio

$$(1) \quad \eta_{\text{actual}} = \frac{\text{capacity of compressor}}{\text{displacement in cfm}},$$

where the displacement is computed as explained in § 86, equation (j). The value of the actual volumetric efficiency, which ranges from 65% to 85%, is obtained only from test of the actual compressor.

118 Conventional Volumetric Efficiency. An equation for the volumetric efficiency, as found from the conventional diagram, is useful in estimating actual values, but more important, it accents certain factors on which volumetric efficiency depends. The volume of air measured at intake conditions on the conventional card, Fig 69, is $V'_1 = V_1 - V_4$, or

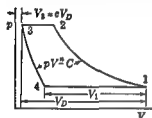


Fig 69

$$\eta_v = \frac{V'_1}{V_D} = \frac{V_1 - V_4}{V_D}$$

From process 3-4,

$$V_4 = V_3 \left(\frac{p_3}{p_4} \right)^{1/\gamma} = cV_D \left(\frac{p_2}{p_1} \right)^{1/\gamma}$$

Also, $V_1 = V_D + cV_D$, where cV_D is the clearance volume V_2 and c is the per cent clearance expressed as a decimal. Therefore we have

$$(58) \quad \eta_v = \frac{V_1 - V_4}{V_D} = \frac{V_D + cV_D - cV_D(p_2/p_1)^{1/\gamma}}{V_D},$$

$$\eta_v = 1 + c - c \left(\frac{p_2}{p_1} \right)^{1/\gamma},$$

which is the *conventional volumetric efficiency*. The actual volumetric efficiency is different from the conventional volumetric efficiency because of fluid friction of flow (the pressure in the cylinder is less than the pressure of free air), and because the cylinder walls, being relatively hot, heat the incoming air (less mass of hot air can occupy a given space). Since, in equation (58), p_2 is greater than p_1 , the volumetric efficiency decreases as the clearance increases, and as the volumetric efficiency decreases, the capacity decreases. The clearance may become so large that no air is discharged by the compressor. This characteristic is often used to control the

output of a compressor by increasing the clearance when a reduced output is desired. Also observe from equation (58) that the volumetric efficiency goes down as the pressure ratio p_2/p_1 goes up. Neither clearance nor volumetric efficiency is a reliable indicator of quality. The user is most concerned about the actual power consumed for the desired capacity.

119. Preferred Compression Curves. Inasmuch as the isentropic curve 1-2', Fig. 70, is steeper than the isothermal 1-2, it takes more work to compress and deliver the air when the compression is isentropic than when the compression is isothermal, the difference being represented by the shaded area. Compression curves with values of n between 1 and k will fall within the shaded area. Thus we see that the work necessary to drive the compressor decreases as the value of n decreases. Note that between specified pressures, Fig. 70,

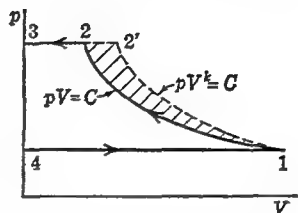
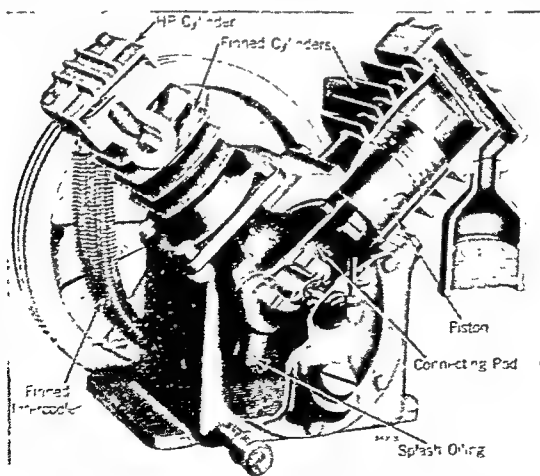


Fig. 70. Comparison of Work for Isothermal and for Isentropic Compression.

Work, isentropic process < work, isothermal process,
 Work, isentropic compressor > work, isothermal compressor.

Polytropic compression and values of n less than k are brought about by circulating cooling water, Fig. 65, or air around the cylinder, Fig. 71. The cooling water or air picks up heat because the work has raised the substance's temperature above that of the environment. In small cheap



Courtesy Ingersoll-Rand Co., New York

Fig. 71. Air-Cooled Compressor. This is a small two-stage compressor driven by an electric motor via a V-belt drive. A part of the finned intercooler is visible on the left.

compressors the cooling will be inadequate and the value of n will be 1.35 or higher. Under favorable circumstances a value of $n = 1.3$ or less may be expected. Values from 1.25 to 1.3 represent the best results for water-jacketed compressors.

A low value of n in a compressor is not necessarily desirable. The best compression process depends upon what the compressed stuff is used for. Stuart and Jackson (31) have discussed this question fully. We note that the adiabatic process results in an increase of enthalpy (by the amount of the work done). Hence if the compressed substance is to be used in a gas turbine, for example, the available portion of the energy involved is later available for work in the turbine and less heat would have to be added in the combustion chamber (Chapter 8).

On the other hand, even though the substance is air used in driving miscellaneous devices, the increase of energy of the air does not help because the air ordinarily loses this energy as heat to the surroundings before it is used. Hence in this situation isothermal compression would be best thermodynamically. In the case of atmospheric air, which always has some water vapor in it, an actual isothermal compression would probably not be desired even if it could be attained because of the troubles which would accompany the condensation of vapor which inevitably takes place. See *aftercooler* § 110.

120 Example A 14x15-in. double-acting air compressor whose clearance is 4% runs at 150 rpm and takes air at 14 psia and 80°F (point 1). Discharge is at 56 psia after a compression which is taken as polytropic with $n = 1.3$. The atmospheric pressure and temperature are 14.7 psia and 70°F respectively. (a) Estimate the free air from the conventional volumetric efficiency. For the substance as it passes through the compressor, determine (b) the heat transferred, (c) the change of enthalpy, (d) the conventional horsepower. (See Fig. 69.)

SOLUTION (a) Because the compressor is double-acting, the number of diagrams completed per minute is $N = 2n = 2 \times 150 = 300$, where n = rpm. The displacement is (sizes of engines are always given bore \times stroke = 14x15)

$$V_D = \left(\frac{\pi D^2}{4}\right)(L)(N) = \frac{\pi(14^2)(15)(300)}{(4)(1728)} = 401 \text{ cfm}$$

With a volumetric efficiency of

$$\eta_v = 1 + c - c \left(\frac{p_2}{p_1}\right)^{1/n} = 1.01 - 0.04 \left(\frac{56}{14}\right)^{1/1.3} = 0.924 = 92.4\%$$

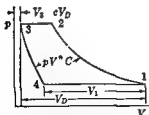


Fig. 69 Repeated

we find the volume drawn in per minute as

$$V_1 = (0.924)(401) = 370 \text{ cfm}$$

measured at 14 psia and 80°F at point 1. If the subscript a refers to atmospheric

air, we have $p_o V_o / T_o = p_1 V'_1 / T_1$; or the approximate volume of free air is

$$V_o = \frac{p_1 V'_1 T_o}{T_1 p_o} = \frac{(14)(370)(530)}{(540)(14.7)} = 346 \text{ cfm.}$$

(b) To get the heat, we might first find the temperature after compression.

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(n-1)/n} = 540 \left(\frac{56}{14} \right)^{(1.3-1)/1.3} = 744^\circ \text{R} = 284^\circ \text{F.}$$

The mass of air delivered is

$$w' = \frac{p_1 V'_1}{R T_1} = \frac{(14)(144)(370)}{(53.3)(540)} = 25.9 \text{ lb./min.}$$

Hence, for $c_v = 0.1715$ for air and $k = 1.4$, and for

$$c_n = \frac{(0.1715)(1.4 - 1.3)}{(1 - 1.3)} = -0.0572 \text{ Btu/lb.,}$$

we get

$$Q = w' c_n \Delta T = (25.9)(-0.0572)(744 - 540) = -302 \text{ Btu/min.,}$$

the negative sign indicating heat rejection.

(c) The change of enthalpy for $c_p = 0.24$ is

$$\Delta H = w' c_p \Delta T = (25.9)(0.24)(744 - 540) = 1268 \text{ Btu/min.}$$

(d) The work, equation (54), is

$$W = -\Delta H + Q = -1268 + (-302) = -1570 \text{ Btu/min.}$$

Using 42.4 Btu/hp-min., we get

$$hp = \frac{1570}{42.4} = 37 \text{ hp.}$$

The use of equation (56) will result in the same answer, but having found Q , the foregoing computation is easier.

121. Efficiencies. The mechanical efficiency of a compressor is

$$(j) \quad \eta_m = \frac{\text{ihp of the compressor}}{\text{bhp of the compressor}} = \frac{W_I}{W_B}.$$

If the compressor is driven by a steam or internal combustion engine, the mechanical efficiency of the compressor system is (32)

$$(k) \quad \eta_{ms} = \frac{\text{ihp of compressor}}{\text{ihp of driving engine}}.$$

For rough estimation and pedagogical purposes we may estimate the horsepower (fhp) used to overcome friction in the compressor as

$$(l) \quad fhp = 0.105 V_D^{2/3},$$

where V_D is the piston displacement in cubic feet per minute. The brake or shaft horsepower of the compressor may be estimated by adding this frictional horsepower to the indicated horsepower of the compressor.

The *adiabatic compression efficiency* η is the theoretical power required as obtained from a conventional diagram with *isentropic* compression divided by the actual indicated power of the compressor

$$(m) \quad \eta = \frac{\text{work (or hp) of conventional card isentropic compression}}{\text{indicated work (or ihp) of compressor}}$$

which is the compression efficiency most commonly used. The *isothermal compression efficiency* is similarly defined except that the numerator of (m) would be the work of the conventional card with isothermal compression. The *overall efficiency* is the product of the mechanical efficiency and the compression efficiency $\eta_o = \eta_m \eta$. Thus the *adiabatic overall efficiency* is

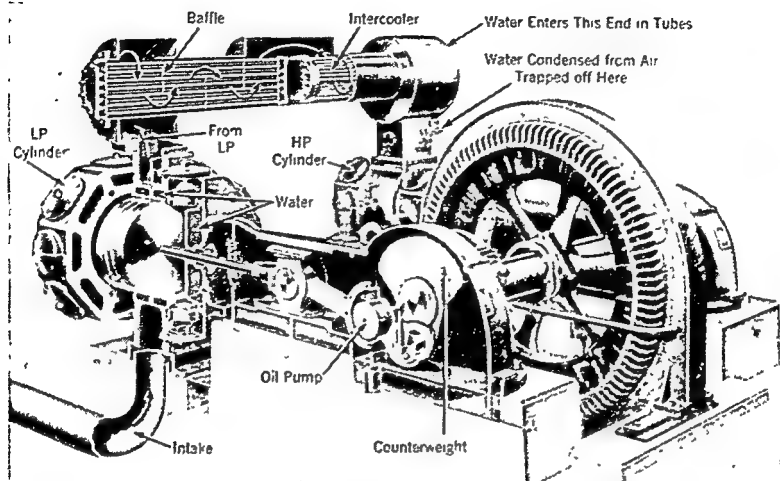
$$(n) \quad \eta_o = \frac{[\text{eq. (j) or (k)}] [\text{eq. (m)}]}{\frac{\text{hp corresponding to isentropic compression}}{(\text{ihp of engine) or (bhp of compressor)}}}$$

where the denominator depends on whether the compressor is driven by an engine or an electric motor. The ASME Test Code (36) calls the ratio in equation (n) the *shaft efficiency* when the denominator is the brake power.

122 Multistage Compression The volumetric efficiency is affected not only by the clearance but also by the ratio of pressures p_2/p_1 [equation (58)]. Thus if the compression is carried out in two or more cylinders (Fig. 72) the volumetric efficiency of the multicylinder machine will be greater than that of a single-cylinder machine of the same clearance and the same total pressure range. Moreover, since the air gets very hot when compressed to high pressures, the final temperatures may be so high as to cause trouble with the lubrication of the cylinder and piston. Furthermore, it is possible to save considerable power by using two or more stages instead of one if the final pressure is above about 60 to 100 psi, if the installation is permanent, and if the required displacement is greater than about 300 cfm.

To obtain the saving in power, we use an intercooler to lower the air temperature between stages, Fig. 72.

The following discussion applies to both Fig. 73 and Fig. 74, in which are pictured the events of the conventional indicator cards of a two-stage machine, with the high-pressure (HP) card superposed on the low-pressure (LP) card. Suction in the LP cylinder begins at A and the volume V'_1 is



Courtesy Ingersoll-Rand Co., New York

Fig. 72. Two-Stage Compressor, Electric Drive. The synchronous motor is mounted on the compressor shaft and therefore does not have separate shaft and bearings. This drive has a high power factor and efficiency. It is said that about 95% of the output of the motor is utilized in the cylinders. The intercooler is "two-stage." Cold water enters intercooler at HP end, and after circulating through the LP end of the intercooler, goes to the cylinder jackets. Air flows around baffles; see arrows. Two-stage compressors are made with the cylinders at an angle (Fig. 71), cylinders at right angles (one horizontal and one vertical), and cylinders in a tandem. In sizes over 1000 hp, this compressor is made in twin tandem style, two more cylinders added opposite to those shown, but with a combination of LP and HP cylinders in tandem arrangement. Output is controlled by clearance pockets, two on each end of each cylinder. See Fig. 67. Compressors are often driven by reciprocating steam engines on the same shaft.

drawn in; compression 1-2 occurs, and the LP cylinder then discharges the air along 2-B. This discharged air passes through an intercooler, being cooled by circulating cold water. The temperature to which the air is cooled is governed by the temperature of the cooling water available. It is relatively easy to bring the temperature of the air to within 20°F of the temperature of the water. In the conventional analysis, it is commonly assumed that the air has the same temperature upon entering the HP cylinder as it had upon entering the LP cylinder. This assumption places point 3, the end of the suction stroke $E-3$ in the HP cylinder, on an isother-

mal line through 1 (Fig 73 and 74) The air is compressed to the final pressure along 3-4, discharged along 4-F, after which the clearance air re-expands, F-E. The same mass of air is involved at points 1, 2, 3, and 4, also the mass of air discharged is equal to the mass drawn into the LP cylinder, if leakage is neglected and if steady flow has been achieved

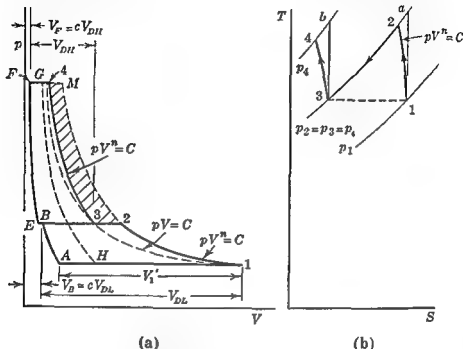


Fig 73. Conventional Cards, Two Stage, No Pressure Drop In (a), 1-M-G-H outlines the conventional card for single stage compression to p_4 for a given per-cent clearance. If the per-cent clearance is the same in both cylinders, re-expansion in the high-pressure cylinder starts at some point F instead of G. Re-expansion in the low pressure cylinder starts at B, where $V_B = V_G$. For a two-stage machine, suction starts at A. For a single-stage machine, suction starts at H. The capacity of the two-stage compressor is greater than that of the single-stage by the amount $V_H - V_A$. Observe that it is possible to make the clearance so large that no air would be delivered. The work saved by the two-stage compression is represented by the shaded area 2-M-4-3. Only the thermodynamic processes (not the suction and discharge) are shown on the TS plane in (b). Isentropic compressions should be 1-a and 3-b. If the LP cylinder had an isothermal compression, the state point would follow the dotted line 1-3.

The equation (h) gives the work of an indicator card like 1-2-B-A or 3-4-F-E, and only for diagrams like these. We may therefore apply this equation to each of these cards, and the total work of the two-stage compressor will be the work of the LP stage plus the work of the HP stage, thus (w' = mass passing through),

$$(o) \quad W = \frac{nw'RT_1}{1-n} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] + \frac{nw'RT_3}{1-n} \left[\left(\frac{p_4}{p_3} \right)^{(n-1)/n} - 1 \right].$$

Typically, the design is based on the assumption that the same amount of work is done in each cylinder. This condition also happens to result in minimum work for compressing a particular mass of air.* Thus, for the

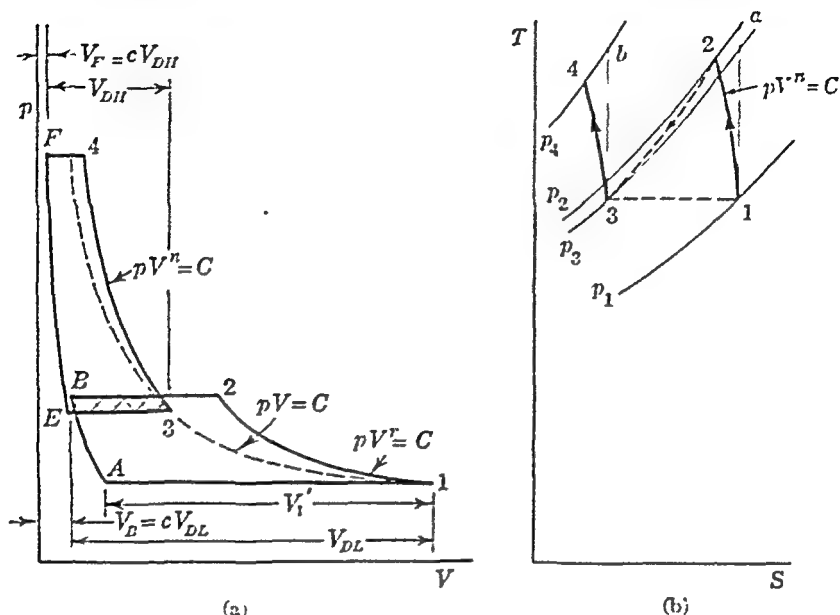


Fig. 74. *Conventional Cards, Two-Stage, with Pressure Drop.* The only change from Fig. 73 is the pressure drop from 2 to 3 in the intercooler. The cards are otherwise conventional in all respects; the compression is taken as polytropic. The shaded area between B and 3 in (a) represents lost (or repeated) work due to the pressure drop. The process through the intercooler is steady flow but irreversible (therefore shown dotted).

case of $T_1 = T_3$ and of $p_2 = p_3 = p_x$, say, we have the work of the first stage equal to that of the second stage, or

$$\frac{nR'T_1}{1-n} \left[\left(\frac{p_x}{p_1} \right)^{(n-1)/n} - 1 \right] = \frac{nR'T_1}{1-n} \left[\left(\frac{p_4}{p_x} \right)^{(n-1)/n} - 1 \right],$$

from which we find

$$(p) \quad \frac{p_x}{p_1} = \frac{p_4}{p_x} \quad \text{or} \quad p_x = (p_1 p_4)^{1/2},$$

* This may be proved for the case of no pressure drop between cylinders and for $T_1 = T_3$ by letting an equation (6) for $p_2 = p_3 = p_x$, a variable intermediate pressure to be determined. Then differentiate W from equation (6) with respect to this pressure p_x and equate the result to zero. The value of p_x found after this differentiation is $p_x = (p_1 p_4)^{1/2}$, the same as that found by equating the work of the first stage to that of the second stage.

which is the proper value for the intermediate pressure for the conditions specified (Fig 73). A pressure drop in the intercooler could be spread on each side of this ideal value.

The conventional cards show very clearly how work is saved, the saving being indicated by the area 2-M-4-3, Fig 73(a). Actual indicator diagrams taken from a two stage machine are shown in Fig 75.

For three or more stages of compression (more than three stages are not often used), the method of analysis is similar to that given for the two stage machine, and since it is anticipated that the reader will have no difficulty in finding the work of the conventional cards, no further detail will be given here. Let p_s = the intermediate pressure between the LP cylinder and the intermediate cylinder, p_v = the intermediate pressure between the intermediate cylinder and the HP cylinder, p_i = the initial or intake pressure, and let p_f = the final discharge pressure. Then, either by differentiation or by the condition of equal works we find the optimum values

$$(q) \quad p_s = (p_i^2 p_f)^{1/3} \quad \text{and} \quad p_v = (p_i p_f^2)^{1/3},$$

which are the values of the discharge pressures from the low (p_s) and intermediate (p_v) cylinders for minimum total work of the conventional cards without pressure drops in intercoolers. Three stages may show a net saving in costs when the final pressure is above about 150 psi, the precise transitional pressure being governed of course, by local conditions. For very high pressures, many stages are used. For example, one installation pumping hydrogen and nitrogen to 15,000 psi uses seven stages.

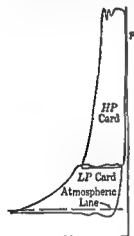


Fig 75 Actual Cards, Two Stage Compressor

123. Energy Diagram for Two-Stage Compression. It will be worth while to keep our thermodynamics in perspective with an energy diagram of the system which consists of the air between the entry and exit boundaries. In Fig 76, the subscripts have the same meanings as before, Fig 73 and 74, except that subscript 5 indicates the state of the substance as it leaves the receiver or aftercooler. We may take each device in the group as an energy diagram and obtain the same work equations as before, after the method of § 112, except that the number for the work will be a positive number inasmuch as it has been assumed as flowing in the correct direction in Fig 76. Similarly, Q is shown in its correct direction. To be specific, the law of conservation of energy applied to the LP cylinder, with $\Delta K = 0$ as usual, gives

$$W_{LP} = h_2 - h_1 + Q \text{ Btu/lb,}$$

where the number for each symbol is substituted as a positive number and W_{LP} is positive. Similarly, for the intercooler, we have ($\Delta K = 0$)

$$\begin{array}{ccc} Q_{2-3} = h_2 - h_3 & \text{and} & Q_{2-3} = \Delta h = h_3 - h_2, \\ \text{[ENERGY DIAGRAM]} & & \text{[CONVENTIONAL]} \end{array}$$

where the value of Q_{2-3} from the energy diagram will be a positive number since it is taken in its true direction, and the value from the conventional form [see equation (i), p. 72] will be negative, the sign indicating that heat is flowing from the system. The transferred heat quantities shown in Fig. 76 are obtained on test by observing the temperatures of the water entering and leaving each device. With these heats known, we may use the system

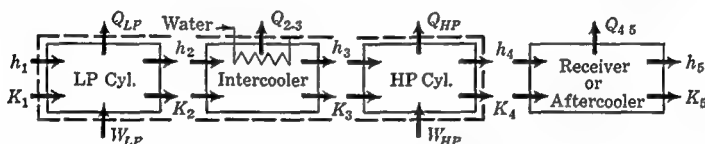


Fig. 76. Energy Diagram—Two-Stage Air Compressor. Any processes, reversible or irreversible. There is a choice to be made in sketching energy diagrams: whether to show energy flux (Q and W) in their known directions and handling each one as a positive number, or whether to show them in conventional directions (opposite to the directions shown here), and let each one be an algebraic number, in which case the sign obtained from the basic form of the equation takes care of the situation. If there is any doubt as to which direction W or Q flows, use the conventional forms. The best plan is to know well enough what you are doing so that you are at home with either idea.

of the air passing through, outlined by heavy dashes, and write the equation for energy entering equal to energy leaving and get the total work as

$$W = h_4 - h_1 + Q_{LP} + Q_{2-3} + Q_{HP} \text{ Btu/lb.},$$

where the Q 's are reduced to Btu per pound of substance flowing through the compressor. Since temperature is an easy property to determine (but not so easy if precision is important), the energy diagram approach is frequently appropriate in test work. If the substance is an ideal gas, $\Delta h = c_p \Delta T$.

124. Example. A two-stage, double-acting, air compressor, operating at 150 rpm, takes in air at 14 psia and 80°F. The size of the LP cylinder is 14x15 in., the stroke of the HP cylinder is the same as that of the LP cylinder, and the clearance of both cylinders is 4%. The LP cylinder discharges the air at a pressure of 56 psia. The air passes through the intercooler and enters the HP cylinder at 80°F and a pressure of 53.75 psia, after which it is discharged from the compressor at a pressure of 215 psia. The value of n in both cylinders is 1.3. Neglect the effect of the piston rods on the crank end. Atmospheric pressure and temperature are 14.7 psia and 70°F, respectively. (a) For the gas in the intercooler, how much heat is removed and what is its change in available energy? (b) What should be the diameter of the HP cylinder? (c) What conventional horsepower is required? (d) Determine the change of available energy of the gas as it passes through the HP cylinder.

SOLUTION Comparison of this example with that of § 120 shows that the LP stage in this problem is identical with the single stage of § 120. Therefore from § 120 we obtain the following additional data (see Fig 77)

$$\begin{array}{lll} w' = 25.9 \text{ lb/min} & T_2 = 744^\circ\text{R} & V'_1 = 370 \text{ cfm} \\ \eta_v(\text{LP}) = 92.4\% & N = 300 \text{ cards/min}, & hp_{\text{LP}} = 37 \end{array}$$

(a) The heat removed is $Q = \Delta H$ (see also Fig 76) and the air is cooled from 744°R to $80^\circ\text{F} = 540^\circ\text{R}$

$$Q = w'c_p(T_3 - T_2) = (25.9)(0.24)(540 - 744) = -1268 \text{ Btu/min}$$

The change of available energy is $Q - T_0\Delta s$ [equation (51) p 112] and $Q = \Delta h$ during steady flow through the intercooler ($\Delta h = 0$). Thus for the air

$$F_o = h_3 - h_2 - T_0(s_3 - s_2) \text{ Btu/lb}$$

Let $T_0 = 530^\circ\text{R}$ the atmospheric temperature and Δs may be obtained from $T ds = c_p dT - (1/\gamma) dp$ as [see equation (a) p 66]

$$\begin{aligned} \Delta s &= c_p \ln \frac{T_3}{T_2} - \frac{R}{J} \ln \frac{p_3}{p_2} = -c_p \ln \frac{T_2}{T_3} + \frac{R}{J} \ln \frac{p_1}{p_2} \\ &= -0.24 \ln \frac{744}{540} + \frac{53.3}{778} \ln \frac{56}{53.75} = -0.0744 \text{ Btu/lb}^\circ\text{R} \end{aligned}$$

Thence using $\Delta h = \Delta H/w' = -1268/25.9 = -49 \text{ Btu/lb}$

$$E_o = -49 - (530)(-0.0744) = -9.6 \text{ Btu/lb}$$

or $(-9.6)(25.9) = -248.5 \text{ Btu/min}$ a decrease. Ordinarily there is little or no opportunity to use this available energy. The net loss in the intercooler is not

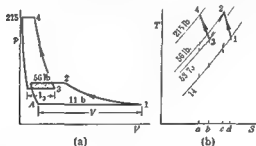


Fig 77

so great as computed because the cooling water gains availability. However, in the end the heat ends up in the sink.

(b) The volume V'_2 passing through the HP cylinder Fig 77 is that corresponding to the weight 25.9 lb/min

$$V'_2 = \frac{w RT_2}{p_2} = \frac{(25.9)(53.3)(540)}{(53.75)(144)} = 96.3 \text{ cfm}$$

The conventional volumetric efficiency of the HP cylinder is

$$\eta_v = 1 + c - c \left(\frac{p_1}{p_2} \right)^{1/\gamma} = 1.04 - 0.04 \left(\frac{215}{53.75} \right)^{1/1.4} = 0.924,$$

the same as for the LP stage, because the pressure ratio is the same (as well as c), $56/14 = 215/53.75 = 4$. Since the HP displacement is

$$V_{DH} = \frac{V'_3}{\eta_v} = \frac{96.3}{0.924} = 104.2 \text{ cfm}$$

by definition of volumetric efficiency, we have

$$V_{DH} = \left(\frac{\pi D^2}{4}\right) (L)(N) = \left(\frac{\pi D^2}{4}\right) \left(\frac{15}{12}\right) (300) = 104.2 \text{ cfm},$$

from which $D = 0.595 \text{ ft.} = 7.14 \text{ in.}$, or say, $7\frac{1}{4} \text{ in.}$, the diameter of the HP cylinder.

(c) We have already found the conventional power required for the LP cylinder (37 hp). For the HP cylinder, we have

$$\begin{aligned} W &= \frac{np_3 V'_3}{J(1-n)} \left[\left(\frac{p_4}{p_3} \right)^{(n-1)/n} - 1 \right] \\ &= \frac{(1.3)(53.75)(144)(96.3)}{(778)(1-1.3)} \left[\left(\frac{215}{53.75} \right)^{(1.3-1)/1.3} - 1 \right] = -1570 \text{ Btu/min.}, \end{aligned}$$

or $1570/42.4 = 37 \text{ hp}$. Therefore, the total horsepower required is $37 + 37 = 74 \text{ hp}$ (Actual horsepower would be of the order of 85–100.)

(d) Since the pressure ratio (4), the value of n (1.3), and the initial temperature (80°F) are the same in LP and HP cylinders, the final temperature is the same, $T_4 = T_2 = 744^\circ\text{R}$, Fig. 77. For 1 lb. and for $c_n = -0.0572$ (from §120), the change of entropy of the air during the reversible polytropic process is $\int c_n dT/T$,

$$s_4 - s_3 = c_n \ln \frac{T_4}{T_3} = -0.0572 \ln \frac{744}{540} = -0.0193 \text{ Btu/lb.}^\circ\text{R.}$$

Now the input of available energy to the air is the work W (as a positive number); but some of this input was lost with the transfer of heat $Q = -302 \text{ Btu/min}$ (from §120). The amount of the loss is

$$Q - T_c \Delta S = -302 - (25.9) (530) (-0.0193) = -37 \text{ Btu/min.}$$

Then the net increase of available energy is $W - 37 = 1570 - 37 = +1533 \text{ Btu/min.}$

125. Gas Tables. Specific heats were assumed to be constant in the foregoing discussion. However, if accuracy is important, the effect of the variation of specific heat should be included. This can be done: by using equations for c_p , such as those in Table II, if they are available; by using a known average value for the temperature range involved, as from Fig. 19; by using a chart of the properties of the substance (see the *Enthalpy-Entropy Chart for Air in Problems on Thermodynamics*;^{*} or by using tabulated properties of substances. (See Chapter 10.) The Keenan and Kaye *Gas Tables* contain tabulated properties of air, of certain products of combustion, nitrogen, oxygen, carbon dioxide, hydrogen, carbon monoxide, and also many other useful tables. Since these tables are becoming a common item

^{*} By Faires, Brewer, Simmang

in an engineer's library, we shall write of them briefly. Tables IV and V, herewith are short extracts from the Keenan and Kaye tables. The symbols h and u in the air table stand for enthalpy and internal energy as usual, but they are pseudo-absolute values computed from

$$h = \int_0^T c_p dT \quad \text{and} \quad u = h - \frac{p}{J} = h - \frac{RT}{J},$$

where the integration is from absolute zero temperature to T . Thus, the $\int c_p dT$ between limits of 1 and 2 is simply the difference of the values read from the table $h_2 - h_1$. All the other symbols in Table IV represent point functions too. The symbol ϕ called the *entropy function*, is defined by

$$\phi = \int_0^T \frac{c_p dT}{T}$$

The value of ϕ given in the air table unlike ϕ as given in all the other tables is the basic value minus one. If one value of ϕ obtained from the table is subtracted from another the difference is the definite integral from 1 to 2 of $\int c_p dT/T$. Again from $T ds = c_p dT - v dp/J$, equation (29), we have

$$(r) \quad \Delta s = \int_1^2 \frac{c_p dT}{T} - \frac{R}{J} \ln \frac{p_2}{p_1} = \phi_2 - \phi_1 - \frac{R}{J} \ln \frac{p_2}{p_1}$$

Thus if the change of entropy between any two states is desired determine it in accordance with equation (r). (There is a table of values of the log term in Keenan and Kaye.) For a constant pressure process only $\Delta s = \Delta \phi$. If the entropy is constant we get from equation (r)

$$\phi_2 - \phi_1 = \frac{R}{J} \ln \frac{p_2}{p_1} \quad \text{or} \quad \phi = \frac{R}{J} \ln p_r$$

[ENTROPY CONSTANT]

the defining relation for p_r which is called the *relative pressure*,

$$p_r = \text{antilog} \frac{J\phi}{R} = e^{J\phi/R}$$

All values of p_r in the tables are ones obtained from the foregoing equation modified by a constant factor in order to have a convenient range of numbers in the end. For an isentropic process,

$$(s) \quad \left(\frac{p_1}{p_2} \right)_s = \frac{p_{r1}}{p_{r2}},$$

where the subscript s is a reminder that the entropy is constant for this

relation. The value of the *relative volume* v_r in the tables is computed from

$$(t) \quad v_r = \frac{RT}{p_r},$$

where the units of R were taken so that v_r is the specific volume in cubic feet per pound when p_r is the pressure in psia for tables based on 1 lb. From equation (t), we see that

$$(u) \quad \left(\frac{v_1}{v_2}\right)_s = \frac{v_{r1}}{v_{r2}},$$

the constant entropy carries over from the definition of p_r .

If the values of the foregoing properties of air are desired on a mol basis, multiply by 28.970 (use 29 for slide-rule work), the equivalent molecular weight of air. The student may interpolate in our tables, but he should realize that the interval between tabulated values is too large for accuracy. While, as stated, these various properties are accurate only at low pressures (ideal gases), the error involved for pressures of several hundreds of psi is negligible. Use Tables IV and V in accordance with any of the fundamental energy equations previously presented.

The properties of the other substances given in other Keenan and Kaye tables are on a pound-mol basis (\bar{h} Btu/mol in Table V, for instance). Thus, an enthalpy value in Table V should be divided by the corresponding molecular weight if it is desired in Btu per pound.

The *internal energy* in Btu per mol is found from the definition of enthalpy $\bar{h} = \bar{u} + p\bar{v}/J = \bar{u} + \bar{R}T$ Btu per mol, where $\bar{R} = 1.986$ Btu per mol-°R. Thus, to get the internal energy of the gases from Table V, subtract the value given in the last column $\bar{R}T$ from the enthalpies \bar{h} at the same temperature.

126. Example. Let one pound of air be compressed adiabatically and in steady flow from 14 psia and 525°R with a compression ratio of 5. The compression efficiency is 75%. Determine the work done, the discharge temperature and pressure, the increase in entropy and in unavailable energy for a sink temperature of 500°R. What value of m in $T_2/T_1 = (V_1/V_2)^{m-1}$ defines the relations of the end states?

SOLUTION. The actual work of this problem is obtained by first finding the isentropic work 1-2, Fig. 78, and then applying the efficiency. By interpolating in our Table IV, the reader should check the table values used below, which are taken directly from Keenan and Kaye and are therefore somewhat more accurate. We have at 525°R,

$$\begin{aligned} h_1 &= 125.47 \text{ Btu/lb.}, & p_{r1} &= 1.2560, \\ v_{r1} &= 154.84, & \phi &= 0.59403. \end{aligned}$$

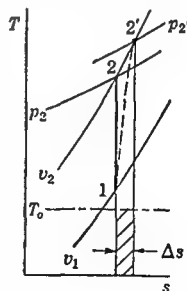


Fig. 78.

Table IV. PROPERTIES OF AIR AT LOW PRESSURES (ONE POUND)

Reproduced from Keenan and Kaye Gas Tables, with permission of authors and publisher, John Wiley

h						h					
T°R	Btu/lb	p	Btu/lb	s	Btu/lb-°R	T°R	Btu/lb	p	Btu/lb	s	Btu/lb-°R
360	85.47	0.3363	61.29	398.6	0.5036J	1480	358.63	50.34	258.84	10.743	0.84704
380	90.75	0.4061	64.70	348.6	0.51863	1490	363.89	53.04	262.44	10.836	0.85062
400	95.53	0.4858	68.11	30.0	0.52890	1500	369.17	55.86	266.34	9.948	0.85416
420	100.32	0.5760	71.52	270.1	0.54058	1520	374.47	58.78	270.26	9.578	0.85767
440	105.11	0.6778	74.93	240.6	0.55172	1540	379.77	61.83	274.20	9.226	0.86113
460	109.90	0.7913	78.36	215.33	0.56235	1560	385.08	65.00	278.13	8.890	0.86456
480	114.69	0.9182	81.77	193.65	0.57255	1580	390.40	68.30	282.09	8.569	0.86794
500	119.48	1.0590	85.20	174.90	0.58233	1600	395.74	71.73	286.06	8.263	0.87130
520	124.27	1.2147	88.62	158.58	0.59173	1620	401.09	75.29	290.04	7.971	0.87462
540	129.06	1.3860	92.04	144.32	0.60078	1640	406.45	78.99	294.03	7.691	0.87791
560	133.86	1.5742	95.47	131.78	0.60950	1660	411.82	82.83	298.02	7.424	0.88116
580	138.66	1.7800	98.90	120.70	0.61793	1680	417.20	86.82	302.04	7.168	0.88439
600	143.47	2.005	102.34	110.88	0.62607	1700	422.59	90.95	306.06	6.924	0.88768
620	148.28	2.248	105.78	102.12	0.63395	1720	428.00	95.24	310.09	6.690	0.89074
640	153.09	2.514	109.21	94.30	0.64159	1740	433.41	99.69	314.13	6.465	0.89387
660	157.92	2.801	112.67	87.27	0.64902	1760	438.83	104.30	318.18	6.251	0.89697
680	162.73	3.111	116.12	80.96	0.65621	1780	444.26	109.08	322.24	6.045	0.90003
700	167.58	3.446	119.58	75.25	0.66321	1800	449.71	114.03	326.32	5.847	0.90308
720	172.39	3.806	123.04	70.07	0.67002	1820	455.17	119.16	330.40	5.658	0.90609
740	177.23	4.193	126.51	65.38	0.67663	1840	460.63	124.47	334.50	5.476	0.90908
760	182.08	4.607	129.99	61.10	0.68312	1860	466.12	129.95	338.61	5.302	0.91203
780	186.94	5.051	133.47	57.20	0.68942	1880	471.60	135.64	342.73	5.134	0.91497
800	191.81	5.526	136.97	53.63	0.69558	1900	477.09	141.51	346.85	4.974	0.91788
820	196.69	6.033	140.47	50.35	0.70160	1920	482.60	147.89	350.98	4.819	0.92076
840	201.56	6.573	143.98	47.34	0.70747	1940	488.12	153.87	355.12	4.670	0.92362
860	206.46	7.149	147.50	44.57	0.71323	1960	493.64	160.37	359.28	4.527	0.92645
880	211.35	7.761	151.02	42.01	0.71886	1980	499.17	167.07	363.43	4.390	0.92926
900	216.26	8.411	154.57	39.64	0.72438	2000	504.71	174.00	367.61	4.258	0.93205
920	221.18	9.102	158.12	37.44	0.72979	2020	510.26	181.16	371.79	4.130	0.93481
940	226.11	9.834	161.68	35.41	0.73509	2040	515.82	188.54	375.98	4.008	0.93756
960	231.06	10.610	165.26	33.62	0.74030	2060	521.39	196.16	380.18	3.890	0.94029
980	236.02	11.430	168.83	31.76	0.74540	2080	526.97	204.02	384.38	3.777	0.94299
1000	240.98	12.298	172.43	30.12	0.75042	2100	532.55	212.1	388.60	3.667	0.94564
1020	245.97	13.215	176.04	28.59	0.75536	2120	538.14	220.5	392.77	3.560	0.94822
1040	250.95	14.182	179.66	27.17	0.76019	2140	543.74	229.6	396.98	3.456	0.95088
1060	255.96	15.203	183.29	25.82	0.76496	2160	549.35	238.4	401.18	3.354	0.95351
1080	260.97	16.278	186.93	24.58	0.76964	2180	554.96	247.8	405.40	3.254	0.95611
1100	265.99	17.413	190.58	23.40	0.77426	2200	560.58	256.8	409.63	3.156	0.95868
1120	271.03	18.604	194.25	22.30	0.77880	2220	566.20	266.5	413.86	3.060	0.96122
1140	276.08	19.858	197.94	21.27	0.78326	2240	571.82	275.9	418.10	2.966	0.96373
1160	281.14	21.18	201.63	20.293	0.78767	2260	577.45	285.4	422.34	2.874	0.96621
1180	286.21	22.56	205.33	19.377	0.79201	2280	583.08	295.0	426.58	2.784	0.96867
1200	291.30	24.01	209.06	18.514	0.79628	2300	588.71	304.7	430.83	2.696	0.97111
1220	296.41	25.53	212.78	17.700	0.80050	2320	594.35	314.5	435.08	2.610	0.97352
1240	301.52	27.13	216.53	16.932	0.80466	2340	600.00	324.4	439.33	2.526	0.97591
1260	306.65	28.80	220.28	16.205	0.80876	2360	605.64	334.4	443.58	2.444	0.97828
1280	311.79	30.55	224.05	15.518	0.81280	2380	611.29	344.5	447.83	2.364	0.98063
1300	316.94	32.39	227.83	14.868	0.81680	2400	616.94	354.7	452.08	2.286	0.98296
1320	322.11	34.31	231.63	14.253	0.82075	2420	622.60	365.0	456.33	2.210	0.98528
1340	327.29	36.31	235.43	13.670	0.82464	2440	628.26	375.4	460.58	2.136	0.98759
1360	332.48	38.41	239.25	13.118	0.82848	2460	633.92	385.9	464.83	2.064	0.98989
1380	337.68	40.69	243.08	12.593	0.83229	2480	639.58	396.5	469.08	1.994	0.99217
1400	342.90	42.88	246.93	12.095	0.83604	2500	645.24	407.2	473.33	1.926	0.99443
1420	348.14	45.26	250.79	11.622	0.83975	2520	650.90	418.0	477.58	1.860	0.99668
1440	353.37	47.75	254.66	11.172	0.84341	2540	656.56	428.9	481.83	1.796	0.99891
1460	358.63	50.34	258.54	10.743	0.84704	2560	662.22	439.9	486.08	1.734	0.99999
1480	363.89	53.04	262.44	10.336	0.85062	2580	667.88	451.0	490.33	1.674	0.99999
1500	369.17	55.86	266.34	9.948	0.85416	2600	673.54	462.2	494.58	1.616	0.99999
1520	374.47	58.78	270.26	9.578	0.85767	2620	679.20	473.5	498.83	1.560	0.99999
1540	379.77	61.83	274.20	9.226	0.86113	2640	684.86	484.9	503.08	1.506	0.99999
1560	385.08	65.00	278.13	8.890	0.86456	2660	690.52	496.4	507.33	1.454	0.99999
1580	390.40	68.30	282.09	8.569	0.86794	2680	696.18	508.0	511.58	1.404	0.99999
1600	395.74	71.73	286.06	8.263	0.87130	2700	701.84	519.7	515.83	1.356	0.99999
1620	401.09	75.29	290.04	7.971	0.87462	2720	707.50	531.5	520.08	1.310	0.99999
1640	406.45	78.99	294.03	7.691	0.87791	2740	713.16	543.4	524.33	1.266	0.99999
1660	411.82	82.83	298.02	7.424	0.88116	2760	718.82	555.4	528.58	1.224	0.99999
1680	417.20	86.82	302.04	7.168	0.88439	2780	724.48	567.5	532.83	1.184	0.99999
1700	422.59	90.95	306.06	6.924	0.88768	2800	730.14	579.7	537.08	1.146	0.99999
1720	428.00	95.24	310.09	6.690	0.89074	2820	735.80	592.0	541.33	1.110	0.99999
1740	433.41	99.69	314.13	6.465	0.89387	2840	741.46	604.3	545.58	1.076	0.99999
1760	438.83	104.30	318.18	6.251	0.89697	2860	747.12	616.7	549.83	1.044	0.99999
1780	444.26	109.08	322.24	6.045	0.90003	2880	752.78	629.2	554.08	1.014	0.99999
1800	449.71	114.03	326.32	5.847	0.90308	2900	758.44	641.7	558.33	0.986	0.99999
1820	455.17	119.16	330.40	5.658	0.90609	2920	764.10	654.3	562.58	0.960	0.99999
1840	460.63	124.47	334.50	5.476	0.90908	2940	769.76	666.9	566.83	0.936	0.99999
1860	466.12	129.95	338.61	5.302	0.91203	2960	775.42	679.6	571.08	0.914	0.99999
1880	471.60	135.64	342.73	5.134	0.91497	2980	781.08	692.3	575.33	0.894	0.99999
1900	477.09	141.51	346.85	4.974	0.91788	3000	786.74	705.1	579.58	0.876	0.99999
1920	482.60	147.89	350.98	4.819	0.92076	3020	792.40	718.0	583.83	0.860	0.99999
1940	488.12	153.87	355.12	4.670	0.92362	3040	798.06	731.0	588.08	0.846	0.99999
1960	493.64	160.37	359.28	4.527	0.92645	3060	803.72	744.1	592.33	0.834	0.99999
1980	499.17	167.07	363.43	4.390	0.92926	3080	809.38	757.2	596.58	0.824	0.99999
2000	504.71	174.00	367.61	4.258	0.93205	3100	815.04	770.4	600.83	0.816	0.99999
2020	510.26	181.16	371.79	4.130	0.93481	3120	820.70	783.7	605.08	0.810	0.99999
2040	515.82	188.54	375.98	4.008	0.93756	3140	826.36	797.1	609.33	0.806	0.99999
2060	521.39	196.16	380.18	3.890	0.94029	3160	832.02	810.6	613.58	0.804	0.99999
2080	526.97	204.02	384.38	3.777	0.94299	3180	837.68	824.1	617.83	0.804	0.99999
2100	532.55	212.1	388.60	3.667	0.94564	3200	843.34	837.7	622.08	0.806	0.99999
2120	538.14	220.5	392.77	3.560	0.94822	3220	849.00	851.3	626.33	0.810	0.99999
2140	543.74	229.6	396.98	3.456	0.95088	3240	854.66	865.0	630.58	0.816	0.99999
2160	549.35	238.4	401.18	3.354	0.95351	3260	860.32	878.7	634.83	0.824	0.99999
2180	554.96	247.8	405.40	3.254	0.95611	3280	865.9				

Table V. *ENTHALPY (BTU/MOL) OF GASES AT LOW PRESSURE*

Taken with permission from Keenan and Kaye, *Gas Tables*, John Wiley; values of \bar{h} Btu/mol; $\bar{R} = 1.986$ Btu/mol-°R; $536.69^\circ\text{R} = 459.69 + 77^\circ\text{F}$.

Temp. °R	N ₂ 28 016	O ₂ 32	H ₂ O 18 016	CO ₂ 44 01	H ₂ 2 016	CO 28 01	Prod. 400 °C Ideal	Prod. 200 °C Ideal	P^0/J $= \bar{R}T$
500	3,472 2	3,466 2	3,962 0	3,706 2	3,386 1	3,472 1	3,486 7	3,511 2	993
520	3,611 3	3,606 1	4,122 0	3,880 3	3,523 3	3,611 2	3,627 4	3,653 7	1,033
536.69	3,727 3	3,723 0	4,253 8	4,027 5	3,638 1	3,727 3	3,744 6	3,772 7	1,067
540	3,750 3	3,746 2	4,282 4	4,056 8	3,660 9	3,750 3	3,768 0	3,796 3	1,072
560	3,880 5	3,886 6	4,442 8	4,235 8	3,798 8	3,889 5	3,900 2	3,939 4	1,112
580	4,028 7	4,027 3	4,603 7	4,417 2	3,937 1	4,028 7	4,030 4	4,082 7	1,152
600	4,167 9	4,168 3	4,764 9	4,600 9	4,075 6	4,168 0	4,191 9	4,226 3	1,192
700	4,864 9	4,870 3	5,575 4	5,552 0	4,770 2	4,866 0	4,901 7	4,947 7	1,390
800	5,564 4	5,602 0	6,396 9	6,552 9	5,467 1	5,568 2	5,617 5	5,676 3	1,589
900	6,268 1	6,337 9	7,230 9	7,597 6	6,165 3	6,276 4	6,340 3	6,413 0	1,787
1000	6,977 9	7,057 5	8,078 9	8,682 1	6,864 5	6,992 2	7,072 1	7,159 8	1,986
1100	7,695 0	7,850 4	8,942 0	9,802 6	7,564 6	7,716 8	7,812 9	7,916 4	2,185
1200	8,420 0	8,625 8	9,820 4	10,953 3	8,265 8	8,450 8	8,563 4	8,683 6	2,383
1300	9,153 9	9,412 9	10,714 5	12,136 9	8,968 7	9,194 6	9,324 1	9,461 7	2,582
1400	9,896 9	10,210 4	11,624 8	13,344 7	9,673 8	9,948 1	10,095 0	10,250 7	2,780
1500	10,648 9	11,017 1	12,551 4	14,576 0	10,381 5	10,711 1	10,875 6	11,052 2	2,979
1600	11,409 7	11,832 5	13,494 9	15,829 0	11,092 5	11,483 4	11,665 6	11,859 6	3,178
1700	12,178 9	12,655 6	14,455 4	17,101 4	11,807 4	12,264 3	12,464 3	12,678 6	3,376
1800	12,956 3	13,485 8	15,433 0	18,391 5	12,526 8	13,053 2	13,271 7	13,507 0	3,573
1900	13,741 6	14,322 1	16,427 5	19,697 8	13,250 9	13,849 8	14,087 2	14,344 1	3,773
2000	14,534 4	15,164 0	17,439 0	21,018 7	13,980 1	14,653 2	14,910 3	15,189 3	3,972
2100	15,334 0	16,010 9	18,466 9	22,352 7	14,714 5	15,463 3	15,740 5	16,042 4	4,171
2200	16,139 8	16,862 6	19,510 8	23,699 0	15,454 4	16,279 4	16,577 1	16,902 5	4,369
2300	16,951 2	17,718 8	20,570 3	25,056 3	16,199 8	17,101 0	17,419 8	17,769 3	4,568
2400	17,767 9	18,579 2	21,645 7	26,424 0	16,950 6	17,927 4	18,268 0	18,642 1	4,766
2500	18,589 5	19,443 4	22,735 4	27,801 2	17,707 3	18,758 8	19,121 4	19,520 7	4,965
2600	19,415 8	20,311 4	23,839 5	29,187 1	18,469 7	19,594 3	19,970 7	20,404 6	5,164
2700	20,246 4	21,182 9	24,957 2	30,581 2	19,237 8	20,434 0	20,842 8	21,293 8	5,362
2800	21,081 1	22,057 8	26,088 0	31,982 8	20,011 8	21,277 2	21,709 8	22,187 5	5,561
2900	21,919 5	22,936 1	27,231 2	33,391 5	20,791 5	22,123 8	22,581 4	23,086 0	5,759
3000	22,761 5	23,817 7	28,386 3	34,806 6	21,576 9	22,973 4	23,456 6	23,988 5	5,958
3100	23,606 8	24,702 5	29,552 8	36,227 9	22,367 7	23,826 0	24,335 5	24,893 3	6,157
3200	24,455 0	25,590 5	30,730 2	37,654 7	23,164 1	24,681 2	25,217 8	25,805 6	6,355
3300	25,306 0	26,481 6	31,918 2	39,086 7	23,965 5	25,539 0	26,102 9	26,719 2	6,554
3400	26,159 7	27,375 9	33,116 0	40,523 6	24,771 9	26,399 3	26,991 4	27,636 4	6,752
3500	27,015 9	28,273 3	34,323 5	41,965 2	25,582 9	27,261 8	27,882 9	28,556 8	6,951
3600	27,874 4	29,173 9	35,540 1	43,411 0	26,398 5	28,126 6	28,779 9	29,479 9	7,150
3700	28,735 1	30,077 5	36,765 4	44,860 6	27,218 5	28,993 5	29,651 5	30,406 0	7,348
3800	29,597 9	30,984 1	37,998 9	46,314 0	28,042 8	29,862 3	30,523 3	31,334 8	7,547
3900	30,462 8	31,893 6	39,240 2	47,771 0	28,871 1	30,732 9	31,400 9	32,266 2	7,745
4000	31,329 4	32,806 1	40,489 1	49,231 4	29,703 5	31,605 2	32,282 9	33,199 6	7,944
4100	32,198 0	33,721 6	41,745 4	50,695 1	30,539 8	32,479 1	33,160 8	34,113 1	8,143
4200	33,068 1	34,639 9	43,008 4	52,162 0	31,379 8	33,354 4	34,041 8	35,034 4	8,341
4300	33,939 9	35,561 1	44,278 0	53,632 1	32,223 5	34,231 2	34,923 8	35,963 8	8,540
4400	34,813 1	36,485 0	45,553 9	55,105 1	33,070 9	35,109 2	35,806 8	36,900 8	8,738
4500	35,687 8	37,411 8	46,835 9	56,581 0	33,921 6	35,988 6	36,689 3	37,836 3	8,937
4600	36,563 8	38,341 4	48,123 6	58,059 7	34,775 7	36,869 3	37,571 0	38,771 0	9,136
4700	37,441 1	39,273 6	49,416 9	59,541 1	35,633 0	37,751 0	38,451 0	39,703 0	9,334
4800	38,319 5	40,209 6	50,715 5	61,024 9	36,493 4	38,633 9	39,331 8	40,633 8	9,533
4900	39,199 1	41,146 1	52,019 0	62,511 3	37,356 9	39,517 8	40,213 8	41,563 8	9,731
5000	40,079 8	42,086 3	53,327 4	64,000 0	38,223 3	40,402 7	41,100 7	42,500 7	9,930
5100	40,961 6	43,029 1	54,640 3	65,490 9	39,092 8	41,288 6	42,000 7	43,440 7	10,129
5200	41,844 4	43,974 3	55,957 4	66,984 0	39,975 1	42,175 5	43,000 7	44,380 7	10,327
5300	42,728 3	44,922 2	57,278 7	68,479 1	40,860 2	43,063 2	44,000 7	45,320 7	10,526
5400	43,612 9	45,872 1	58,603 9	69,975 9	41,747 7	43,951 8	45,000 7	46,260 7	10,724

For a compression ratio of 5 during an isentropic process,

$$\left(\frac{v_1}{v_2}\right)_s = 5 = \frac{v_{r1}}{v_{r2}}, \quad \text{or} \quad v_{r2} = \frac{154.84}{5} = 30.968$$

Locate this value of v_{r2} in the air table and read other needed properties at point 2 which is on an isentropic line, Fig 78

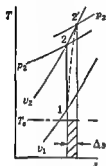


Fig 78 Repeated

$$T_2 = 989.5^\circ\text{R},^* \quad h_2 = 238.37 \text{ Btu/lb}, \quad p_{r2} = 11.837$$

Then, from equation (54), $s = C$,

$$W_s = h_1 - h_2 = 125.47 - 238.37 = -112.9 \text{ Btu/lb},$$

work done on the air. Also from equation (54), the actual adiabatic work is $h_1 - h_x$, Fig 78, from the definition of adiabatic compression efficiency (§ 121), the actual indicated work is $W'' = (h_1 - h_2)/\eta_c$. Equating these two values of the actual work, we have

$$h_1 - h_x = \frac{h_1 - h_2}{\eta_c} = \frac{-112.9}{0.75} = 125.47 - h_x,$$

from which $h_x = 275.97 \text{ Btu/lb}$. For this value of h , we interpolate to find

$$T_x = 1139.6^\circ\text{R} \quad \text{and} \quad \phi_x = 0.78317$$

Now we have from $p/v = C$ (keep the stated compression ratio $v_1/v_2 = 5$)

$$\frac{p_1 v_1}{T_1} = \frac{p_x v_x}{T_x} \quad \text{or} \quad p_x = \frac{T_x v_1 p_1}{T_1 v_x} = \frac{(1139.6)(5)(14)}{525} = 151.8 \text{ psia},$$

the discharge pressure. Note that the relative pressure at state 2', Fig 78 is irrelevant because 2' is not on the isentropic line from 1. (However, we can find the pressure at 2 from

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \quad \text{or} \quad p_2 = 14 \left(\frac{11.84}{1.256} \right) = 132 \text{ psia},$$

which is p_2 for an isentropic compression ratio of 5.) The increase in entropy to 2', equation (r) is

$$\Delta s = \phi_x - \phi_1 - \frac{R}{J} \ln \frac{p_x}{p_1} = 0.78317 - 0.59403 - \frac{53.3}{778} \ln \frac{151.8}{14} = 0.0259 \text{ Btu/lb } ^\circ\text{R},$$

and (§ 98) the increase in unavailable energy which is that portion of the 100% available energy (work) input that becomes unavailable, is ($T_o = 500^\circ\text{R}$, Fig 78),

$$T_o \Delta s = (500)(0.0259) = 12.95 \text{ Btu/lb}$$

In order to find m for the given compression ratio of 5, we have

$$\frac{T_x}{T_1} = \left(\frac{v_1}{v_x} \right)^{k-1} = \frac{1139.6}{525} = 5^{k-1},$$

* It will interest you to note that for the same compression ratio (5) and the same low temperature (525°R) the high temperature is 1000°R in § 89, compared with 989.5°R here. Constant specific heats and $k = 1.4$ were used in § 89.

or $m = 1.481$. If the entropy change were computed from equation (r), § 71, for this value of m , it would be somewhat different from 0.0259 found above, because c_v is taken constant in equation (r). However, if mean values of c_v and k for the particular temperature range are used, the foregoing answer for Δs will be closely approximated. Similar remarks are appropriate with respect to the work computed from equation (e) of § 112.

127. Closure. The reader should fix in mind the difference between an engine diagram on the pV plane and a thermodynamic cycle. The intake and exhaust of an engine may be shown on the pV plane because changes of pressure or volume, or both, are involved. A constant pressure intake is not a constant pressure heating. Idealized, the intake is no more than a change of location of a body of substance with no change of thermodynamic properties. While it is important to understand what an engine diagram is, be sure to observe how the law of conservation of energy is simply applied to *any* engine or process (such as the heat exchanger).

It may have occurred to the reader that the air compressor plus the air engine plus the atmosphere (sink) plus another heat reservoir (source) can be combined to form a thermodynamic cycle. Such a cycle is actually used for refrigeration purposes and for the purpose of manufacturing power, depending upon the way the heat flows during certain processes. An air cycle for developing power, the gas turbine, is the subject of the next chapter. For additional information on fans, compressors, and blowers, see references (31 to 36).

THE GAS TURBINE AND TURBO-JET

128 Introduction The first gas turbine to produce useful work was probably a windmill wherein there is no precompression and no combustion. The characteristic features of a gas turbine as we think of the name today include a compression process and a heat addition (or combustion) process. These features are not new although a practical machine is a relatively recent development. Joule and Brayton* independently proposed the cycle which is the ideal prototype of the actual unit. An unsuccessful turbine unit was built as far back as 1872 and by 1906 a unit which produced net power had been built (39). There were two principal obstacles to be overcome as revealed by thermodynamic analysis. Such an analysis showed that in order for practical amounts of power to be delivered (1) the temperature at the beginning of expansion must be high (until a few years ago the highest permissible temperatures were about 700-800°F) and (2) the compressor must operate at a high efficiency. Metallurgical developments in recent years (for example the use of an expensive alloy of cobalt-chromium-nickel for the turbine blades of the J-47) are raising the highest permissible temperatures (sometimes 1600°F and more 2000°F if short life is acceptable as for some military purposes). A better knowledge of aerodynamics has been responsible for improving the efficiency of both the centrifugal and axial flow compressors. An axial flow compressor is much the reverse of a turbine but it takes a more precise knowledge of aerodynamic characteristics to design an efficient compressor (38). Gas turbines driven by the exhaust of internal combustion engines have long been used for super

* See footnote p. 16. George Brayton, a contemporary of Otto, was a Boston engineer. However, his engine was a reciprocating type rather than a turbine.

charging such engines (turbo-superchargers). Of further promise on the temperature problem is the use of ceramic-metal combinations.

129. Operation of a Simple Gas Turbine Power Plant. Air enters the compressor at condition 1, Fig. 79. After compression, it enters the combustors, some of it going around the outside of the combustion chamber proper and the remainder furnishing oxygen for burning the fuel, which is continuously injected into the combustion chamber. Because of their temperature rise, the gases expand (Charles' law) and enter the turbine in state 3, Fig. 79. After expansion through the turbine, the exhaust to the atmosphere is in some condition 4. In an ordinary power plant arrangement, the work of the turbine W_t is great enough to drive the compressor W_c and deliver brake work W_B to drive, say, a generator or propeller; $W_t = W_B + W_c$. An external source of power is needed to start a gas turbine unit. Figure 80, a cutaway of a turbo-prop engine, shows in some

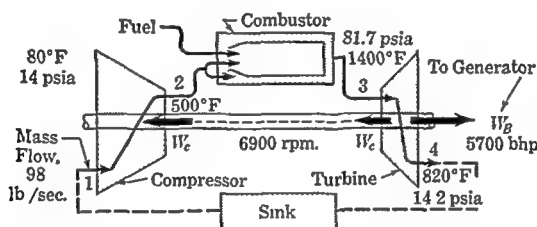


Fig. 79. Diagrammatic Layout of a Gas Turbine Unit. The temperatures and pressures given are typical of actual values.

detail the actual appearance of the simple gas turbine plant, where the excess of power produced by the turbine drives the propeller (instead of a generator). Additional driving force (thrust) is obtained in this application from the change of momentum of the gases leaving the tail pipe as compared with the entering momentum of the air (perhaps 15 to 20% "jet" propulsion).

130. Air Standard Brayton Cycle. In Fig. 79, let the combustor be changed to a heat exchanger and let the same amount of heat be added to the air at constant pressure from an external reservoir of heat. Then, let the exhausted air at 4 be led through another heat exchanger via which heat is transferred to the sink. Following the dotted path in Fig. 79, the same air, cooled to its original intake temperature, now re-enters the compressor at 1 and starts the cycle anew. This is a closed cycle, spoken of as the *equivalent air-standard cycle*, and it is pictured on the pV and TS planes in Fig. 81. Such closed systems for a gas turbine, with air and other working substances, have been built and operated, but the open system is currently receiving the more attention. In the ideal cycle, the compression 1-2 and expansion 3-4 are isentropic; the heat supply 2-3 and rejection 4-1

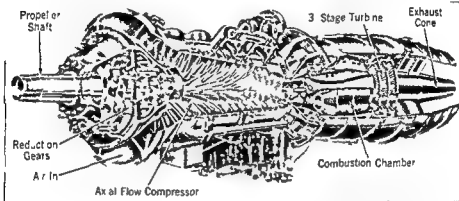
are at constant pressure The work of this cycle, with constant average c_p , is

$$(a) \quad W = \Sigma Q = wc_p(T_2 - T_1) + wc_p(T_3 - T_4) \\ = wc_p(T_2 - T_1) - wc_p(T_4 - T_3)$$

The thermal efficiency is

$$(b) \quad e = \frac{W}{Q_A} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

This equation may be put into various significant forms First, using the



Courtesy Pratt & Whitney Aircraft East Hartford Conn

Fig 80 Turbo prop Engine The compressor is a 13 stage axial flow, the turbine a 3 stage, 4200 rpm Turbo prop engines are well adapted for airplane speeds of some 400-450 mph and for "middle" distances (up to about 1500 mi) The axial flow compressor presents a smaller frontal area for a particular capacity than the centrifugal type—advantageous on an airplane

definition of the pressure ratio $r_p = p_2/p_1$ (§ 87) and the TP relation for an isentropic, we have

$$(c) \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = r_p^{(k-1)/k},$$

$$\text{and} \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{(k-1)/k} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = r_p^{(k-1)/k},$$

whence $T_2/T_1 = T_3/T_4$ Rearranging we get

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \quad \text{or} \quad \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1 = \frac{T_4 - T_1}{T_1} = \frac{T_3 - T_2}{T_2},$$

$$(d) \quad \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \frac{T_4}{T_3}$$

Using this relation in equation (b), we get the thermal efficiency of the Brayton cycle as

$$(e) \quad e = 1 - \frac{T_1}{T_2} = \frac{T_2 - T_1}{T_2} = \frac{T_3 - T_4}{T_3}.$$

From this equation, we may conclude that the efficiency of the Brayton cycle increases as T_2 increases and as T_4 decreases. Now let the compression ratio for isentropic compression be $r_k = V_1/V_2$, by definition (§ 87). Then the TV relation for the isentropic is

$$(f) \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1} = r_k^{k-1}.$$

Using r_p from equation (c) and r_k from equation (f), in equation (e) we find

$$(g) \quad e = 1 - \frac{1}{r_k^{k-1}} = 1 - \frac{1}{r_p^{(k-1)/k}}.$$

The pressure ratio r_p is more commonly used for gas-turbine units than is the compression ratio r_k . Equation (e) reminds one of the Carnot efficiency, but it is not so. A Carnot cycle operating through the same temperature

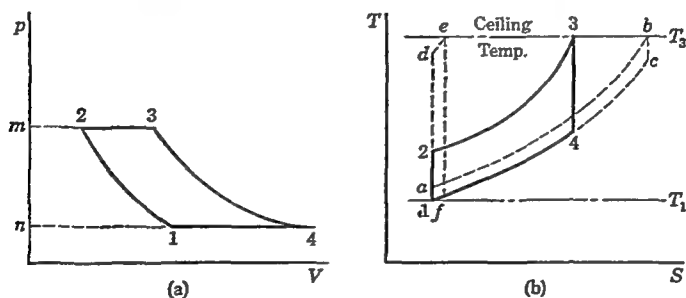


Fig. 81. *Brayton Cycle.* Also known as the *Joule cycle.*

limits, T_3 and T_1 , Fig. 81, has a greater efficiency, $(T_3 - T_1)/T_3$. An examination of equation (g) suggests that to improve the thermal efficiency of the gas turbine, it is necessary to increase the compression ratio, a surmise which is strictly true for the ideal cycle, but which must be qualified for actual cycles, as we shall see later. One of the facts of life which the gas-turbine engineer must contend with is a temperature ceiling, as previously stated. With this limitation, we would not likely choose the most efficient Brayton cycle, because the mep and the amount of work obtainable from a particular size of engine affects the decision. In Fig. 81(b), let the cycle under consideration be 1-d-e-f, wherein the compression ratio (1 to d) is large and the efficiency of the cycle is approaching the Carnot efficiency. Although this diagram is not drawn to any particular scale, it is easy to see that the work done is becoming quite small. Also, if the compression ratio decreases, as to 1-a, the work of the cycle 1-a-b-c becomes quite small (and the efficiency is lower too).

and rearrange to get

$$e' = \left(\frac{\eta_c T_2 - T_1 r_p^{(k-1)/k} / \eta_c}{T_2 - T_1 - T_1 (r_p^{(k-1)/k} - 1) / \eta_c} \right) \left(1 - \frac{1}{r_p^{(k-1)/k}} \right)$$

Notice that the last parentheses enclose a term which is the efficiency of an ideal Brayton cycle with a pressure ratio of r_p . The purpose of arriving at this form is to show that the efficiency of the actual cycle depends upon the high and low temperatures as well as on the pressure ratio. If it is desired to know the maximum efficiency for a particular temperature range, differentiate the foregoing equation with respect to r_p and equate to zero. There is little or nothing to be done about T_1 , so let it be say, 540°R . Now, for a particular turbine inlet temperature (say, $1200^\circ\text{F} = 1660^\circ\text{R}$), assume various pressure ratios and plot a curve. You will find a curve such as the one labeled 1200°F in Fig 83 which points up an important fact. For each turbine inlet temperature, there is a certain pressure ratio which results in maximum

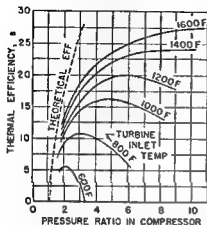


Fig 83 Efficiency vs Pressure Ratio (39) Each curve is for a particular value of T_1 , as labeled, and for the simple cycle corresponding to Fig 82. Combustor efficiency is 100%.

thermal efficiency that is the actual thermal efficiency does not go up indefinitely with pressure ratio as in the case of the ideal cycle. See the dotted curve in Fig 83.

134 Example The intake of the compressor of an air standard Brayton cycle is 40 000 cfm at 15 psia and 40°F . The compression ratio $r_k = 5$ and the temperature at the turbine inlet is 1440°F . Neglect the pressure drop between compressor and turbine and let the exit pressure of the turbine be 15 psia. (a) For the ideal cycle determine the net horsepower output and the thermal efficiency. (b) For the case of an engine efficiency $\eta_e = 85\%$ and a compression efficiency $\eta_c = 83\%$ (these are almost as high as they go) compute the net output and the thermal efficiency. What is the percentage reduction in power? (c) Determine the mep of the ideal cycle. (d) What is the available energy in the exhaust air from the ideal cycle with respect to a sink at $p_o = 15$ psia and $T_o = 500^\circ\text{R}$?

SOLUTION Refer to the ideal cycle 1-2-3-4 Fig 81 (See also Fig 82). In the compressor the high temperature is not so high but that we may assume that $k \approx 1.4$ as for cold air. Thus the pressure at the end of compression is [answers within brackets are as obtained from Keenan and Kaye (14) check them from Table IV]

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^k = (15)(5)^{1.4} = (15)(9.5) \quad 142.5 \text{ psia} \quad [142]$$

and we note that the pressure ratio is $r_p = 9.5$ [9.45]. See Fig. 83. This problem may now be solved by using the gas tables, by using the air chart (in the problem book), or by using the equations and computing desired property changes. If the properties are computed, average c_p and k for the approximate temperature range (or variable specific heat) should be used. (However, the cold-air values would illustrate the thermodynamic procedure, if one wishes to practice using thermodynamics the easiest way.) We shall use the air chart as far as possible and check by the *Gas Tables* [indicated by brackets]. The work of the compressor is

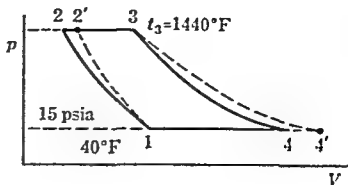


Fig. 84.

$$W_c = -(h_2 - h_1) = -(227 - 119) = -108 \text{ Btu/lb.} \quad [-107.7]$$

The turbine expansion is from 142.5 psia [142] and $1440^\circ\text{F} = 1900^\circ\text{R}$ (point 3, Fig. 84) to 15 psia.

$$W_t = h_3 - h_4 = 477 - 255 = 222 \text{ Btu/lb.} \quad [222.6]$$

$$W_{\text{net}} = W = 222 - 108 = 114 \text{ Btu/lb.} \quad [114.9]$$

For a mass of air of

$$w = \frac{pV}{RT} = \frac{(15)(144)(40,000)}{(53.3)(500)} = 3240 \text{ lb./min.,}$$

we find the net ideal horsepower as

$$hp = \frac{(114)(3240)}{42.4} = 8710 \quad [8790].$$

The energy chargeable against the air cycle is (Fig. 82)

$$Q_A = h_3 - h_2 = 477 - 227 = 250 \text{ Btu/lb.;}$$

hence, $e = W/Q_A = 114/250 = 45.6\%$. [46%]

(b) The actual works and the corresponding net horsepower output are

$$W'_c = \frac{-108}{0.83} = -130 \text{ Btu/lb.,} \quad W'_t = (0.85)(222) = 188.7 \text{ Btu/lb.}$$

$$W' = 188.7 - 130 = 58.7 \text{ Btu/lb.,} \quad hp = \frac{(58.7)(3240)}{42.4} = 4490 \text{ hp.}$$

The actual work delivered by the shaft would be of the order of 2% less than this value—allowing for the mechanical efficiency. To get the actual thermal efficiency, we need the enthalpy at state 2', Fig. 82, in order to compute $Q'_A = h_3 - h_{2'}$. From the value of W'_c above,

$$W'_c = h_1 - h_{2'} = -130, \quad \text{or} \quad h_{2'} = 119 + 130 = 249 \text{ Btu/lb.}$$

$$e' = \frac{W'}{Q'_A} = \frac{58.7}{477 - 249} = 25.7\%.$$

$$\text{Percentage loss} = \frac{114 - 58.7}{114} = 48.5\%.$$

This large loss from the work of the ideal cycle ignores losses from incomplete combustion and from the pressure drop through the combustor and other passages

(c) The 'displacement' of the ideal cycle is the largest volume minus the smallest $V_4 - V_1$. To keep the numbers small, use one pound, $v_4 - v_1$. Hence,

$$v_1 = \frac{V_1}{w} = \frac{40\,000}{3210} = 12.34 \text{ cu ft/lb}, \quad v_2 = \frac{v_1}{r_2} = \frac{12.34}{5} = 2.468 \text{ cu ft/lb},$$

$$v_4 = \frac{RT_4}{p_4} = \frac{(53.3)(1070)}{(15)(144)} = 26.4 \text{ cu ft/lb} \quad [26],$$

where in computing v_4 the value of T_4 has been estimated from the air chart as 1070°R [1053]. The mep of the ideal cycle Fig 84, is

$$p_m = \frac{W}{V_D} = \frac{W}{v_4 - v_2} = \frac{(114)(778)}{(26.4 - 2.47)(144)} = 25.8 \text{ psi}$$

Compare with the mep obtained for the Carnot cycle (11 psi, § 89) with the same compression ratio. Of course the actual mep which is a hypothetical number for a turbine unit inasmuch as a turbine does not provide an indicator card, is only some 51.5% of the ideal $[(58.7/114)(25.8) = 13.29]$. Because of other losses in the actual engine not considered in these calculations, the brake mep of the engine would be even less than 13.29 psi. This low value of the mep can be tolerated in an airplane engine only because of the high rotative speeds which may be used, 4200 rpm for the turbo-prop engine of Fig 80 7600 rpm and more for a turbo-jet engine Fig 92. Some gas-turbine units turn 36 000 rpm some faster. The sometimes close agreement between the answers obtained from the chart and those from the gas tables is fortuitous because the small chart cannot be read accurately.

(d) The heat that would be rejected to the sink at $T_s = T_1$ from state 4 Fig 84, is $h_4 - h_s = h_4 - h_1$, of which $T_s(s_4 - s_s) = T_s(s_4 - s_1)$ is unavailable (§ 98) (The sink state has been taken the same as state 1.) Therefore, the available portion of the rejected heat is

$$E_a = h_4 - h_s - T_s(s_4 - s_s)$$

The entropy term is approximately

$$s_4 - s_s = c_p \ln \frac{T_4}{T_s} = 0.24 \ln \frac{1070}{500} = 0.182 \text{ Btu/lb}^\circ\text{R} \quad [0.1814]$$

Since h_s is the same as h_1 we have

$$E_a = 255 - 119 - 500 \times 0.182 = 45 \text{ Btu/lb}$$

The high temperature of the exhaust and this availability suggests that something further might be gained from the energy of the exhaust, although it is unlikely that more work can be obtained directly from this energy. However, see § 137.

135 Heating Value of Fuels. The air standard is an invaluable tool in cycle analysis, inasmuch as the effects of changes in variables can be easily appraised. If the effect is reduced to a number or percentage, the answer, while not being precise, is often closely indicative of what actually happens because the actual cycle varies somewhat in proportion to the air cycle

However, there is a question as to how much energy to charge against the actual engine in obtaining its thermal efficiency—the higher heating value or the lower heating value.

The *heating value* of a fuel is that amount of heat given up by the products of combustion on being cooled to the initial temperature after complete combustion at constant pressure (or constant volume). This is not a single simple number because of the different ways in which the test may be run and because of H_2O , which is formed from some fuels. Fuels used are commonly hydrocarbons, such as fuel oil, kerosene, and gasoline, the chemical formula of which is in the form C_xH_y . When these fuels burn (react with oxygen), the hydrogen forms H_2O . If the products of combustion are *hot* (above about $125^\circ F$), this H_2O is vapor (steam); if the products have been cooled to normal atmospheric temperatures, the H_2O is condensed, or largely condensed, and the H_2O is water. During condensation it gives up the latent heat of evaporation. Thus, considering this factor only, we see that there may be at least two heating values for fuels containing hydrogen, the *higher heating value* q_h when the H_2O formed from the fuel is condensed, and the *lower heating value* q_l when the fuel is burned so that the H_2O does not condense.* Since tests are run sometimes at constant volume and sometimes at constant pressure, this gives two more heating values.

In the actual engine, the exhaust gases are quite hot and the steam does not come close to condensing. Since this is so, it is reasoned that it would be unfair to the engine to charge against it the higher heating value; hence, the tendency is to use the *lower heating value at constant pressure* in computing the thermal efficiency of actual gas-turbine engines. (The practice in this country has been to use the higher heating value for the same purpose in computing the thermal efficiency of internal combustion engines, but there is some tendency now to use the lower. With such a confused state of affairs, courtesy demands that the kind of heating value used always be stated.)

136. The Combustion Process. For pedagogical reasons, we shall adopt a simple approach to the combustion process at this time.* As a close approximation, we may take the lower heating value q_l at constant pressure as the energy to be released by the ideal chemical reaction. Thus, for w_f lb. fuel per lb. air and for one pound of entering air, Fig. 85, we have

$$(o) \quad w_f q_l + h_i = (1 + w_f) h_p + Q \text{ Btu/lb. air,} \quad [\text{APPROX}]$$

where h_i Btu per lb. of air is the initial enthalpy of the air, h_p Btu per lb. of products is the enthalpy of the departing products, both measured from the same datum as the heating value q_l which is usually $77^\circ F$ and where Q is the heat transferred (shown as *positive if departing*). In this application, the value of Q is small enough that the combustion can often be considered as

* See the author's *Thermodynamics* (Chapter 13) for a little more detail.

adiabatic $Q = 0$ The approximation involved in (o) relates to the fact that the energy entering with the fuel is $w_f(E_c + h_f)$, in which E_c may be called the stored chemical energy and h_f is the enthalpy of the fuel in its entering state. Since h_f is relatively small $w_f(E_c + h_f)$ is closely approximated by $w_f q_i$. Thus for adiabatic combustion ($Q = 0$), we have

$$(p) \quad (1 + w_f)h_p - h_s = w_f q_i$$

For the *air standard* (reactants and products the same), one could ignore the quantity of fuel and use the further approximation

$$(q) \quad \Delta h = \int c_p dT = w_f q_i$$

Fig 85 Simplified Energy Diagram—Combustor

The foregoing equations imply that all the fuel is burned and that all of the energy released appears in the enthalpy of the outgoing products, that is that the combustor efficiency η_f is 100%. We may define combustor efficiency as

$$(r) \quad \eta_f = \frac{\text{actual energy taken on by the gases } (H_p - H_s)}{\text{energy released during ideal and complete combustion}}$$

Reasonable approximations are in order. The actual heat added in the air standard cycle of Fig 82 is $h_3 - h_2$ (mass of fuel neglected). If the ideal energy released is $w_f q_i$ then the efficiency is $\eta_f = (h_3 - h_2)/w_f q_i$. If the combustor efficiency is η_f and if the combustion process is adiabatic the result is that unburned fuel in the amount of $(1 - \eta_f)w_f$ passes out of the combustor with a heating value of q_i Btu per lb fuel thus an energy term leaving the combustor should be added to Fig 85, in the amount of

$$\text{Unburned energy of fuel} \approx (1 - \eta_f)w_f q_i \text{ Btu/lb air,} \quad [Q \quad 0]$$

and the mass $1 + w_f$ times h_p should be modified accordingly that is h_p is multiplied by 1 plus the mass of fuel actually burned. The combustor efficiency at rated loads or thereabouts should be greater than 95% (38)

We may now write the actual thermal efficiency as

$$e = \frac{\text{actual work}}{\text{energy chargeable against the cycle, } E_c} \approx \frac{W'}{w_f q_i},$$

where w_f is the actual amount of fuel *supplied*. If $W' = W_B$ the brake work the result would be the brake thermal efficiency e_b . If w_f is in lb fuel per hp hr then $w_f q_i$ Btu/hp hr is called the *heat rate*, and

$$e = \frac{2544}{(w_f q_i)}$$

The pressure drop in the combustor should fall between the limits of 2% to 10% of the entering pressure

137. Regenerative Heating—Ideal Cycle. Having observed that the temperature of the turbine exhaust 4, Fig. 86, is higher than the temperature at the end of compression 2, we might happen to think of applying Ericsson's notion of regeneration. In this event, the exhaust gas at 4 and the discharged air at 2 could each be led to a heat exchanger (regenerator) so that the hot exhaust 4 gives up heat to the air 2. (See the regenerator in Fig. 87.) Theoretically, if the heat exchanger were large enough and the flow were slow enough, the air from the compressor could be heated reversibly to temperature 4 at state *b*, Fig. 86, while the exhaust cools to temperature 2 at state *a* (§ 90). Some of the formerly discharged heat $h_4 - h_a$ is exchanged *within* the system and the heat to the sink is now $h_a - h_1$. Moreover, it is necessary to add only the heat equal to $h_3 - h_b$, instead of $h_3 - h_2$ as formerly. Consequently, less fuel is needed and this additional piece of equipment should materially increase the efficiency of the ideal cycle, which it does. From Fig. 86, we find the thermal efficiency as ($W = \Sigma Q$, $T_b = T_4$, and $T_a = T_2$)

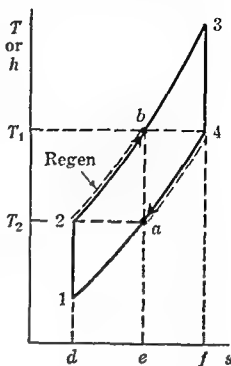


Fig. 86. Regeneration—Ideal Cycle. With the *s* axis at absolute zero, area 4-*a*-*e*-*f* = 2-*b*-*e*-*d*, each representing heat.

$$e = \frac{\Sigma Q}{Q_A} = \frac{c_p(T_3 - T_b) + c_p(T_1 - T_a)}{c_p(T_3 - T_b)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$(s) \quad e = 1 - \frac{T_1}{T_3} \left(\frac{T_2/T_1 - 1}{1 - T_4/T_3} \right) = 1 - \frac{T_1}{T_3} r_p^{(k-1)/k},$$

where we have used $T_2/T_1 = T_3/T_4 = r_p^{(k-1)/k}$ [equation (c), § 130]. With a fixed initial temperature T_1 , equation (s) shows that with a regenerator, the thermal efficiency increases as T_3 increases and decreases as the pressure ratio increases. Note in contrast that without the regenerator [equation (g)] the cycle efficiency increases as the pressure ratio increases. With regeneration, the cycle 1-*a*-*b*-*c*, Fig. 81, approaches the Carnot efficiency. Regeneration is impossible with cycle 1-*d*-*e*-*f*, Fig. 81.

138. Effectiveness of Regenerator. Study Figs. 87 and 88 until the regenerative action is clearly in mind. The state points with the prime marks indicate actual points, except that pressure drops have not been shown. There will certainly be a pressure drop of both the air and the exhaust gases in flowing through the regenerator, and it is important to keep this pressure drop small. The resistance of the regenerator to flow could easily be so great as to offset the theoretical gain of efficiency due to regen-

eration As a consequence, regenerators are likely to be relatively large and costly

The *effectiveness of the regenerator* is defined as

$$(t) \quad \eta_r = \frac{\text{actual amount of heat transferred}}{\text{amount that could be transferred reversibly}}$$

Considering the actual points 2 and 4', Fig 88, a reversible transfer of heat (§ 90) would result in the air being heated from 2' to d , and the exhaust

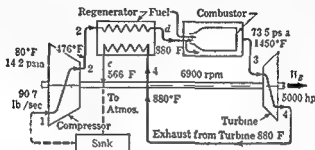


Fig 87 Diagrammatic Layout with Regeneration

gases being cooled from 4' to c . Actually, however, the air is heated only to some state d and the gases are cooled to some state c' . Thus the effectiveness of the regenerator in terms of the states shown in Fig 88 is

$$(u) \quad \eta_r = \frac{h_d - h_2}{h_4 - h_c}, \quad [t_c = t_2]$$

which applies with or without pressure drop in the heat exchanger (regenerator) if d' is the actual state of the departing air (because $Q_{a,c} = h_d - h_2$ for steady flow, $\Delta h = 0$) and with a negligible mass of fuel. If the products are considered to have the same properties as air and if the variation of specific heat is to be ignored use $\Delta h = c_p \Delta T$ and cancel c_p . If the analysis of the cycle is being made for the actual substances we note that the denominator of equation (u) applies to the products of com-

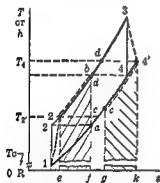


Fig 88 Imperfect Regeneration without Fluid Friction Area $c-2-d-f =$ area $k-4-c-g$

bustion. If the prior discussion is understood, the student should have no trouble in writing equations for heats added and rejected and for the work of a cycle with regenerative heating.

At a particular ceiling temperature, the efficiency curve plotted against pressure ratio for a Brayton cycle with regenerator and with fluid friction rises to a peak at some pressure ratio and then decreases. Such a pressure ratio is about 3.5 for a turbine inlet temperature of 1500°F.

139. Other Variations of the Brayton Cycle. Intercooling in the compression process is used to save work, theoretically just as described in

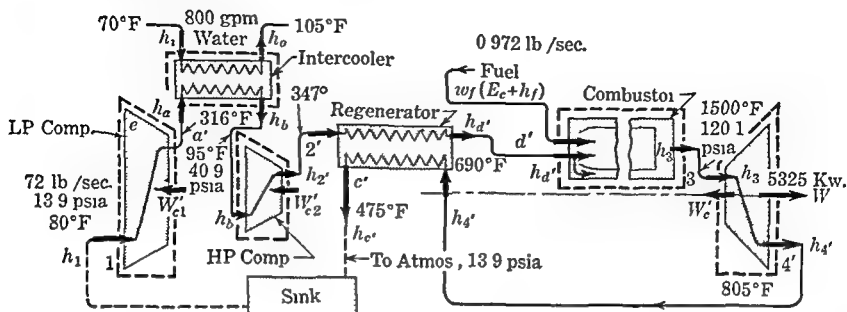


Fig. 89. Intercooling and Regeneration. The properties shown are typical values as rounded off from actual test data. Consider w_f as lb. fuel per lb. air. (See Fig. 90.)

§§ 122 and 123. This feature, together with regeneration, is diagrammatically pictured in Fig. 89. The various energy quantities can be written by inspection of the individual systems in Fig. 89. For negligible changes in kinetic energy (Figs. 89 and 90):

$$W'_c = h_1 - h_{a'} + h_b - h_{2'} \text{ Btu/lb. air,}$$

$$W'_t = h_3 - h_4, \text{ Btu/lb. products,}$$

but the substance in the turbine is the product of combustion (not air) and the mass flow in the turbine is $1 + w_f$ lb. for each pound of air entering the compressor, where w_f = lb. fuel per lb. air. Therefore, we have

$$W'_t = (1 + w_f)(h_3 - h_4)$$

$$= W'_c + W' \text{ Btu/lb. of entering air.}$$

$$Q'_{LR} = h_{a'} - h_b + (1 + w_f)(h_{c'} - h_1)$$

Btu/lb. of entering air.

The compression process may be broken into more than two stages if it should be economic and advantageous.

In addition to intercooling during compression, the turbine may be divided into two turbines. In between these two turbines, we may arrange to burn more fuel. That is, the gases pass through another combustor and are reheated to

approximately the original maximum temperature. This idea, together with intercooling and regeneration, is depicted on the T 's plane in Fig. 90.

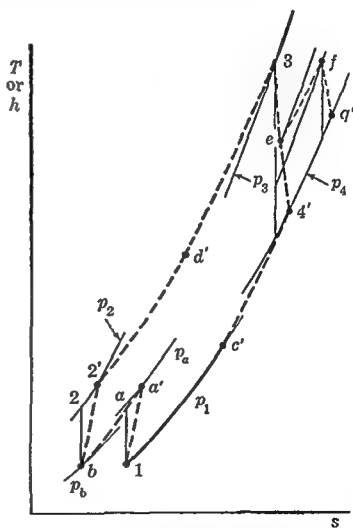


Fig. 90. Intercooling, Regeneration, and Reheating. The cycle 1-a'-b-2'-d'-3-4'-c'-1 shows the T 's representation of the cycle in Fig. 89. The reheating is $e'f$, so that the two turbine expansions are 3-e' and $f q'$.

approximately the original maximum temperature. This idea, together with intercooling and regeneration, is depicted on the T 's plane in Fig. 90.

In this figure, the dotted line $e'f$ represents the second combustor process and $f'q'$ represents the second turbine expansion. A common plan is to design the high-pressure turbine ($3e'$) with just enough power to drive the compressor, and to use the lower pressure turbine ($f'q'$) on a separate shaft to drive the generator (or do other work), but separate shafts are not necessary in order to use this idea. If the plan is used, however, the work of the HP turbine is equal to the work of compression plus whatever energy E_f is needed to overcome friction in these two components, Fig 90,

$$(1 + w_f)(h_3 - h_{e'}) = h_a - h_t + h_t - h_4 + E_f \text{ Btu/lb air,}$$

where w_f may be taken as zero if you wish to ignore the mass of fuel, which is of the order of 1% of the mass of air in gas turbines

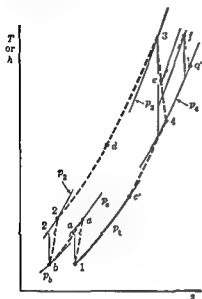


Fig 90 Repeated

140 Jet Propulsion Let us consider a turbo-jet engine, Fig 92. If this engine can drive a plane at a speed of v , fps in still air, the relative effect is the same as if the engine is on a stationary test block and receives air at an initial speed of v , fps. If the entrance to the compressor is properly designed it utilizes the kinetic energy of the entering air to compress it, that is, the entrance is a diffuser (§ 228). This compression is called the *ram effect* (when the engine is moving through the air) and is pictured by 10 , Fig 91, as an isentropic compression. If the air is moving slowly at point o , $K_o = 0$ just as it enters the compressor, it carries energy h_o , and

$$(v) \quad h_o = h_i + K, \text{ Btu/lb,}$$

where K_i is the initial kinetic energy (or, for the moving plane, the kinetic energy of the air stream corresponding to the speed of the plane, which is to say that it is the *kinetic energy relative to the plane*). The enthalpy h_o , for the stream at rest, is called the *stagnation enthalpy*. Since the diffuser is not 100% efficient in converting kinetic energy into enthalpy, the actual rise in pressure is not so large as the



Fig 91 Ideal Turbojet Cycle. The kinetic energy represented by $10de$ is relative to the engine, which may be moving

isentropic rise. Let o' (not shown on Fig. 91 and somewhat below o) be the actual state to which the air is compressed; then the

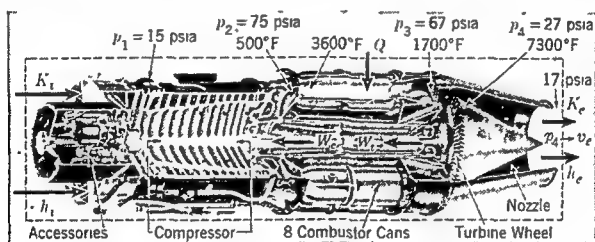
$$\text{Ram or pressure coefficient} = \frac{p_{o'} - p_i}{p_o - p_i};$$

that is, the ratio of the actual pressure rise to that which would occur during isentropic compression.

At o , compression begins in the compressor and is completed at b (Fig. 91). The process in the combustor is the same as before, and gases enter the turbine in state c , expanding to d and doing enough work $abcd$ to drive the compressor. (The areas in Fig. 91 are not to scale.) Leaving the turbine in state d , the gases expand in the nozzle according to the energy relation (Fig. 91)

$$h_d - h_e = K_e - K_d; \quad [\text{NOZZLE}]$$

that is, the drop in enthalpy Δh is converted into kinetic energy ΔK . If the gases enter the nozzle at low velocity, which is likely, we may let $K_d = 0$. All our discussion is on the assumption that the jet gases expand in the nozzle to exactly atmospheric pressure at that location. If the engine is stationary, the energy diagram is as shown in Fig. 92. No net work is



Courtesy General Electric, Schenectady, N. Y.

Fig. 92. Turbo-jet Engine. The energy diagram ignores the mass of fuel. There are 12 stages of compression, $r_k = 5.05$. Normal thrust on test is 4730 lb. at 7630 rpm with fuel consumption of 1.04 lb. per hr.-lb.; cruising thrust is 3700 lb. at 7000 rpm with fuel consumption of 1.03 lb. per hr.-lb. The temperatures and pressures given are roughly typical.

done. The work quantities W_i and W_e are an interchange of work within the system. If the engine is in an airplane moving at a speed of v , fps in still air, level flight, an observer *on the plane* "sees" the same energy diagram. In this situation, the velocities v_i and v_e are initial and exit velocities relative to the plane, the velocities as the observer sees them. The "heat added" to the air in the combustors is the energy released by the chemical reaction of the fuel and oxygen, as this observer sees the power system, and according to our approximation is taken as $Q = w_f q_i$, where w_f is the mass of fuel (per pound of air, if the energy equation is set up on this basis; note that $1 + w_f \approx 1$ lb.):

$$h_i + K_i + w_f q_i = (1 + w_f)(K_e + h_e) \text{ Btu/lb. air.}$$

Since this equation does not contain a work term we may resort to another principle to find the work.

141 Work from the Impulse-Momentum Principle The stream moving past turbine blades does work by virtue of the continuous change of momentum of the stream as you have learned in your course on mechanics. The force exerted by an airplane or ship propeller exists by virtue of the change of momentum of the fluid brought about by the propeller. The propulsive force of jets exists for the same reason. From your mechanics you recall $a = dv/dt$ and $F = ma/g_0$ which latter equation is Newton's law that the resultant force F in any direction is equal to the mass times the acceleration in that direction. Together these relations result in

$$(w) \quad F dt = (w/g_0) dv$$

which is the famous impulse($F dt$) momentum($w dv/g_0$) principle where w/g_0 is the mass in slugs. If the mass rate of flow is constant we have from equation (w)

$$(59) \quad F = \frac{w}{g_0} (v_e - v) \quad \text{or} \quad F = \frac{(w_a + w_f)v_e}{g_0} - \frac{w_a v}{g_0}$$

where upon integration the time interval is taken as $\Delta t = 1$ sec (that is w is the constant mass rate of flow in pounds per second) v_e and v fps are the exit and initial velocities respectively. Allowing for the addition of fuel to the stream we find the second equation where w_a lb per sec of air and w_f lb per sec of fuel are flowing steadily. The right hand side of the first equation (59) is the rate of change of momentum of a stream in steady flow undergoing a velocity change of Δv . We recall that momentum is a vector quantity (48) but the vectors are in the same direction in jet engines. In applying equation (59) to a stream passing through a moving body an airplane use the velocities as those *relative to the moving body*. Since the mass of fuel used in a turbo-jet engine is of the order of 1% of the mass of air small error is introduced if the mass of fuel is ignored in computing F . (A large amount of air in excess of that needed for combustion is necessary to keep the temperature rise within limits.) Therefore the propulsive force of a jet engine is approximately

$$(59) \quad F = \frac{w}{g_0} (v_e - v) \text{ lb}$$

which is the force of the fluid on the plane. From one viewpoint F is the force necessary to accelerate the stream from a relative velocity of v to a relative velocity of v_e but action and reaction are equal so it is also a force on the plane. If the plane is moving with a speed of v_p fps the work done by F lb is Fv_p ft lb per sec or from equation (59)

$$(x) \quad W = Fv_p = \frac{wv_p}{g_0} (v_e - v) \text{ ft lb/sec},$$

where v_i is the initial relative velocity of the air with respect to the plane. If w lb. per sec. is the *actual* mass of air and if the velocities are *actual* relative velocities, this equation gives the actual propulsive work, or rate of work, in moving the plane at a constant speed v_p (mass of fuel neglected). Or it would be the instantaneous rate of work if the plane should be accelerating with an instantaneous speed of v_p . Orient yourself to the fact that no work is done by F at $v_p = 0$; and that for a particular value of the thrust F , the greater v_p , the greater the work. Thus, a jet engine producing a thrust force of 5000 lb. in the direction of motion, would be developing horsepower as follows, for example:

$$\text{At } v_p = 550 \text{ fps, hp} = \frac{(5000)(550)}{550} = 5000;$$

$$\text{At } v_p = 1100 \text{ fps, hp} = \frac{(5000)(1100)}{550} = 10,000;$$

(550 fps \approx 370 mph; 550 ft-lb. per hp-sec.) At an altitude where $p = 8$ psia, $T = 500^\circ\text{R}$, and for a plane speed of 500 fps and $v_i = 2500$ fps, the 5000-lb. thrust would require a flow of about 111,000 cfm; check it for yourself, using equation (59) and $pV = wRT$. A reciprocating internal combustion engine can be connected to a dynamometer; then its horsepower output can be measured and reduced to standard conditions. Applying a known efficiency of transmission and propeller, we may convert this power into thrust at appropriate plane speeds. Thus, as we shall soon see, the economy of the reciprocating engine may be expressed in terms of parameters like pounds of fuel per horsepower-hour, called *specific fuel consumption*. In view of the distinctive characteristic of jet engines described above (power is zero when the engine is at rest), other ratios for expressing the fuel consumption are used, one of which is

$$(y) \quad \text{Specific fuel consumption} = \frac{\text{lb. fuel/hr.}}{\text{lb. thrust}}$$

At a particular speed and thrust, the specific fuel consumption may be expressed in terms of some work unit like horsepower-hour for the purpose of comparison with engines which deliver shaft work. Jet engines are tested at rest and the initial air stream velocity $v_i = 0$. From equation (59), we see that since the thrust depends on v_i , it is different under static test than when the engine is in motion. Therefore, any value of specific fuel consumption or of thrust should include the conditions to which the value applies. A typical fuel rate is 1.08 lb. per hr. of fuel for 1-lb. thrust at sea level static test.

The solution of problems on turbo-jets should follow suggestions presented for the gas-turbine engine. With certain specified data, one would start at the entrance state and work through the cycle, piece by piece, either for the ideal cycle, or for the actual cycle given the appropriate efficiency numbers.

142. Efficiencies of Turbo-jets. The *propulsive efficiency* η_p is defined as the work of the propulsive force *divided by* the energy converted to work or kinetic energy (100% available energy) in the system. The work of the propulsive force for the plane flying in still air, $v_p = v$, and for $w = 1$ lb per sec is, from equation (x),

$$W = \frac{v_p(v_e - v_p)}{g_0} \text{ ft-lb /sec-lb}$$

The energy developed in the system which is 100% available may be found in either of two ways. First, the observer on the plane sees the increase in relative kinetic energy but no work, for one pound,

$$(z) \quad E_s = \frac{v_e^2 - v_p^2}{2g_0} = \frac{(v_e + v_p)(v_e - v_p)}{2g_0} \text{ ft lb /sec-lb}$$

Second, the observer on the ground "sees" absolute zero kinetic energy entering the system (still air) an absolute kinetic energy leaving the system of $v_e^2/(2g_0)$, where v_e is the absolute velocity of the departing gases,

$$v_e = v_e - v_p,$$

and he "sees" work W being done in moving the plane against a resistance, that is in overcoming air resistance, etc. Thus the 100% available energy in sight is

$$\begin{aligned} E_s = W + \frac{v_e^2}{2g_0} &= \frac{v_p(v_e - v_p)}{g_0} + \frac{(v_e - v_p)^2}{2g_0} \\ &= \frac{(v_e - v_p)(v_e + v_p)}{2g_0} \text{ ft lb /sec-lb,} \end{aligned}$$

the same as before. (The ground observer also "sees" $h_1 = h$, enthalpy entering the system, $h_2 = h_e$ enthalpy departing, and he sees a decrease in the stored energy of the system of wq_1 Btu. The reader should sketch this energy diagram with W leaving the system and show that $(v_e = v_p)$)

$$h_e - K_e = W + K_1,$$

where K_1 is the absolute exit kinetic energy and the other terms are as previously defined. However, the impulse-momentum principle is usually employed for computing W , as explained above. Now we write the expression for the propulsive efficiency as

$$\eta_p = \frac{W}{E_s} = \frac{v_p(v_e - v_p)2g_0}{g_0(v_e + v_p)(v_e - v_p)} = \frac{2v_p/v_e}{1 + v_p/v_e},$$

where E_s is defined by (z). This equation shows that this efficiency is zero when $v_p = 0$ and is 100% when $v_p = v_e$. Since the condition $v_p = v_e$ is the condition for zero available energy generated [equation (z)], there would be no work done to drive the plane at 100% efficiency. According to this equation, high efficiencies are obtained when the speed of the jet v_e is a little

larger than the plane speed v_p , but in this event the rate of flow of air would necessarily be very large [equation (59)] in order to obtain large thrust, which means a larger engine.

The thermal efficiency of the jet engine is the propulsive work W divided by the energy E_c chargeable against the engine, taken as $E_c = w_f q_i$;

$$e = \frac{W}{E_c} = \frac{W}{w_f q_i} = \frac{w v_p (v_e - v_p)}{g_o J w_f q_i} = \frac{F v_p}{J w_f q_i},$$

where w_f is lb. fuel per lb. air and q_i is Btu per lb. of fuel. The field of jet engines is so recently developed that one finds a number of different efficiency ratios defined—and different names for the same ratio. We shall define one more efficiency, the engine efficiency for the whole unit considered as the engine,

$$(60) \quad \eta = \frac{W'}{W} = \frac{\text{actual work}}{\text{work of the corresponding ideal cycle or ideal engine}},$$

where the ideal cycle in this application is *ibce*, Fig. 91, and the corresponding work may be found in any one of several ways already explained.

The thermal efficiency of turbo-jet engines is likely to be low as compared with reciprocating types, but, especially for airplanes, there are some offsetting advantages, such as the smaller frontal area to produce less air resistance and less weight per horsepower developed in normal flight. Also, since the efficiency of a propeller drops rapidly after some speed, say about 400–450 mph, the turbo-jet *drive* actually becomes more efficient than a reciprocating-engine-propeller drive at some high speed. In commercial planes, speeds of 500–600 mph with turbo-jet drives appear to be economic on long range flight. At and above plane speeds of Mach 1, the velocity of sound in the air surrounding the plane, jet propulsion is being used. The practical limiting speed of travel with turbo-jet engines is expected to be about Mach 1.5 to 2.

Other means of jet propulsion than the turbo-jet are ram jets and rockets. The ram jet becomes practical when the speed of body is high, well above Mach 1 (say, Mach 3, or about 2000 mph). At these high speeds, the ram effect produces enough compression of the entering air to develop a cycle as previously described, the ideal prototype being the Brayton cycle. Typical data for a speed through air of 2000 fps are: combustion temperature, 3600°F; jet temperature exhaust, 2200°F; jet exhaust velocity, 4000 fps. The German V-1 buzz bomb was propelled by a ram-jet engine in which the firing was intermittent, giving rise to the buzzing. The rocket is a jet propulsion device which not only carries its own fuel but also the reactant for the fuel, as liquid oxygen. Speeds up to 11,000 fps are known to have been obtained with hydrogen as the fuel and liquid oxygen as the reactant.

143. Closure. The student of thermodynamics is generally interested in actual performance data, and typical values have been indicated in the

text and on illustrations in this chapter. Other data, taken at random from the literature, are given below. Design values for gas turbines (56)

Compression efficiency = 85%,	Engine efficiency = 83%,
Combustion pressure loss = 3%	Nozzle efficiency = 98%,
Combustion efficiency = 98%	Regenerator effectiveness = 50%
Ram coefficient = 85%,	Regenerator pressure loss = 5%,
Heating value q_1 = 18 550 Btu/lb	Intercooler pressure loss = 3%

Some characteristic data as given by one of the General Electric bulletins are

Fuel oil	Simple Cycle	Regenerative Cycle
Rating	5300 kw	4000 kw
Heat rate	18 950 Btu/kw hr	16 600 Btu/kw hr
Thermal efficiency	18%	20.5%
Air required (80°F)	114 lb/sec	97 lb/sec
Weight	72.5 tons	200 tons

Inasmuch as there are books written on the subjects of gas engines and jet engines, it is easily understood that this presentation is necessarily limited in scope. Development of gas turbine engines for various purposes (for example, as an automotive drive) is being carried on actively in many laboratories, and progress has been and probably will continue to be exceptionally rapid. It is easy to understand that the various ideas of this chapter might be combined in many different ways, giving a number of modifications of the Brayton cycle. Moreover, combinations of the gas-turbine engines with other apparatus, such as steam cycles, might become advisable. Since this is so, the beginner's endeavor should be to master each idea so that he can intelligently analyze any combination or so that he might perhaps devise a useful combination.

The gas turbine and the steam turbine are much the same. For further readings on gas turbines and jet propulsion, see references (38-47).

9

INTERNAL COMBUSTION ENGINES

144. Introduction. The internal combustion engine is relatively new. The earliest attempts to build such an engine were based on the use of gunpowder. Barsanti and Matteucci built a free-piston engine in 1857, which operated as follows: An explosion drove a piston vertically upward. As it started down under the action of gravity, it engaged a ratchet which was so connected as to turn a shaft. Such a clumsy machine was doomed to failure, although Otto and Langen successfully marketed a number of free-piston engines about 1867. In 1860, Lenoir proposed and built an engine without compression. This engine drew in a charge of gas and air at atmospheric pressure for half a stroke, at which point the mixture was burned. The resulting rise in pressure provided the motive force to complete this stroke, return the piston to the end of the next stroke to exhaust the burned gases, and to bring the piston again to the point of the burning of the new charge. While this engine was used for a while, its efficiency was too low for it to be an economic source of power.

Although Beau de Rochas, a Frenchman, worked out the theory and gave the conditions for high efficiency in 1862, it remained for Nicholas A. Otto (1832-1891) to build a successful engine in 1876 after he had independently invented the same cycle. This engine was called the silent Otto engine, but the word "silent" should not be taken in a literal sense. Otto was born in Holzhausen, Germany, and was a partner in a gas-engine manufacturing plant at the time of his famous invention.

To circumvent Otto's patents, Sir Dugald Clerk, born in Glasgow in 1854, invented the two-stroke-cycle engine, which was first exhibited in 1881.

In these early stages of the internal combustion engine, rotative speeds of the order of 200 rpm were typical. The German Gottlieb Daimler (1834-1899) was the first to conceive of small, relatively high-speed engines for greater power from a particular size, say 1000 rpm (vs 4000 rpm and more for today's automotive engines), and he made them work by improved hot-bulb ignition. The "high-speed" engine made the automobile a practicable idea.

We have seen that the highest temperature in the gas-turbine cycle, occurring in the combustor, is sharply limited. The combustor is subjected to this operating temperature continuously. On the other hand, since the reciprocating internal combustion engine (ICE) is subjected to the highest temperature intermittently, just after the fuel has been fired, temperature has not been much of a problem. We may use air-fuel ratios such that the amount of air is close to that which is ideally required for combustion and let the temperature go where it may. Since the highest temperature exists for only a small portion of the cycle, the interval during the remainder of the cycle can be used for water or for air cooling the cylinder in order to prevent the metal from becoming dangerously hot.

As you know, energy is supplied to the internal combustion engine by the combustion of a fuel within the cylinder. The widespread use of internal combustion engines in automobiles, on the farm, in industrial plants, on ships, in power plants, is common knowledge. Because these engines are used so much, the ideal cycles for them are particularly significant. The fuels used are natural or manufactured gas, gasoline, kerosene, oil, etc., alcohol, and others. The most common fuels are gas, gasoline, and fuel oil.

145. The Four-Stroke Cycle. The *four-stroke cycle* is one wherein four strokes of the piston, two revolutions, are required to complete a cycle. The sequence of events, pictured in Fig 93, are the same for any four-stroke ICE, namely

- 1 A *suction stroke*, drawing fuel and air into an Otto engine, § 146, or drawing air only into a Diesel engine, § 150,
- 2 A *compression stroke*, Fig 93(b),
- 3 Ignition of a fuel already in the cylinder, as by a spark plug, or the self ignition of fuel, which ideally is injected into the cylinder at the end of the compression stroke (the burning of the fuel releases energy for use by the system),
- 4 An *expansion stroke* or *power stroke*, during which positive work is done, and
- 5 An *exhaust stroke* during which most of the products of combustion are pushed from the cylinder, then the cycle repeats.

The end positions of a piston in any reciprocating machine are called *head-end dead center*, or in the automotive industry the *top dead center* (TDC) [Fig 93 (a) and (c)], and the *crank end dead center* or *bottom dead center* (BDC) [Fig 93 (b) and (d)]. Figure 94 reveals some of internal construction of an automotive engine.

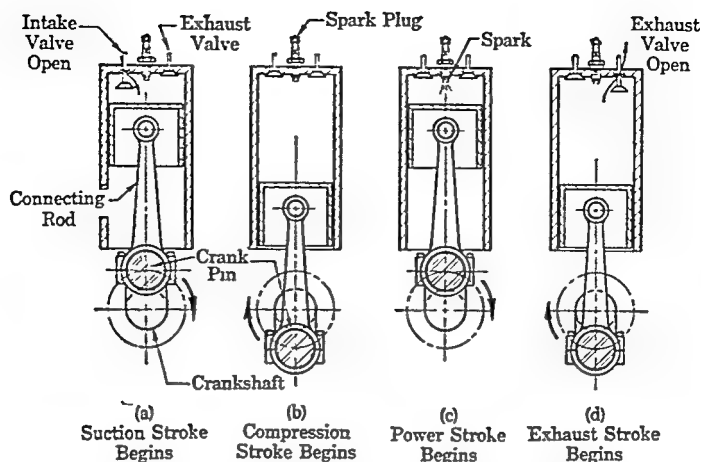
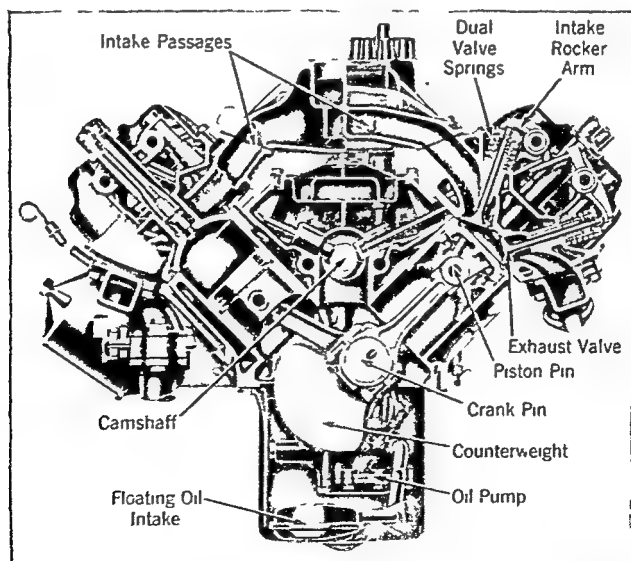


Fig. 93. Four-Stroke Cycle. This diagrammatic representation shows a spark plug which ignites the fuel after compression. The same sequence of events occurs in the 4-stroke-cycle Diesel engine, except that the air is compressed to a temperature high enough to cause the fuel to burn without spark ignition.



Courtesy Chrysler Corp., Detroit

Fig. 94. Four-Stroke Automotive Engine. A $3\frac{1}{16} \times 3\frac{1}{4}$ in. engine, rated at a maximum bhp = 140 at 4400 rpm.

146 The Otto Cycle The Otto cycle, which is the ideal prototype of most small internal combustion engines, is one wherein it is imagined that the combustion process takes place instantaneously at top dead center to give a constant volume combustion of the fuel (or constant volume process of heat added in the equivalent air cycle). The Otto engine may be analyzed either as a flow device or as a closed cycle.

We note that ideally (no pressure drops, etc.), the suction stroke 0-1 and discharge stroke 1-0, Fig 95, cancel one another, so to speak. The positive work under 0-1 is equal to the negative work under 1-0, and these works correspond to the ideal flow works done in getting the air into and

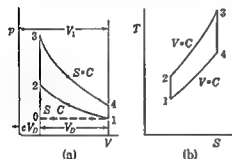


Fig 95 Otto Cycle

out of an open system. Moreover the ideal open system and the ideal closed system reject the same amount of heat, according to the first law, because in each case, if the cycles are comparable, the same heat is added and the same work is done, hence from $Q_A - Q_R = W$, each must reject the same heat Q_R to the sink. Therefore we see that the analysis of an ideal open air cycle is the same as that of an ideal, closed air cycle and such an analysis is called an *air-standard analysis*.

The Otto cycle is shown on the pV and TS planes in Fig 95 and the reader should relate these processes to the events in the actual engine (Fig 93). 1-2 is the compression process which in the ideal case is, as usual, an isentropic process, 2-3 is the instantaneous heating of the air (combustion) at constant volume, 3-4 is the expansion which is ideally isentropic, and 4-1 is the instantaneous rejection of heat at constant volume (equivalent to a valve opening at 4 with heat rejected to the atmosphere after the gases have left the engine in the open cycle). During nonflow constant volume processes, $Q = \Delta U$, regardless of working substance. Since it is common practice to analyze these cycles for constant specific heats, we have, for this assumption and for the closed cycle Fig 95(b),

$$Q_A = U_3 - U_2 = w c_v (T_3 - T_2) \text{ Btu,}$$

$$Q_R = U_1 - U_4 = w c_v (T_1 - T_4) = -w c_v (T_4 - T_1) \text{ Btu}$$

The net work W is ΣQ so that

$$(a) \quad W = U_3 - U_2 - (U_4 - U_1) = w c_v (T_3 - T_2) - w c_v (T_4 - T_1) \text{ Btu}$$

The thermal efficiency of the Otto cycle is

$$e = \frac{W}{Q_A} = \frac{U_3 - U_2 - (U_4 - U_1)}{U_3 - U_2} = \frac{w c_v (T_3 - T_2) - w c_v (T_4 - T_1)}{w c_v (T_3 - T_2)},$$

or

$$(b) \quad e = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad [c_p \text{ AND } c_v \text{ CONSTANT}]$$

The expressions in terms of internal energies are basic; but the analysis is continued for the air standard. To simplify equation (b), use the TV relation for an isentropic process, equation (35). Thus, $T_4/T_3 = (V_3/V_4)^{k-1}$ and $T_1/T_2 = (V_2/V_1)^{k-1}$; or, since $V_3 = V_2$ and $V_4 = V_1$,

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{k-1} = T_3 \left(\frac{V_2}{V_1} \right)^{k-1}, \quad \text{and} \quad T_1 = T_2 \left(\frac{V_2}{V_1} \right)^{k-1}.$$

Substituting these values of T_4 and T_1 into (b), we find

$$(c) \quad e = 1 - \frac{T_3(V_2/V_1)^{k-1} - T_2(V_2/V_1)^{k-1}}{T_3 - T_2} = 1 - \left(\frac{V_2}{V_1} \right)^{k-1}.$$

Let the adiabatic compression ratio V_1/V_2 be represented by the symbol r_k ; then for constant specific heats,

$$(61) \quad e = 1 - \frac{1}{r_k^{k-1}}.$$

Since we may wish to study something about the cycle related to pressures and volumes (mep, for example), we should keep in practice in writing work equations from the pV plane; thus, from Fig. 104(a),

$$(d) \quad W = \frac{p_2 V_2 - p_1 V_1}{1 - k} + \frac{p_4 V_4 - p_3 V_3}{1 - k} \text{ ft.-lb.,}$$

which, with $pV = wRT$ and $c_v = (R/J)/(k - 1)$, can be shown to be the same as equation (a) except for units.

147. Ideal Standards of Comparison. In equation (61), we have arrived at an important characteristic of the Otto cycle, to wit, that its efficiency with constant specific heats depends only on the value of k and the compression ratio r_k (Fig. 96). The efficiency of the real engine is subject to many other variables, of course, including simple thermodynamic factors such as the initial temperature and the temperature at 3 (amount of heat released). Nevertheless, one of the principal aims in the development of spark-ignition engines over the years has been to increase the compression ratio, whose value is now limited largely by the detonation characteristics of the fuel. If we use the air standard as a basis of comparison, we may consider the air as "cold air," $k = 1.4$, and the corresponding standard is called the *cold-air standard*. Let $r_k = 6$ and let the actual value of the thermal efficiency be $e' = 21\%$ by test; then we might say that the actual

engine is

$$\frac{e'}{e} = \frac{0.21}{1 - 1/r_k^{(k-1)}} = \frac{0.21}{1 - 1/6^{0.4}} = 41\%$$

as efficient as the ideal engine

The cold air standard is unfair to the engine, however, because operation is inevitably with rather hot gas most of the time. To keep the computations simple, some value of k which is roughly the average throughout the cycle is sometimes assumed, and the cycle is analyzed as explained above except that k is some value like 1.3 with the corresponding [equation (25)] value of c_v (and c_p if needed, Diesel cycle). If a hot air value of k is used, the standard is said to be a *hot-air standard*. Again, let $r_k = 6$ and $e' = 21\%$ by test, then as before, except that $k = 1.3$, we may say that the actual engine is

$$\begin{aligned} \frac{e'}{e} &= \frac{0.21}{1 - 1/r_k^{(k-1)}} \\ &= \frac{0.21}{1 - 1/6^{0.3}} = 50.5\% \end{aligned}$$

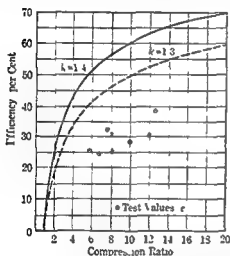


Fig 96 Efficiency Vs Compression Ratio Otto Cycle. The solid curve is for cold air, $k = 1.4$, dotted curve for hot air, $k = 1.3$. The test values, computed on the lower heating value and taken at random from the literature, suggest that actual efficiency tends to improve with ideal efficiency.

equations, Table II. A much less tedious approach is to use the air table, p. 146, and obtain a *variable specific-heat air standard*.

Still another alternative is to consider the actual fuel-air mixture and the products of combustion, including the phenomenon of dissociation, if any. This method results in the most realistic standard of comparison, and certainly there could be little reason left for the real engine to complain about being compared with such a standard, which we shall call the *real mixture standard*. Such calculations as are involved would be quite tedious and time consuming were it not for available aids—charts giving properties of real mixtures allowing for dissociation, and the *Gas Tables* (which do not account for dissociation).

as efficient as the ideal. This higher ratio of the efficiencies is a better indicator (than 41%) of the possible margin of improvement as the actual engine might be made to approach the ideal, and it is a fairer indicator of how good the actual engine is.

Another alternative is to integrate $\int c_v dT$ with variable specific heat

148. Clearance Volume. The compression ratio is varied by varying the *clearance volume*, which is the volume of the combustion space when the piston is on TDC position, Fig. 95. It is usually expressed as the clearance fraction or *per-cent clearance*, c . Thus, the clearance volume is cV_D , where V_D is the displacement volume. The compression ratio is

$$(e) \quad r_k = \frac{V_1}{V_2} = \frac{V_D + cV_D}{cV_D} = \frac{1 + c}{c},$$

from which the clearance can be computed from the compression ratio, or the compression ratio can be found from the clearance.

149. Example. An ideal Otto engine with 25% clearance operates on one pound of air with $k = 1.3$. In Fig. 95, $p_1 = 14$ psia, $t_1 = 120^\circ\text{F}$, and $t_3 = 4740^\circ\text{F}$. (a) What is the displacement volume? (b) Find t_2 , p_2 , and p_3 . (c) Find Q_A , Q_R , and e . (d) Find the mep of the ideal cycle and the percentage approach to perfection (commonly called engine efficiency), if the actual thermal efficiency is 24%.

SOLUTION. (a) First find V_1 and r_k .

$$V_1 = \frac{wRT_1}{p_1} = \frac{(53.3)(580)}{(14)(144)} = 15.33 \text{ cu. ft.}$$

$$r_k = \frac{(1 + c)}{c} = \frac{1.25}{0.25} = 5 = \frac{V_1}{V_2}.$$

From this we find

$$V_2 = \frac{V_1}{r_k} = \frac{15.33}{5} = 3.066 \text{ cu. ft.}$$

$$V_D = V_1 - V_2 = 15.33 - 3.066 = 12.26 \text{ cu. ft.}$$

(b) Use the TV relation for an isentropic process and Charles' law.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}, \quad \text{or} \quad T_2 = (580)(5^{1/3-1}) = 940^\circ\text{R} = 480^\circ\text{F}.$$

From $p_1V_1^k = p_2V_2^k$,

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^k = (14)(5^{1/3}) = 113.4 \text{ psia.}$$

From Charles' law ($T_3 = 4740 + 460 = 5200^\circ\text{R}$),

$$p_3 = p_2 \left(\frac{T_3}{T_2}\right) = (113.4) \left(\frac{5200}{940}\right) = 627 \text{ psia.}$$

(c) First, we must find c , corresponding to the value of $k = 1.3$.

$$c_v = \frac{R}{J(k-1)} = \frac{53.3}{(778)(1.3-1)} = 0.228.$$

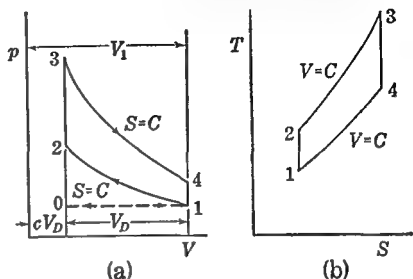


Fig. 95. Repeated.

$$Q_A = u c_v (T_3 - T_2) = (0.228)(5200 - 940) = 971 \text{ Btu}$$

$$T_4 = T_3 \left(\frac{1}{r_k} \right)^{k-1} = \frac{5200}{(5^{0.3})} = 3210^\circ \text{R}$$

$$Q_R = u c_v (T_1 - T_4) = (0.228)(580 - 3210) = -600 \text{ Btu}$$

$$e = \frac{\Sigma Q}{Q_A} = \frac{971 - 600}{971} = 38.2\%$$

$$e = 1 - \frac{1}{r_k^{k-1}} = 1 - \frac{1}{5^{0.3}} = 38.3\% \text{ (check)}$$

$$(d) \quad p_m = \frac{W}{V_D} = \frac{e Q_A}{V_D} = \frac{(0.383)(971)(778)}{(12.26)(144)} = 164 \text{ psi}$$

$$\eta = \frac{\text{actual thermal efficiency}}{\text{ideal thermal efficiency}} = \frac{0.24}{0.383} = 62.7\%$$

We have now solved examples of three cycles with the same isentropic compression ratio $r_k = 5$ the Carnot § 89 the Brayton § 134, and the Otto herewith. Let us compare the corresponding meps even though they are not computed on the same basis (Carnot cold air Brayton variations of specific heat allowed for, Otto hot air) Carnot 11 psi Brayton 26.2 psi Otto 164 psi. Recalling that the relative sizes of the engines are roughly inversely proportional to the meps for a particular power and speed we see immediately the advantage of the Otto cycle at relatively low speeds. The possibility of running gas turbines at much higher speeds than the reciprocating ICE compensates for the relatively low mep of the Brayton cycle. Actual meps are naturally less than those of the ideal cycles.

150. Diesel Cycle Rudolf Diesel* was primarily interested in developing an internal combustion engine to operate on coal as a fuel. The final outcome, however, was a four stroke-cycle engine (§ 145) in which air only is taken into the cylinder on the suction stroke and a liquid fuel is later injected, the injection starting theoretically at the end of the compression stroke and continuing at such a rate that burning proceeds at constant pressure 2-3 Fig. 97. Otherwise the cycle operates as the Otto cycle does and the air standard (closed) cycle is 1-2-3-4 Fig. 97. The ideal open air cycle would be the same except that 0-1 is the suction and 1-0 is the discharge. Thus, for constant specific heats we have

$$Q_A = w c_p (T_3 - T_2) \text{ Btu}$$

$$Q_R = u c_v (T_1 - T_4) = -u c_v (T_4 - T_1) \text{ Btu}$$

$$W = \Sigma Q = u c_p (T_3 - T_2) - u c_v (T_4 - T_1) \text{ Btu}$$

$$(f) \quad e = \frac{W}{Q_A} = 1 - \frac{c_v (T_4 - T_1)}{c_p (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)}$$

This expression (f) is usable but it may be placed in a more convenient and

* Rudolf Diesel (1858-1913) born in Paris of German parents who later moved to London because of the Franco-German War (1870) educated in Germany obtained in 1893 a patent on the type of engine which now bears his name. After some difficulty in financing the project he built an engine which blew up at the first injection of fuel. Diesel narrowly escaped being killed. Four years of tedious and costly experiment elapsed before he produced a successful engine. He inexplicably disappeared in 1913 while crossing the English Channel during a storm.

revealing form by eliminating the temperatures. We may express three of the temperatures in terms of the fourth, say, in terms of T_1 . Thus, along the isentropic 1-2, Fig. 97, $T_2/T_1 = (V_1/V_2)^{k-1}$. However, V_1/V_2 is defined as the compression ratio, r_k . Therefore

$$(g) \quad T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1} = T_1 r_k^{k-1}.$$

Along the constant pressure line 2-3, Charles' law holds and $T_3/T_2 = V_3/V_2$. Let $V_3/V_2 = r_c$, a ratio termed the fuel cutoff ratio. We find then

$$(h) \quad T_3 = T_2 \left(\frac{V_3}{V_2} \right) = T_1 r_k^{k-1} r_c$$

by using equation (g). For the isentropic process 3-4, $T_4/T_3 = (V_3/V_4)^{k-1}$. But from 2-3, $V_3 = (T_3/T_2)V_2 = r_c V_2$. Using this value of V_3 , the value of T_3 from equation (h), and using the definition of the compression ratio namely $r_k = V_1/V_2$, we get

$$(i) \quad T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{k-1} = T_1 r_c^k.$$

Substituting into equation (f) the values of T_2 , T_3 , and T_4 just found, we have

$$(62) \quad e = 1 - \frac{T_1 r_c^k - T_1}{k(T_1 r_k^{k-1} r_c - T_1 r_k^{k-1})} \\ = 1 - \frac{1}{r_k^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)} \right].$$

Observe that this expression for the efficiency of the Diesel cycle differs from that of the Otto cycle [equation (61)] only in the bracketed factor. This factor is always greater than 1, because r_c is always greater than 1. Thus, for a particular compression ratio r_k , the Otto cycle is more efficient. (See also Fig. 98.) However, if the compression ratio is too high in an Otto engine, a knocking occurs due to self-ignition of the fuel. Since the Diesel engine compresses air only, the compression ratio is higher than in an Otto engine. Therefore, an actual Diesel engine with, say, $r_k = 15$ is more efficient than an actual Otto engine with $r_k = 9$. In passing, we may note the relation between the compression ratio r_k , cutoff ratio r_c , and expansion ratio r_e ($V_4 = V_1$, Fig. 97):

$$(j) \quad r_k = \frac{V_1}{V_2} = \left(\frac{V_3}{V_2} \right) \left(\frac{V_1}{V_3} \right) = r_c r_e.$$

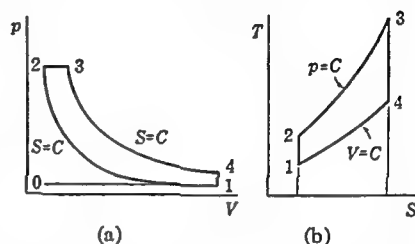


Fig. 97. Diesel Cycle. Between the same temperature limits, the constant volume curve on the TS plane is steeper than the constant pressure curve (§ 67). However, both curves get steeper as the temperature increases. Thus, the temperature at 3 is so high that the constant pressure curve is steeper than the constant volume curve, which is at the lower temperatures between 1 and 4.

Study of equation (62) shows that as r_c increases, the bracketed factor increases, and the efficiency decreases (Fig 99). Therefore, the lower fuel cutoff ratios are conducive to higher efficiencies but larger ratios result in greater power. However, there is a limit to the amount of fuel which

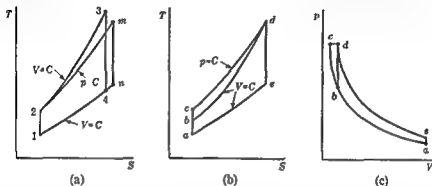


Fig 98 Comparison of Otto and Diesel Cycles These cycles may be compared in many different ways and these sketches, which are qualitative with respect to areas may be of interest. In (a), they are sketched for the same compression ratio and the same heat added, that is, area under 2-3 (Otto) is equal to area under 2-m (Diesel). We see that the Diesel rejects more heat $n-1$ than the Otto does $4-1$, a visual demonstration that the Otto is more efficient. But the compression ratio is not the same in the two cycles as they are actually used, so (b) and (c) are sketched for the same temperature and pressure d after combustion (which is not true either, except by chance). At any rate, you can see how the areas are affected.

can be injected without excessive 'smoking', hence the compromise usually is such that cutoff seldom occurs later than 10% of the stroke, corresponding to a cutoff ratio of about 2.4 usually earlier.

As in the Otto cycle the value of k in the cold air standard is 1.4. Lower values, say about 1.35 would be used in the hot-air standard. A preferred

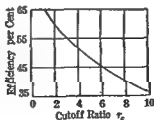
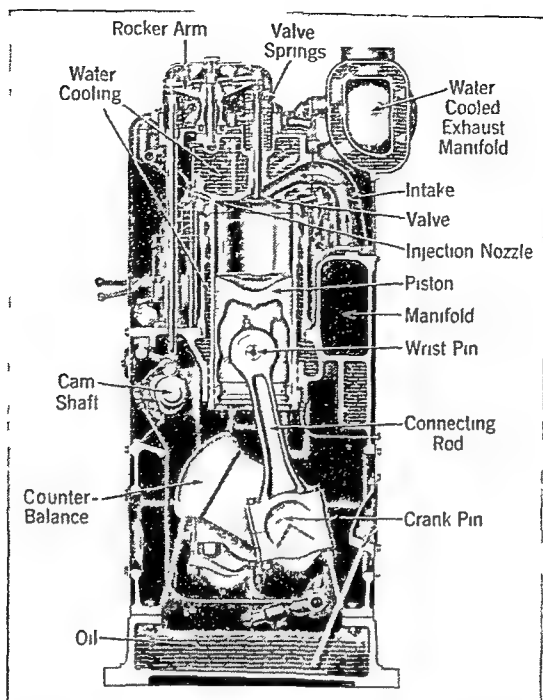


Fig 99 Diesel Efficiency Vs Cutoff Ratio—Constant Compression Ratio

standard of comparison would be one based upon an analysis of the real mixture in the engine and accounting for the variation of specific heat. An end-view cross section of a four stroke Diesel engine is shown in Fig 100.



Courtesy Ingersoll-Rand Co., New York

Fig. 100. Four-Stroke Diesel Engine. Study the picture for detail. A hole at the top of the drilled connecting rod serves to spray the piston head with oil for cooling. The bore and stroke are $10\frac{1}{2} \times 12$ in., assembled with 5 to 8 cylinders; 375 to 600 bhp; supercharged, 570–900 bhp; supercharged with cooling of supercharged air, 600–1000 bhp; all at 720 rpm. Figure 103 is an external view of this engine.

151. Fuel Cutoff Ratio. The question arises as to what is the ideal Diesel cycle corresponding to a real engine operating at a particular load. The point 2 at the end of isentropic compression is defined by the compression ratio, which is a characteristic number of the engine and is usually known. To locate state 3, Fig. 97, we note that the first law applied to the process 2-3 gives

(Stored energy)₂ + energy entering = (stored energy)₃ + energy departing as work,

$$u_2 + (w_f h_f + w_f E_c) = (1 + w_f) u_3 + \frac{p_3(1 + w_f) v_3 - p_2 v_2}{J},$$

$$(k) \quad h_2 + w_f h_f + w_f E_c = (1 + w_f) h_3,$$

where h_2 is the specific enthalpy of the gases in cylinder at 2, w_f is lb fuel

per lb air, h_f is the enthalpy of the entering fuel (h_f is often negligible in

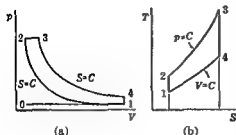


Fig 97 Repeated

which case it may be dropped), E_c is the chemical energy released by combustion in Btu per lb of fuel and h_3 is the specific enthalpy of the products of combustion. The enthalpies h_2 , h_f , h_3 and the energy E_c should be reckoned from the same datum (14) when you are accounting for the real mixtures. With the proper aids in the way of tables or charts (14, 51), equation

$$(k) \text{ is easy to use to solve for } h_3, \text{ then other state properties at 3 can be determined. However, the air standard can be closely defined if in equation (k), we let } w_f E_c = w_f q_1 \text{ where } q_1 \text{ Btu per lb is the lower heating value of the fuel at constant pressure, and if we let the other terms be represented by the approximation } w_T(h_3 - h_2),$$

$$(l) \quad w_f q_1 = w_T(h_3 - h_2) = u_T \int c_p dT = u_T c_p (T_3 - T_2),$$

[AIR STANDARD]

where $w_T = w_a + w_f$, the total mass of working substance, which may be on the basis of $w_a = 1$ lb of air, and where c_p in the final form is some mean value for the cycle based on the hot air standard. For the air standard T_2 is computed from the properties at 1. Since from Charles' law $V_3/V_2 = T_3/T_2$, the cutoff ratio for the corresponding ideal cycle is $r_c = V_3/V_2 = T_3/T_2$.

152 The Two-Stroke Cycle Many small gasoline engines and many Diesels large and small, operate on a two stroke cycle. Since the exhaust stroke in the real engine is for the purpose of *scavenging* the cylinder (ridding the cylinder of the products of combustion), it is only necessary to provide other means of scavenging in order to be able to complete the cycle in two strokes (one revolution). An idealized sort of indicator card is shown in Fig 101. Compression has been completed at point c , combustion occurs cd , followed by an expansion. In the two stroke cycle exhaust begins early at some point e and scavenging is accomplished by blowing air (or air and fuel) into the cylinder. To allow time for scavenging or for the introduction of fuel mixtures the valves usually remain open until the piston has moved to some position corresponding to b , where compression begins.

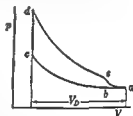


Fig 101 Two Stroke Cycle

In four-stroke cycles, there is *one* power stroke for *two* revolutions, in two-stroke cycles, there is *one* power stroke for *one* revolution. However,

the two-stroke-cycle engine, instead of developing 100% more power, develops only some 70% to 90% more than the four-stroke-cycle engine of the same displacement, because of (1) poorer scavenging, (2) a smaller mass of combustible mixture in a given size of cylinder, (3) a greater loss of unburned fuel, (4) a small power consumption in compressing the air which scavenges the cylinder, and (5) the loss of pressure by early exhaust.

Compressed air for scavenging is obtained from crankcase compression, as in some outboard motors and lawn-mower engines, or from separate blowers.

153. Brake Power. The name *brake work* came about because in the early days of small, slow-speed engines the power output was dissipated in the friction of a brake. Brakes are still used for this purpose over suitable ranges of power and speed. A type known as a **prony brake** is shown diagrammatically in Fig. 102. When the brake is clamped to the flywheel,

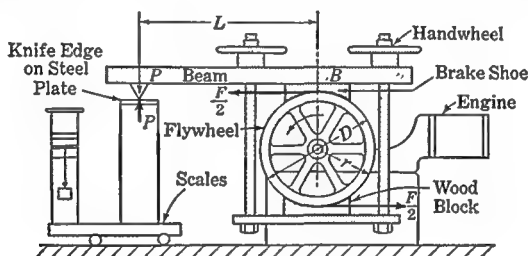


Fig. 102. Prony Brake.

the frictional force F ($= F/2 + F/2$ at the brake shoes) tends to turn the brake with the flywheel. However, the knife edge on the beam rests on scales and prevents motion of the brake. The force P , which is weighed by the scales, consists of the reaction produced by friction and a portion of the weight of the brake, called the *tare*—unless there is a counterweight to balance the brake about the centerline of the flywheel. To find the tare, support the brake on an *edge* at B and weigh the tare on the scales. Thus, the net force on the scales produced by the frictional moment is $(P - \text{tare})$.

The information obtained from a prony brake test is used to compute the brake horsepower. Thus, the frictional force F acting through one revolution of the flywheel does work $W = (F\pi D)/12$ ft.-lb., where the flywheel diameter D is expressed in inches (usual practice). The work multiplied by n revolutions per minute gives the work in ft.-lb. per min. Dividing this result by 33,000 converts to horsepower; thus

$$(63) \quad hp = \frac{F\pi Dn}{(12)(33,000)} = \frac{F2\pi rn}{(12)(33,000)} = \frac{Tn}{63,000},$$

where $T = Fr = FD/2$ in.-lb. is the frictional torque on the flywheel. The sum of the moments of the forces on the brake about the center of the fly-

wheel gives

$$(m) \quad T r = \frac{F D}{2} = (P - \text{tare}) L = T,$$

where L in inches is the moment arm of the force P . Thus knowing the dimension L on the brake we may observe the value of P indicated by the scales and the number of rpm of the engine and calculate the brake horsepower (bhp) from equation (63). Sometimes the tare is neglected some times it is balanced by a counterweight.

Other instruments for measuring shaft work are a hydraulic brake and a dynamometer. Because the ordinary type of indicator (Fig. 62) has too much inertia to respond accurately at high speed it became customary to measure the power output of automotive and other similar engines by dynamometer only. This practice brought about the common use of the **brake mean effective pressure** (bmep) p_{mB} which is simply the pressure computed from the mep equation (53) using bhp instead of *thp* (review §§ 86 and 111),

$$(53C) \quad p_{mB} = \frac{33\,000 \text{ bhp}}{LAN}$$

While dynamometer tests are still routine industrial practice high speed indicators utilizing the oscilloscope and also optical and photographic effects have been developed so that it is now possible to obtain from high speed engines the valuable information revealed by an indicator card.

154 Example At 3000 rpm a six cylinder four stroke gasoline engine $3\frac{1}{2} \times 4$ in (always bore \times stroke) develops 50 bhp. What is the bmep?

SOLUTION To find the number of cycles or power strokes per minute N we note that there are 2 revolutions/cycle for each cylinder or $\frac{1}{2}$ cycle/cylinder in one revolution which is $6 \times \frac{1}{2} = 3$ cycles/revolution for the six cylinder engine therefore $N = (3)(3000 \text{ rpm}) = 9000 \text{ cpm}$. From equation (53C)

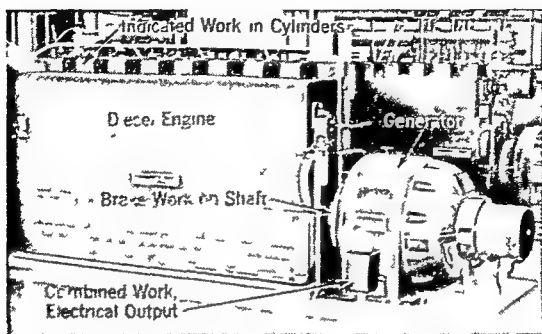
$$p_{mB} = \frac{(33\,000)(80)}{(5\frac{1}{2})(\pi 3.25^2/4)(9000)} = 106 \text{ psi}$$

155 Thermal Efficiencies The thermal efficiency $e = W/Q_A$ has already been defined a number of times. In general terms for a power cycle or engine

$$(47A) \quad \text{Thermal efficiency} = \frac{\text{work output of the system}}{\text{energy chargeable against the system}}$$

For ideal cycles and engines the numerator and denominator are ideal values which are defined for each ideal system taken up. In this expression the work may be any energy which is 100% available: electricity, for example. There is no one *actual* thermal efficiency, because there are several

places where work and power may be measured (Fig. 103). You know about the indicated and brake horsepower. The combined work or combined power, measured by electrical instruments on the instrument panel, say, in kilowatts (kw), is the output of the generator. If we let Q'_A represent



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Fig. 103. Meaning of Indicated Work, Brake Work, and Combined Work; W_I , W_B , W_K

the actual energy to be charged against a system, we have three thermal efficiencies, the indicated e_i , the brake e_b , and the combined e_c as follows:

$$(47B) \quad e_i = \frac{W_I}{Q'_A}, \quad e_b = \frac{W_B}{Q'_A}, \quad e_c = \frac{W_K}{Q'_A}.$$

The symbol Q'_A will be defined for each actual engine analyzed. As you know from preceding chapters, we charge the heating value of the fuel against the gas-turbine unit and the internal combustion engine, preferably the lower, $w_f q_l$, where q_l is Btu per lb. of fuel* and w_f is in units suitable for the units of the numerator of equation (47B). Thus, if W is in Btu per lb. of air, then w_f must be lb. of fuel per lb. of air; etc. If w_f is the *specific fuel consumption* in pounds of fuel per unit of work, the term $w_f q_l$ Btu per unit of work is called the *heat rate*, which is defined as the energy supplied to the system per unit of work. The unit of work is generally the hp-hr. (2544 Btu) or the kw-hr. (3412 Btu). From these values, we obtain other important equations for thermal efficiency:

$$(64) \quad e = \frac{2544}{w_f q_l} \left(\frac{\text{Btu/hp-hr.}}{\text{Btu/hp-hr.}} \right), \quad \text{or} \quad e = \frac{3412}{w_f q_l} \left(\frac{\text{Btu/kw-hr.}}{\text{Btu/kw-hr.}} \right)$$

or

$$(64A) \quad e_i = \frac{2544}{w_{fi} q_l}, \quad e_b = \frac{2544}{w_{fb} q_l},$$

where the specific fuel consumption w_{fi} is in lb. per ihp-hr. and w_{fb} is in

* The higher heating value is frequently used in this country for ICE (See § 157.)

lb per bhp hr If w_{fk} is in lb per kw-hr output of the generator, we have

$$(64B) \quad e_k = \frac{3412}{w_{fk} q_1}$$

156 Engine Efficiencies. The engine efficiency η has been defined in connection with gas-turbine units (§§ 132 and 142), in general terms, let it be*

$$(60) \quad \text{Engine efficiency } \eta = \frac{\text{actual work of a system}}{\text{work of the corresponding ideal system}}$$

Applied to reciprocating internal combustion engines, this definition results in three engine efficiencies corresponding to the three actual works, W_I , W_B , and W_K . Thus for the *brake engine efficiency* η_b , the *indicated engine efficiency* η_i , and the *combined engine efficiency* η_k , we have

$$(60A) \quad \eta_i = \frac{W_I}{W}, \quad \eta_b = \frac{W_B}{W}, \quad \text{and} \quad \eta_k = \frac{W_K}{W},$$

where the ideal work W is in the same units as the numerator and is computed for the corresponding ideal system. The corresponding ideal system for an Otto engine is one which has the same compression ratio as the actual engine and the same energy supplied ($Q_A - Q'_A$). For a Diesel engine, the corresponding ideal system is an ideal Diesel cycle whose r_k and Q_A are the same as for the actual. In each case, one must of course, decide upon what standard of comparison to use, cold air, hot air, variable specific heat, or real mixture. In those cycles where the actual Q'_A and the ideal Q_A are the same, as in Otto and Diesel cycles (but not gas turbine-unit cycles), the engine efficiencies are also ratios of the thermal efficiencies ($W = eQ_A$)

$$(a) \quad \eta_i = \frac{e_i}{e}, \quad \eta_b = \frac{e_b}{e}, \quad \eta_k = \frac{e_k}{e} \\ \text{[WHEN } Q_A = Q'_A]$$

Since $W = p_m V_D$ [equation (49), p. 97] we get other ratios from equation (60) as follows

$$(c) \quad \eta_i = \frac{p_{mi}}{p_m}, \quad \eta_b = \frac{p_{mB}}{p_m}, \quad \eta_k = \frac{p_{mK}}{p_m},$$

where p_m is the mep of the corresponding ideal cycle and the numerators are determined from equation (53). See equation (53C), § 153. The reader can find other ratios which give the engine efficiency, for example, horse-powers and fuel rates.

Since the engine efficiency can often be estimated closely from previous experience with a certain kind of engine, it is a convenient design factor for use in determining the size of an actual engine to produce a specified amount

* This is different from the previous editions of this book, but it accords with ASME Codes.

of power. The engine efficiency was applied to a turbine in § 132, equation (n).

157. Actual Thermal Efficiency. As mentioned in Chapter 8 on gas turbines, there is some question as to what heating value to use in computing the efficiency. Although the lower heating value is commonly used in other countries, the practice in this country has been to use the higher heating value, which accords with the argument that the engine is at fault if it cannot use any part of the latent heat of the H_2O formed by combustion. However, the fact remains that none of the H_2O is likely to condense in the ICE: hence, it is said to be unfair to the engine to charge it with the higher value. Consider the definition of thermal efficiency, $(Q_A - Q_R)/Q_A$. There is agreement that the Q_A in the numerator should be the lower heating value; that is, the lower value is used to determine the temperature at 3 after combustion. Now in order to be consistent thermodynamically, we should use the lower value for the Q_A in the denominator too. It is suggested that in this course the lower heating value be used for all combustion engines, as now done by some ICE designers—unless the instructor directs otherwise. Thus, the thermal efficiency is the work divided by the energy $E = w_f q_l$ chargeable against the engine,

$$(p) \quad e = \frac{W}{w_f q_l}$$

where e is a dimensionless ratio and the numerator and denominator must therefore have the same units. For example, if w_f is the *specific fuel consumption* in lb. of fuel per hp-hr., a value commonly obtained on test from the total fuel used in a particular time when the engine is steadily delivering a certain horsepower, if q_l Btu per lb. of fuel is the lower heating value of the fuel, then

$$(q) \quad e = \frac{2544 \text{ (Btu/hp-hr.)}}{w_f \left(\frac{\text{lb. fuel}}{\text{hp-hr.}} \right) \times q_l \left(\frac{\text{Btu}}{\text{lb. fuel}} \right)} = \frac{2544}{w_f q_l}$$

158. Mechanical Efficiency. The mechanical efficiency η_m is a number which tells of the mechanical losses in a machine. For the generator (Fig. 103) it includes internal electrical losses;

$$(r) \quad \eta_m = \frac{W_K}{W_B} \quad [\text{GENERATOR}]$$

For a reciprocating engine of any type which delivers work,

$$(s) \quad \eta_m = \frac{W_E}{W_I} \quad [\text{RECIPROCATING ENGINE}]$$

See § 121 for η_m for a compressor. The foregoing works may be expressed in any convenient work or power unit or in terms of any numbers which are

proportional to the W 's, but remember that engine efficiency is a dimensionless ratio. The difference (ihp) - (bhp), or $W_I - W_B$, represents the loss due to mechanical friction of the moving parts of the engine, expressed in horsepower, it is called the *friction horsepower* (fhp), $fhp = (1 - \eta_m)(ihp)$. Be sure to note that mechanical efficiency is not a fixed number characteristic of the machine but that it depends upon operating conditions, especially output, speed, and lubrication.

159. Volumetric Efficiency. There are a number of factors which account for power loss in an internal combustion engine, among which are the obvious mechanical-friction loss and the loss which accompanies the heat rejected to the cooling medium, which is the cooling water in the automobile engine. Another factor is that combustion is not instantaneous as assumed in an ideal Otto cycle (nor at constant pressure as assumed in the Diesel cycle), nor is combustion complete—some unburned components escape in the exhaust. A significant factor is the time and work involved in pumping the working substances into and out of the cylinder, a loss which becomes relatively large at high speeds. This is a fluid friction loss. There is also a loss in multicylinder engines because the manifold is imperfect, the same mixture (air-fuel ratio) is not delivered to every cylinder.

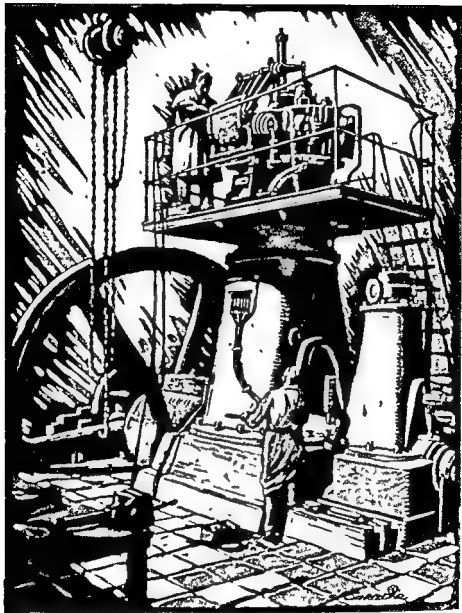
Other factors remaining the same, the power obtained from an engine which draws in air and fuel depends upon the mass of combustible mixture drawn into the cylinders—given a mixture with the correct air-fuel ratio—and anything which reduces the mass of fuel entering the engine reduces the power output below what could have been obtained. For example (1) in the real engine, we have found that, because of the fluid friction of flow or throttling around the valves and in the passages, the suction pressure is less than atmospheric pressure, and therefore the mass of gas ($w = pV/RT$) is less than if atmospheric pressure were maintained, (2) the internal surfaces and passages of the engine are relatively hot, so that the mixture is heated as it passes into the cylinder. In accordance with Charles' law, the increase in temperature further reduces the weight of mixture that the given displacement can contain, (3) the gases in the *clearance space* of the real engine are at a pressure *above* atmospheric at the end of the exhaust stroke and must expand during the suction stroke to the intake pressure before a new charge begins to enter, (4) the pressure of the atmosphere decreases with altitude, so that the mass of mixture drawn in at high altitudes is still further decreased below that which would be drawn in at sea level.

The mass of charge brought into the cylinder is sometimes defined in terms of *volumetric efficiency* (that is what we call it, § 117, but it is a ratio of masses in an ICE), which is defined as (49)

$$(t) \quad \eta_v = \frac{\text{mass of air drawn into the engine}}{\text{mass of air that would occupy the displacement volume at } p_a \text{ and } T_a}$$

where p_e and T_e are the air pressure and temperature at intake, say, in the test room. The value of the denominator is $w = (p_e V_D)/(R_e T_e)$. The numerator and denominator in equation (t) must be in the same units. Usually we use mass per engine cycle or mass per minute. The displacement volume *per engine cycle* is the volume swept out by the piston *in one stroke*; this number, multiplied by the number of cycles per minute, gives the displacement per minute (§ 86). In a particular engine, volumetric efficiency is affected by the speed. Suppose, for example, that the valve timing has been adjusted for maximum volumetric efficiency at 2000 rpm. Then, in general, as the speed increases, the volumetric efficiency decreases because of the greater throttling effect (fluid friction) at higher speeds. The volumetric efficiency as defined by equation (t) can be made greater than unity by use of a supercharger.

160. Closure. There are economic and advantageous uses for all kinds of prime movers. The Otto, or spark-ignition, type is especially suited to low power (less than several hundred horsepower) where rotative speeds up to say 4000–6000 rpm are adaptable. Large Otto engines are more prone to detonation than small ones, because the flame front has farther to travel in large combustion chambers, so that this is a factor in limiting the size of such engines. Diesel or compression ignition engines overlap Otto engines on size, because they burn a cheaper fuel. For this reason, they are widely used on trucks and buses although they are more expensive per horsepower. Diesels are used in much larger units than Otto engines (up to several thousand horsepower, maybe 8000), and are excellent power plants for marine use, locomotives, and small electric generating stations. Internal combustion engines are appropriate for mills which operate on a seasonal basis, because of the ease of maintenance and start-up. To keep perspective, the ICE does not compete in central station power plants, in the largest of which, single compound steam-turbine units of more than 100,000 hp are often found.



Courtesy Johnson Publishing Co Cleveland

Rudolf Diesel and the Diesel Engine

10

LIQUIDS AND VAPORS

161. Introduction. Up to this point, we have considered substances which were entirely gaseous or entirely liquid. However, one of the most common situations in practice concerns a mixture of a liquid and its vapor, called a *two-phase system*, such as the water and steam in a boiler. Frequently, either the liquid is being evaporated or the vapor is being condensed. Also, it often happens that a mixture of a vapor with some liquid suspended in it enters a process or emerges from one. In any event, we must be able to determine the properties of liquid-vapor mixtures, which is the subject of this chapter.

In case you have not studied the preceding chapter, which is not essential for an understanding of the remainder of this book, we should note the difference between a vapor (imperfect gas) and an ideal gas. When any gas is cooled, it approaches a state where it will begin to condense. When the substance is about to condense or when the pressure is relatively high, or both, the molecules are relatively close together, so that the molecular forces are large and the volume of the molecules themselves is a significant portion of the total space containing the substance. In an ideal gas, the molecules exert no forces on one another and have zero volume. The outcome is that, except at quite low pressures, the substances which we call vapors behave very differently from ideal gases, and equations of state which express their actions are likely to be complicated. To avoid the use of these complicated equations, engineers have developed tables which give significant properties of several substances. For illustrative purposes, we shall use steam most often, but the methods outlined apply to other imperfect gases for which thermodynamic properties are available.

If a vapor is heated to a temperature far from that at which condensation will occur, or if the pressure is reduced to low values, it will act nearly in accordance with the ideal gas laws and may be treated as a perfect gas.

162. Vaporizing a Liquid at Constant Pressure. To understand the states in which a liquid, a vapor, and a mixture of a liquid and vapor may exist, consider the phenomenon of a liquid being heated while the pressure remains constant. Let there be one pound of water in a cylinder, Fig 104(a), and let the weight w be such that the pressure on the water is 100 psia. Suppose further that the temperature of the water is 32°F . Now let heat be added to the water. The temperature increases, and during most of the time the volume of the liquid increases. The increase in volume causes the piston and weight to move up, thus, work is done in moving the piston against this pressure. This work, however, is only a very small portion of the heat added to the liquid during the rise in temperature, that is, the temperature change substantially is a measure of the change of internal energy.

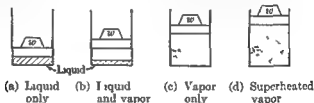


Fig 104. Heating at Constant Pressure

The temperature of the liquid is soon such that it begins to boil. The temperature at which a liquid boils depends upon the pressure on it. For each pressure, there is a precise temperature that marks the boiling point of a particular liquid. This temperature is called the *saturation temperature*, and when a liquid is at this temperature, it is called a *saturated liquid*. The saturation temperature is a function of the pressure. As you know, water boils at 212°F when the pressure is atmospheric, 14.696 psia. At 100 psia, water boils at 327.81°F . At 100 psia, ammonia boils at 56.05°F . These data are taken from vapor tables (§ 103). Another characteristic of the boiling process is that the temperature of the liquid and vapor remains constant at the saturation temperature as long as there is any liquid present*. Thus, while the water in the cylinder of Fig 104(b) and (c) is evaporating, the temperature remains at 327.81°F .

In Fig 104(b), part of the water has evaporated. The cylinder has in it a mixture of steam and water. A mixture of a vapor and its liquid is called a *wet mixture* or a *two-phase system*. The state of a wet mixture at a given pressure is expressed by its *quality*, or *percentage moisture*. The *quality* x of a mixture is the per cent by weight which is vapor. Thus, if the quality is $x = 75\%$, then in one pound of mixture, 0.75 lb is vapor and 0.25 lb is liquid. The *percentage moisture* is the per cent by weight of the mixture

* This statement assumes a condition of internal equilibrium and is substantially true, but actually, small differences of temperature in the various parts of the mixture are bound to exist.

which is liquid, so that 25% moisture means that, in one pound, 0.25 lb. is liquid and 0.75 lb. is vapor.

If the transfer of heat to the mixture continues, all the liquid will eventually be evaporated [Fig. 104(c)]. At the point when the last drop of liquid is evaporated, the temperature of the vapor is the saturation temperature. Vapor which is at the saturation temperature and 100% quality is known as *saturated vapor*. (The mixture in Fig. 104(b) is one of saturated water and saturated steam and is therefore also called a *saturated mixture*.)

Finally, if further heat is added to the vapor, its temperature will rise, its volume will increase [Fig. 104(d)], and the vapor is said to be *superheated*. *Superheated vapor* is vapor at any temperature above the saturation temperature. If thermal equilibrium exists within the superheated vapor, there can be no liquid in it. To define the state of a superheated vapor, we usually state the pressure and the temperature. Often we speak of the *degrees of superheat*, which is the difference between the actual temperature of the superheated vapor and the saturation temperature for the existing pressure. For example, suppose the steam is at a pressure of 100 psia and a temperature of 500°F. Since the saturation temperature of steam at this pressure is 327.81°F (see above), the degrees of superheat are $500 - 327.81 = 172.19^\circ$.

163. Vapor Tables. The computations of the properties of vapors are based upon the data of extensive and carefully conducted experiments. Different sets of vapor tables may give slightly different values for the properties of a saturated vapor or liquid at a particular pressure. Such differences arise because of variations in experimental data from which the values in the tables are computed. However, so many data have now been accumulated for steam that we can say confidently for most values that the true property lies between certain limits which are fairly close together. Both the values quoted on pp. 196-199 from Keenan and Keyes (71) and the steam tables quoted in the *problem book* fall within internationally adopted tolerances. The Keenan and Keyes values are the preferred ones in this country.

The nature of these tables is best explained by quoting briefly from the contents. The first two columns of Tables VI and VII are corresponding saturation values of p and t . Beginning with the third column:

v_f is the volume of 1 lb. of saturated liquid, the *specific* volume of the water at the stated pressure or temperature;

v_{fg} is the *change* of volume undergone when 1 lb. of water evaporates to 1 lb. of steam;

v_g is the *specific* volume of the steam;

h_f is the enthalpy of 1 lb. of water, the *specific* enthalpy;

h_{fg} is the *change* of enthalpy during the vaporization of 1 lb. of liquid;

h_g is the *specific* enthalpy of (1 lb.) saturated steam;

s_f is the entropy of 1 lb of saturated liquid,

s_{fg} is the change of entropy during the evaporation of 1 lb of liquid,

s_g is the specific entropy of saturated vapor

u_f and u_g are the specific internal energies of saturated liquid and saturated vapor respectively (in Table VII only)

In connection with these tables note that the volume of the liquid v_f increases as the pressure and temperature increase. However, the change of volume is practically independent of the change in pressure, unless the change in pressure is very large. This statement is on the assumption that the liquid is nearly incompressible—a satisfactory assumption for the average practical problem. It follows that the noted increase in volume is due to the increase in temperature, so

when the specific volume of the liquid is desired the volume corresponding to the actual temperature should be found

no matter what the pressure may be.* The tables show that the volume of the saturated vapor v_g is equal to the volume of the saturated liquid v_f plus the change in volume during vaporization v_{fg} , that is

$$(a) \quad v_g = v_f + v_{fg}$$

By definition the enthalpy h_f of saturated liquid at 32°F is zero (Table VI). Since the enthalpy of any substance is defined by $h = u + pv/J$ the internal energy u_f for the saturated water at 32°F is

$$u_f = -\frac{pv}{J} = -\frac{(144)(0.08854)(0.01602)}{778} = -0.00026 \text{ Btu}$$

The negative sign indicates merely that the measurement is below the chosen datum.

We see from Table VI that if the pressure is 0.08854 psia water will boil at 32°F. To bring about this evaporation we must add heat to the water the amount needed for saturated water being $h_{fg} = 1075.8$ Btu per lb (Table VI) when the pressure is maintained constant. The heat transferred to one pound of saturated liquid to evaporate it is often called the *latent heat of evaporation*, or simply the *latent heat*. Observe from Tables I and VII that the latent heat of steam h_{fg} decreases as the saturation pressure and temperature increase and that

$$(b) \quad h_g = h_f + h_{fg}$$

As in the case of enthalpy the datum of entropy is saturated water at 32°F, where $s_f = 0$. Observe that

$$(c) \quad s_g = s_f + s_{fg}$$

* At very high pressures the assumption of incompressibility of water is in error and some liquids are more compressible than water. See Table IX and § 168.

164. The pv and Ts Planes. Since we shall now be dealing with mixtures of liquids and vapors, it will be convenient to draw on the pv and Ts planes lines that mark the boundaries of regions representing the various phases. From the values in the tables, we may plot points through which may be drawn curves which are called the *saturated liquid line* and the *saturated vapor line* or *dry vapor line*. See Fig. 105 for the method. We commonly use the terms *liquid line* and *vapor line* for short, and they are so labeled in Fig. 105, but the word *saturated* is always understood. The liquid line meets the vapor line on both pv and Ts planes at the *critical point* (§ 172).

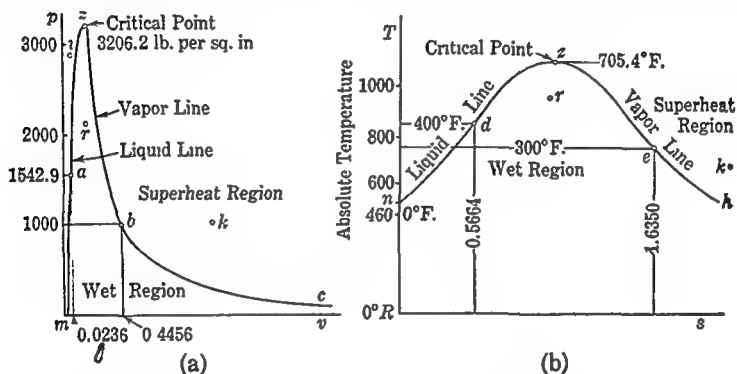


Fig. 105. Liquid and Vapor Lines on pv and Ts Planes. The curves are obtained as follows: Plot the pressures against v , (as taken from vapor tables) to find the saturated liquid line maz in (a). For example, let the pressure be 1542.9 psia; the corresponding $v_l = 0.0236$ cu. ft. (Table VI). Lay off these values in (a) and locate point a . Other points are plotted in a similar manner. A smooth curve through these points will give the liquid line. Points on the vapor line zbc in (a) are found by plotting pressure against v , as taken from the tables. Point b in (a) is for $p = 1000$ psia and $v_g = 0.4456$ cu. ft. (Table VII). The curves on the Ts plane are found by plotting $T^{\circ}R$ against s_l for the liquid line ndz and $T^{\circ}R$ against s_g for the vapor line zeh . See points d and e in (b) and compare the coordinates of these points with values in Table VI. The volume scale in (a) has been distorted because the volume of the liquid is so very small as compared with the volume of the vapor at low pressures.

Whenever a point such as r , which represents the state of a substance, lies within the curves mzc , Fig. 105(a), and nzh , Fig. 105(b), the substance is a wet mixture, part liquid and part vapor. If the state point is on the liquid line, for example, a or d in Fig. 105, the substance is a saturated liquid. If the state point is on the vapor line, for example, b or e , the substance is a saturated vapor. Finally, whenever the substance is a superheated vapor, its state point will lie to the right of the vapor line; for example, point k .

Observe how the saturated vapor line on the pv plane flattens out at low pressures, indicating that the volume increases at an increasing rate at low pressures. Refer to steam tables for quantitative data. In the solution of problems concerning vapors, the Ts plane is particularly helpful. The student should acquire the habit of sketching one or both of these planes for

Table VI SATURATED STEAM TEMPERATURES*

TEMP FAHR	ABS PRESS	SPECIFIC VOLUME			ENTHALPY			ENTROPY		
	Lb Sq In	Sat Liqu d	Evap	Sat Vapor	Sat Liqu d	Evap	Sat Vapor	Sat Liqu d	Evap	Sat Vapor
t	p	v _f	v _g	v _g	h _f	h _{fg}	h _g	s _f	s _g	s _g
32°	0.0834	0.01602	3305	3305	0.00	1073.8	1073.8	0.0000	2.1877	2.1877
35	0.0993	0.01602	2917	2917	3.0°	1074.1	1077.1	0.0061	2.1700	2.1700
40	0.12170	0.01602	2444	2444	8.0	1071.3	1079.3	0.0167	2.1430	2.1430
45	0.14752	0.01602	2036.4	2036.4	13.06	1069.4	1081.5	0.032	2.1167	2.1167
50	0.17811	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.0903
60°	0.2573	0.01601	1206.6	1206.7	28.06	1059.9	1088.0	0.050	2.0393	2.0393
70	0.3631	0.01606	867.8	867.9	38.04	1054.3	1093.3	0.0743	1.9907	1.9907
80	0.5069	0.01608	633.1	633.1	48.07	1048.6	1098.6	0.0977	1.9417	1.9417
90	0.6982	0.01610	468.0	468.0	57.99	1042.9	1100.9	0.1115	1.8927	1.8927
100	0.9492	0.01613	330.3	330.4	67.97	1037.2	1105.2	0.1255	1.8531	1.8531
110°	1.2748	0.01617	265.3	265.4	77.94	1031.6	1109.5	0.1471	1.8106	1.8106
120	1.6974	0.01620	203.25	203.27	87.92	1025.8	1113.7	0.1654	1.7694	1.7694
130	2.2225	0.01623	157.3	157.31	97.90	1019.9	1117.9	0.1816	1.7296	1.7296
140	2.8886	0.01626	122.99	123.01	107.89	1014.1	1122.0	0.1961	1.6910	1.6910
150	3.718	0.01631	97.06	97.07	117.89	1008.2	1126.1	0.2140	1.6537	1.6537
160°	4.741	0.01639	77.27	77.29	127.89	1002.3	1130.2	0.2311	1.6174	1.6174
170	5.992	0.01645	62.01	62.06	137.90	996.3	1134.2	0.2472	1.5822	1.5822
180	7.510	0.01651	50.21	50.23	147.90	990.3	1138.1	0.2630	1.5480	1.5480
190	9.339	0.01657	40.94	40.96	157.90	984.1	1142.0	0.2785	1.5147	1.5147
200	11.526	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4821	1.4821
210°	14.123	0.01670	27.80	27.82	178.05	971.6	1149.7	0.3090	1.4508	1.4508
212	14.690	0.01672	26.78	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.4446
220	17.186	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4101	1.4101
230	20.780	0.01684	19.365	19.38	195.23	958.8	1157.0	0.3387	1.3901	1.3901
240	24.069	0.01692	16.306	16.323	203.31	952.2	1160.5	0.3531	1.3669	1.3669
250°	28.825	0.01700	13.804	13.821	218.48	945.5	1164.0	0.3675	1.3393	1.3393
260	35.420	0.01709	11.746	11.763	238.64	938.7	1167.3	0.3817	1.3043	1.3043
270	41.858	0.01717	10.044	10.061	253.81	931.8	1170.6	0.3958	1.2709	1.2709
280	49.203	0.01726	8.63	8.645	269.06	924.7	1173.8	0.4098	1.2391	1.2391
290	57.556	0.01735	7.444	7.461	285.31	917.5	1176.8	0.4234	1.2238	1.2238
300°	67.013	0.01745	6.449	6.466	299.59	910.1	1179.7	0.4369	1.1980	1.1980
310	77.63	0.01755	5.609	5.626	313.88	902.6	1182.5	0.4501	1.1737	1.1737
320	89.66	0.01765	4.886	4.914	329.23	894.9	1185.2	0.4637	1.1478	1.1478
330	103.06	0.01776	4.289	4.307	340.63	887.0	1187.7	0.4769	1.1233	1.1233
340	118.01	0.01787	3.770	3.788	351.13	879.0	1190.1	0.4900	1.0992	1.0992
350°	134.63	0.01799	3.324	3.342	361.63	870.7	1192.3	0.5029	1.0754	1.0754
360	153.04	0.01811	2.939	2.957	372.18	862.2	1194.4	0.5153	1.0519	1.0519
370	173.37	0.01823	2.606	2.625	382.79	853.5	1196.3	0.5286	1.0297	1.0297
380	195.77	0.01836	2.317	2.335	393.45	844.6	1198.1	0.5413	1.0090	1.0090
390	220.37	0.01850	2.0631	2.0836	404.17	835.4	1199.6	0.5539	0.9892	0.9892
400°	247.31	0.01864	1.8447	1.8633	414.97	826.0	1201.0	0.5664	0.9698	0.9698
410	276.75	0.01878	1.6512	1.6700	425.83	816.3	1202.1	0.5788	0.9506	0.9506
420	308.83	0.01894	1.4811	1.5000	436.77	806.3	1203.1	0.5912	0.9316	0.9316
430	343.72	0.01910	1.3303	1.3499	447.79	796.0	1203.8	0.6033	0.9127	0.9127
440	381.59	0.01926	1.1979	1.2171	458.90	785.4	1204.3	0.6158	0.8939	0.8939
450°	422.6	0.01954	1.0796	1.0993	469.3	774.5	1204.6	0.6280	0.8753	0.8753
460	466.9	0.0196	0.9748	0.9944	479.4	763.2	1204.6	0.640	0.8569	0.8569
470	514.7	0.0198	0.8811	0.9009	489.8	751.5	1204.3	0.6523	0.8383	0.8383
480	566.1	0.0200	0.7972	0.8172	499.4	739.4	1203.7	0.664	0.8198	0.8198
490	621.4	0.0202	0.7221	0.7423	476.0	726.8	1202.8	0.6766	0.7953	0.7953
500°	680.8	0.0204	0.6545	0.6749	487.8	713.9	1201.7	0.6887	0.7738	0.7738
600	1542.9	0.0236	0.2432	0.2668	617.0	548.5	1165.5	0.8131	0.5176	0.5176
700°	3693.7	0.0369	0.0399	0.0761	821.3	312.1	995.4	0.9905	0.1484	0.1484
705.4	3206.2	0.0403	0	0.0903	902.7	0	902.7	1.0380	0	0.0380

* Steam tables abridged from *Thermodynamic Properties of Steam* by Joseph H. Keenan and Frederick G. Keyes. Copyright 1937 by Joseph H. Keenan and Frederick G. Keyes. Published by John Wiley & Sons Inc., New York.

Table VII. SATURATED STEAM: PRESSURES

ABS. PRESS.	TEMP. FAHR.	SPECIFIC VOLUME		ENTHALPY			ENTROPY			INTERNAL ENERGY	
Lb. Sq. In.		Sat. Liquid	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Sat. Vapor
<i>p</i>	<i>t</i>	<i>v_f</i>	<i>v_g</i>	<i>h_f</i>	<i>h_{fg}</i>	<i>h_g</i>	<i>s_f</i>	<i>s_{fg}</i>	<i>s_g</i>	<i>u_f</i>	<i>u_g</i>
1.0	101.74	0.01614	333.6	69.70	1036.3	1106.0	0.1326	1.8456	1.9782	69.70	1044.3
2.0	126.08	0.01623	173.73	93.99	1022.2	1116.2	0.1749	1.7451	1.9200	93.98	1051.9
3.0	141.48	0.01630	118.71	109.37	1013.2	1122.6	0.2008	1.6855	1.8863	109.36	1056.7
4.0	152.97	0.01636	90.63	120.86	1006.4	1127.3	0.2198	1.6427	1.8625	120.85	1060.2
5.0	162.24	0.01640	73.52	130.13	1001.0	1131.1	0.2347	1.6094	1.8441	130.12	1063.1
6.0	170.06	0.01645	61.98	137.96	996.2	1134.2	0.2472	1.5820	1.8292	137.94	1065.4
7.0	176.83	0.01649	53.64	144.76	992.1	1136.9	0.2581	1.5586	1.8167	144.74	1067.4
8.0	182.86	0.01653	47.34	150.79	988.5	1139.3	0.2674	1.5383	1.8057	150.77	1069.2
9.0	188.28	0.01656	42.40	156.22	985.2	1141.4	0.2759	1.5203	1.7962	156.19	1070.8
10	193.21	0.01659	38.42	161.17	982.1	1143.3	0.2835	1.5041	1.7876	161.14	1072.2
14.696	212.00	0.01672	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	180.02	1077.5
15	213.03	0.01672	26.29	181.11	969.7	1150.8	0.3135	1.4415	1.7549	181.06	1077.8
20	227.96	0.01683	20.089	196.16	960.1	1156.3	0.3356	1.3962	1.7319	196.10	1081.9
25	240.07	0.01692	16.303	208.42	952.1	1160.6	0.3533	1.3606	1.7130	208.34	1085.1
30	250.33	0.01701	13.746	218.82	945.3	1164.1	0.3680	1.3313	1.6993	218.73	1087.8
35	259.28	0.01708	11.898	227.91	939.2	1167.1	0.3807	1.3063	1.6870	227.80	1090.1
40	267.25	0.01715	10.498	236.03	933.7	1169.7	0.3919	1.2844	1.6763	235.90	1092.0
45	274.44	0.01721	9.401	243.36	928.6	1172.0	0.4019	1.2650	1.6669	243.22	1093.7
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	249.93	1095.3
55	287.07	0.01732	7.787	256.30	919.6	1175.9	0.4193	1.2316	1.6509	256.12	1096.7
60	292.71	0.01738	7.175	262.09	915.5	1177.6	0.4270	1.2168	1.6438	261.90	1097.9
65	297.97	0.01743	6.655	267.50	911.6	1179.1	0.4342	1.2032	1.6374	267.29	1099.1
70	302.92	0.01748	6.206	272.61	907.9	1180.6	0.4409	1.1906	1.6315	272.38	1100.2
75	307.60	0.01753	5.816	277.43	904.5	1181.9	0.4472	1.1787	1.6259	277.19	1101.2
80	312.03	0.01757	5.472	282.02	901.1	1183.1	0.4531	1.1676	1.6207	281.76	1102.1
85	316.25	0.01761	5.168	286.39	897.8	1184.2	0.4587	1.1571	1.6158	286.11	1102.9
90	320.27	0.01766	4.896	290.56	894.7	1185.3	0.4641	1.1471	1.6112	290.27	1103.7
95	324.12	0.01770	4.652	294.56	891.7	1186.2	0.4692	1.1370	1.6068	294.25	1104.5
100	327.81	0.01774	4.432	298.40	888.8	1187.2	0.4740	1.1286	1.6026	298.08	1105.2
110	334.77	0.01782	4.049	305.66	883.2	1188.9	0.4832	1.1117	1.5948	305.30	1106.5
120	341.25	0.01789	3.728	312.44	877.9	1190.4	0.4916	1.0962	1.5878	312.05	1107.6
130	347.32	0.01796	3.455	318.81	872.9	1191.7	0.4995	1.0817	1.5812	318.38	1108.6
140	353.02	0.01802	3.220	324.82	868.2	1193.0	0.5069	1.0682	1.5751	324.35	1109.6
150	358.42	0.01809	3.015	330.51	863.6	1194.1	0.5138	1.0556	1.5691	330.01	1110.5
160	363.53	0.01815	2.834	335.93	859.2	1195.1	0.5204	1.0436	1.5640	335.39	1111.2
170	368.41	0.01822	2.675	341.09	854.9	1196.0	0.5266	1.0324	1.5590	340.52	1111.9
180	373.06	0.01827	2.532	346.03	850.8	1196.9	0.5325	1.0217	1.5542	345.42	1112.5
190	377.51	0.01833	2.404	350.79	846.8	1197.6	0.5381	1.0116	1.5497	350.15	1113.1
200	381.79	0.01839	2.288	355.36	843.0	1198.4	0.5435	1.0018	1.5453	354.68	1113.7
250	400.95	0.01865	1.8438	376.00	825.1	1201.1	0.5675	0.9588	1.5263	375.14	1115.8
300	417.33	0.01890	1.5433	393.84	809.0	1202.8	0.5879	0.9225	1.5104	392.79	1117.1
350	431.72	0.01913	1.3260	409.69	794.2	1203.9	0.6056	0.8910	1.4966	408.45	1118.0
400	444.59	0.0193	1.1613	424.0	780.5	1204.5	0.6214	0.8630	1.4844	422.6	1118.5
450	456.28	0.0195	1.0320	437.2	767.1	1204.6	0.6336	0.8378	1.4734	435.5	1118.7
500	467.01	0.0197	0.9278	449.4	755.0	1204.4	0.6487	0.8147	1.4634	447.6	1118.6
550	476.94	0.0199	0.8424	460.8	743.1	1203.9	0.6608	0.7934	1.4542	458.8	1118.2
600	486.21	0.0201	0.7698	471.6	731.6	1203.2	0.6720	0.7734	1.4454	469.4	1117.7
650	494.90	0.0203	0.7083	481.8	720.5	1202.3	0.6826	0.7548	1.4374	479.4	1117.1
700	503.10	0.0205	0.6554	491.5	709.7	1201.2	0.6925	0.7371	1.4296	488.8	1116.3
800	518.23	0.0209	0.5687	509.7	688.9	1198.6	0.7108	0.7045	1.4153	506.6	1114.4
900	531.98	0.0212	0.5006	526.6	668.8	1195.4	0.7275	0.6744	1.4020	523.1	1112.1
1000	544.61	0.0216	0.4456	542.4	649.4	1191.8	0.7430	0.6467	1.3897	538.4	1109.4
1200	567.22	0.0223	0.3619	571.7	611.7	1183.4	0.7711	0.5956	1.3667	566.7	1103.0
1400	587.10	0.0231	0.3012	598.7	574.7	1173.4	0.7963	0.5491	1.3454	592.7	1095.4
1500	596.23	0.0235	0.2765	611.6	556.3	1167.9	0.8082	0.5269	1.3351	605.1	1091.2
2000	635.82	0.0257	0.1878	671.7	463.4	1135.1	0.8619	0.4230	1.2849	662.2	1065.6
3000	695.36	0.0346	0.0858	802.5	217.8	1020.3	0.9731	0.1885	1.1615	783.4	972.7
3206.2	705.40	0.0503	0.0503	902.7	0	902.7	1.0580	0	1.0580	872.9	872.9

Table VIII SUPERHEATED STEAM

Abs. Press.		TEMPERATURE—DEGREES FAHRENHEIT									
Lb./Sq. In. (Sat. Temp.)		200°	300°	400°	500°	600°	700°	800°	900°	1000°	1100°
1	v	392.6	452.3	512.0	571.6	631.2	690.8	750.4	809.9	869.5	929.1
(101.74)	A	1150.4	1195.8	1241.7	1288.3	1335.7	1383.8	1432.8	1482.7	1533.5	1585.2
	v	2.0512	2.1153	2.1720	2.2233	2.2702	2.3137	2.3542	2.3923	2.4283	2.4625
5	v	78.16	90.25	102.26	114.22	126.16	138.10	150.03	161.95	173.87	185.79
(162.24)	A	1145.8	1195.0	1241.2	1288.0	1335.4	1383.6	1432.7	1482.6	1533.4	1585.1
	v	1.8718	1.9370	1.9942	2.0456	2.0927	2.1361	2.1767	2.2148	2.2509	2.2851
10	v	38.85	45.00	51.04	57.05	63.03	69.01	74.98	80.95	86.92	92.88
(193.21)	A	1146.6	1193.0	1240.6	1287.5	1335.1	1383.4	1432.5	1482.4	1533.2	1585.0
	v	1.7927	1.8595	1.9172	1.9689	2.0160	2.0.96	2.1002	2.1383	2.1744	2.2086
14 696	v		30.53	34.68	38.78	42.86	46.94	51.00	55.07	59.13	63.19
(212.00)	A		1192.8	1239.9	1287.1	1334.8	1382.2	1432.3	1482.3	1533.1	1584.8
	v		1.8160	1.8743	1.9261	1.9734	2.0170	2.0576	2.0953	2.1319	2.1662
20	v		22.36	25.43	28.46	31.47	34.47	37.46	40.45	43.44	46.42
(237.96)	A		1191.6	1239.2	1286.6	1334.4	1382.9	1432.1	1482.1	1533.0	1584.7
	v		1.7803	1.8396	1.8918	1.9392	1.9829	2.0235	2.0618	2.0978	2.1321
40	v		11.040	12.628	14.168	15.658	17.198	18.702	20.20	21.70	23.20
(267.25)	A		1186.8	1236.5	1286.3	1336.1	1386.9	1437.4	1488.4	1539.4	1590.3
	v		1.6994	1.7608	1.8140	1.8619	1.9058	1.9467	1.9850	2.0212	2.0555
60	v		7.250	8.337	9.403	10.477	11.441	12.449	13.452	14.454	15.453
(282.71)	A		1181.6	1233.6	1283.0	1331.8	1380.9	1430.5	1480.8	1531.9	1583.8
	v		1.6492	1.7133	1.7678	1.8162	1.8605	1.9015	1.9400	1.9762	2.0106
80	v		6.220	7.020	7.797	8.562	9.322	10.077	10.830	11.582	
(312.03)	A		1230.7	1291.1	1350.5	1409.9	1469.7	1529.7	1589.1	1648.4	
	v		1.6791	1.7346	1.7836	1.8281	1.8694	1.9079	1.9442	1.9787	
100	v		4.937	5.589	6.219	6.835	7.446	8.052	8.658	9.259	
(327.61)	A		1227.6	1279.1	1329.7	1379.8	1429.9	1479.8	1529.5	1580.2	
	v		1.6518	1.7035	1.7580	1.8029	1.8443	1.8829	1.9193	1.9538	
120	v		4.081	4.636	5.163	5.683	6.195	6.702	7.207	7.710	
(341.23)	A		1224.4	1277.2	1327.7	1377.8	1428.1	1478.2	1528.2	1582.4	
	v		1.6287	1.6869	1.7370	1.7822	1.8237	1.8623	1.8990	1.9335	
140	v		3.468	3.934	4.413	4.861	5.301	5.738	6.172	6.604	
(353.02)	A		1221.1	1275.2	1326.4	1376.8	1427.3	1478.2	1529.7	1581.9	
	v		1.6087	1.6683	1.7190	1.7645	1.8063	1.8451	1.8817	1.9163	
160	v		3.008	3.443	3.840	4.244	4.631	5.015	5.396	5.775	
(363.53)	A		1217.6	1273.1	1325.0	1375.7	1426.4	1477.5	1529.1	1581.4	
	v		1.5908	1.6519	1.7033	1.7491	1.7911	1.8301	1.8667	1.9014	
180	v		2.649	3.044	3.411	3.764	4.110	4.452	4.792	5.129	
(373.06)	A		1214.0	1271.0	1323.5	1374.7	1425.6	1476.8	1528.6	1581.0	
	v		1.5745	1.6373	1.6994	1.7553	1.7776	1.8167	1.8534	1.8882	
200	v		2.361	2.726	3.060	3.390	3.693	4.002	4.309	4.613	
(381.70)	A		1210.3	1268.9	1322.1	1373.6	1424.8	1476.2	1528.0	1580.5	
	v		1.5594	1.6240	1.6767	1.7232	1.7655	1.8048	1.8415	1.8763	
220	v		2.125	2.465	2.772	3.066	3.352	3.634	3.913	4.191	
(389.86)	A		1206.5	1266.7	1320.7	1372.6	1424.0	1475.5	1527.5	1580.0	
	v		1.5453	1.6117	1.6652	1.7120	1.7545	1.7939	1.8303	1.8656	
240	v		1.9276	2.247	2.533	2.804	3.068	3.327	3.584	3.839	
(397.37)	A		1202.5	1264.5	1319.2	1371.5	1423.2	1474.8	1526.9	1579.6	
	v		1.5319	1.6003	1.6546	1.7017	1.7441	1.7839	1.8209	1.8558	
260	v			2.063	2.330	2.583	2.827	3.067	3.305	3.541	
(404.42)	A			1262.3	1317.7	1370.4	1422.3	1474.2	1526.3	1579.1	
	v			1.5897	1.6447	1.6922	1.7352	1.7748	1.8113	1.8467	
300	v			1.7675	2.005	2.227	2.442	2.652	2.859	3.065	
(417.33)	A			1257.6	1314.7	1369.3	1423.6	1478.2	1532.5	1587.1	
	v			1.5701	1.6268	1.6751	1.7184	1.7582	1.7954	1.8305	
350	v			1.4923	1.7036	1.8980	2.084	2.256	2.445	2.622	
(431.72)	A			1251.5	1310.9	1365.5	1418.5	1471.1	1523.8	1577.0	
	v			1.5491	1.6070	1.6563	1.7002	1.7403	1.7777	1.8130	
400	v			1.2551	1.4770	1.6508	1.8161	1.9767	2.134	2.290	
(444.59)	A			1245.1	1306.9	1362.7	1416.4	1469.4	1522.4	1575.8	
	v			1.5281	1.5894	1.6398	1.6842	1.7247	1.7623	1.7977	

Table VIII. SUPERHEATED STEAM (Continued)

Abs. Press.		TEMPERATURE—DEGREES FAHRENHEIT										
Lb./Sq. In. (Sat. Temp.)		500°	550°	600°	620°	640°	660°	680°	700°	800°	900°	1000°
450 (456.28)	τ 1.1231 λ 1238.4 s 1.5095	1.2155 1272.0 1.5437	1.3005 1302.8 1.5735	1.3332 1314.6 1.5845	1.3652 1326.2 1.5951	1.3967 1337.5 1.6054	1.4278 1348.8 1.6153	1.4584 1359.9 1.6250	1.6074 1414.3 1.6699	1.7516 1467.7 1.7108	1.8928 1521.0 1.7486	
500 (467.01)	τ 0.9927 λ 1231.3 s 1.4919	1.0800 1266.8 1.5280	1.1591 1298.6 1.5583	1.1893 1310.7 1.5701	1.2188 1322.6 1.5810	1.2478 1334.2 1.5915	1.2763 1345.7 1.6016	1.3044 1357.0 1.6115	1.4405 1412.1 1.6571	1.5715 1466.0 1.6982	1.6996 1519.6 1.7363	
550 (476.94)	τ 0.8852 λ 1223.7 s 1.4751	0.9686 1261.2 1.5131	1.0431 1294.3 1.5451	1.0714 1306.8 1.5568	1.0989 1318.9 1.5680	1.1259 1330.8 1.5787	1.1523 1342.5 1.5890	1.1783 1354.0 1.5991	1.3038 1409.9 1.6452	1.4241 1464.3 1.6868	1.5414 1518.2 1.7250	
600 (486.21)	τ 0.7947 λ 1215.7 s 1.4586	0.8753 1255.5 1.4990	0.9463 1289.9 1.5323	0.9729 1302.7 1.5443	0.9988 1315.2 1.5558	1.0241 1327.4 1.5667	1.0489 1339.3 1.5773	1.0732 1351.1 1.5875	1.1899 1407.7 1.6343	1.3013 1462.5 1.6762	1.4096 1516.7 1.7147	
700 (503.10)	τ 0.7277 λ 1243.2 s 1.4722	0.7934 1280.6 1.4722	0.8177 1294.3 1.5084	0.8411 1307.5 1.5333	0.8639 1320.3 1.5449	0.8860 1332.8 1.5559	0.9077 1345.0 1.5665	0.9277 1357.0 1.5765	1.0108 1403.2 1.6147	1.1082 1459.0 1.6573	1.2024 1513.9 1.6963	
800 (518.23)	τ 0.6154 λ 1229.8 s 1.4467	0.6779 1270.7 1.4467	0.7006 1285.4 1.4863	0.7223 1299.4 1.5000	0.7433 1312.9 1.5129	0.7635 1325.9 1.5250	0.7833 1338.6 1.5366	0.7833 1338.6 1.5476	0.8763 1398.6 1.5972	0.9633 1455.4 1.6407	1.0470 1511.0 1.6801	
900 (531.98)	τ 0.5264 λ 1215.0 s 1.4216	0.5873 1260.1 1.4216	0.6089 1275.9 1.4653	0.6294 1290.9 1.4800	0.6491 1305.1 1.5066	0.6680 1318.8 1.5187	0.6863 1332.1 1.5303	0.6863 1332.1 1.5414	0.7716 1393.9 1.5814	0.8506 1451.8 1.6257	0.9262 1508.1 1.6656	
1000 (544.61)	τ 0.4533 λ 1198.3 s 1.3961	0.5140 1248.8 1.3961	0.5350 1265.9 1.4450	0.5546 1281.9 1.4610	0.5733 1297.0 1.4757	0.5912 1311.4 1.4893	0.6084 1325.3 1.5021	0.6084 1325.3 1.5141	0.6878 1389.2 1.5670	0.7604 1448.2 1.6121	0.8294 1505.1 1.6525	
1100 (556.31)	τ 0.4532 λ 1235.7 s 1.4251	0.4738 1255.3 1.4251	0.4738 1255.3 1.4425	0.4929 1272.4 1.4583	0.5110 1288.5 1.4728	0.5281 1303.7 1.4862	0.5445 1318.3 1.4989	0.5445 1318.3 1.4989	0.6191 1384.3 1.5535	0.6866 1444.5 1.5995	0.7503 1502.2 1.6405	
1200 (567.22)	τ 0.4016 λ 1223.5 s 1.4052	0.4222 1243.9 1.4052	0.4222 1243.9 1.4243	0.4410 1262.4 1.4413	0.4586 1279.6 1.4568	0.4752 1295.7 1.4710	0.4752 1295.7 1.4843	0.4909 1311.0 1.4943	0.5617 1379.3 1.5409	0.6250 1440.7 1.5879	0.6843 1499.2 1.6293	
1400 (587.10)	τ 0.3174 λ 1193.0 s 1.3639	0.3390 1193.0 1.3639	0.3390 1218.4 1.3877	0.3580 1240.4 1.4079	0.3753 1260.3 1.4258	0.3912 1278.5 1.4419	0.3912 1278.5 1.4567	0.4062 1295.5 1.4567	0.4714 1369.1 1.5177	0.5281 1433.1 1.5666	0.5805 1493.2 1.6093	
1600 (604.90)	τ 0.2733 λ 1187.8 s 1.3489	0.2936 1215.2 1.3489	0.2936 1215.2 1.3741	0.2936 1215.2 1.3952	0.3112 1238.7 1.3952	0.3271 1259.6 1.4137	0.3271 1259.6 1.4303	0.3417 1278.7 1.4303	0.4034 1358.4 1.4964	0.4553 1425.3 1.5476	0.5027 1487.0 1.5914	
1800 (621.03)	τ 0.2407 λ 1185.1 s 1.3377	0.2597 1214.0 1.3377	0.2597 1214.0 1.3638	0.2597 1214.0 1.3855	0.2760 1238.5 1.3855	0.2907 1260.3 1.4044	0.2907 1260.3 1.4044	0.2907 1260.3 1.4194	0.3502 1347.2 1.4765	0.3986 1417.4 1.5301	0.4421 1480.8 1.5752	
2000 (635.82)	τ 0.1936 λ 1145.6 s 1.2945	0.2161 1184.9 1.2945	0.2161 1184.9 1.3300	0.2161 1184.9 1.3564	0.2337 1214.8 1.3564	0.2489 1240.0 1.3783	0.2489 1240.0 1.3783	0.2489 1240.0 1.3994	0.3074 1335.5 1.4576	0.3532 1409.2 1.5139	0.3935 1474.5 1.5603	
2500 (668.13)	τ 0.1484 λ 1132.3 s 1.2687	0.1686 1178.8 1.2687	0.1686 1178.8 1.3073	0.1686 1178.8 1.3300	0.1848 1214.8 1.3300	0.2000 1240.0 1.3564	0.2000 1240.0 1.3783	0.2000 1240.0 1.3994	0.1686 1178.8 1.4127	0.2294 1303.6 1.4772	0.2710 1357.8 1.5273	
3000 (695.36)	τ 0.0984 λ 1060.7 s 1.1966	0.1060 1060.7 1.1966	0.1060 1060.7 1.2690	0.1060 1060.7 1.2915	0.1060 1060.7 1.3140	0.1060 1060.7 1.3365	0.1060 1060.7 1.3590	0.1060 1060.7 1.3815	0.1760 1267.2 1.3690	0.2159 1365.0 1.4439	0.2476 1441.8 1.4984	
3206.2 (705.40)	τ 0.1583 λ 1250.5 s 1.3503	0.1583 1250.5 1.3503	0.1583 1250.5 1.4309	0.1583 1250.5 1.4534	0.1583 1250.5 1.4759	0.1583 1250.5 1.4984	0.1583 1250.5 1.5209	0.1583 1250.5 1.5434	0.1583 1250.5 1.6159	0.1981 1355.2 1.4309	0.2288 1434.7 1.4874	

Table IX. COMPRESSED WATER

Taken with permission from Table 4 of Keenan and Keyes, *Thermodynamic Properties of Steam*, published by John Wiley & Sons, Inc

Temperature °F =		32°	100°	200°	300°	400°	500°	600°
Saturated liquid	$p =$	0.08854	0.9492	11.526	67.013	247.31	680.8	1542.9
	$v_f =$	0.016022	0.016132	0.016634	0.017449	0.018639	0.020432	0.023626
	$h_f =$	0	67.97	167.99	269.59	374.07	487.82	617.0
	$s_f =$	0	0.12948	0.29382	0.43694	0.56638	0.68871	0.8131
Abs press (Sat temp)								
200 (351.70)	$(v - v_f)10^3$	-1.1	-1.1	-1.1	-1.1			
	$(h - h_f)$	+0.01	+0.54	+0.41	+0.23			
	$(s - s_f)10^3$	+0.01	-0.05	-0.21	-0.21			
1000 (544.61)	$(v - v_f)10^3$	-5.7	-5.1	-5.4	-6.9	-8.7	-6.4	
	$(h - h_f)$	+2.99	+2.70	+2.21	+1.75	+0.84	-0.14	
	$(s - s_f)10^3$	+0.15	-0.53	-1.20	-1.64	-2.00	-1.41	
2000 (635.62)	$(v - v_f)10^3$	-11.0	-9.0	-10.8	-13.8	-19.5	-27.8	-32.6
	$(h - h_f)$	+5.97	+5.31	+4.51	+3.64	+2.03	-0.38	-2.5
	$(s - s_f)10^3$	+0.22	-1.18	-2.39	-3.42	-4.57	-5.58	-4.3
3000 (695.36)	$(v - v_f)10^3$	-16.3	-14.7	-16.0	-20.7	-30.0	-47.1	-67.9
	$(h - h_f)$	+0.00	+7.84	+6.76	+5.49	+3.33	-0.41	-6.9
	$(s - s_f)10^3$	+0.28	-1.79	-3.56	-5.12	-7.07	-9.42	12.4

each problem, showing in each case the liquid and vapor lines, drawn freehand

165. Internal Energy. Ordinarily, vapor tables do not give internal energies, yet they are needed now and then. Values of internal energy are computed from $u = h - pv/J$. For a saturated liquid, we may write

$$u_f = h_f - \frac{p_s v_f}{J} \text{ Btu/lb,}$$

where the values of h_f and p_s (saturation pressure) are taken from the tables in accordance with the actual temperature of the liquid. However, unless the temperature is quite high, the corresponding saturation pressure will be so low that the term $p_s v_f/J$ is negligible. Customarily, we therefore consider that the internal energy of the liquid at low pressures is equal to the enthalpy of the liquid, $u_f \approx h_f$.

The internal energy of saturated vapor is

$$u_g = h_g - \frac{p v_g}{J} \text{ Btu/lb,}$$

where h_g and v_g are found in the tables according to the pressure p (or the corresponding temperature). For example, at 100 psia, the internal energy of saturated vapor is (compare with Table VII)

$$u_0 = 1187.2 - \frac{(100)(144)(4.432)}{778} = 1105.2 \text{ Btu/lb.}$$

166. **Properties of a Wet Mixture.** Consider any liquid under some pressure p psia and at 32°F . If p is greater than the saturation pressure corresponding to 32°F , the liquid is not saturated and is called a *compressed liquid*. If the datum of enthalpy be taken as zero for a saturated liquid, then this compressed liquid does not have zero enthalpy, since $h = u + pv/J$. However, the pv/J term is small unless the pressure is quite large, and for the purpose of a visual aid in remembering the properties, we may consider it as negligible. In this event, the enthalpy of the liquid at t , Fig. 106, is nearly zero, and if it is heated at constant pressure to the boiling point (saturation temperature) at m , the area under the constant-pressure curve tm on the T 's plane represents heat $Q = \Delta h$ for $p = C$. That is, the area under tm represents closely the enthalpy of the liquid at m . The s coordinate of m is the entropy of the liquid exactly. Now if part of the liquid is evaporated, the increase in any property is a part of the change which that property undergoes during complete vaporization. Take any property for illustration, say, s . The

by point 1, Fig. 106, is the entropy s_l of the liquid *plus* the fraction x of the liquid evaporated (x = quality) *times* the entropy s_{fg} of evaporation, or

(d) $s_1 = s_f + xs_{f\theta}$.

In terms of the percentage moisture $y = 1 - x$, start at point n . The entropy at 1 is equal to the entropy s_g of saturated vapor at n minus the fraction y of the liquid which has *not* evaporated *times* the entropy of evaporation, or

(e) $s_1 = s_g - 1/s_{fg}$.

All properties of a two-phase system can be obtained by similar reasoning. For one pound,

(f)	$h = h_o - y h_{fo},$	$h = h_f + x h_{fo},$
(g)	$v = v_o - y v_{fo},$	$v = v_f + x v_{fo}.$
	[HIGH QUALITY]	[LOW QUALITY]

The expressions with y give more accurate slide-rule answers when the quality is high (above 75%); the expressions with x gives more accurate answers when the quality is low (below 25%). There is little difference in the inter-

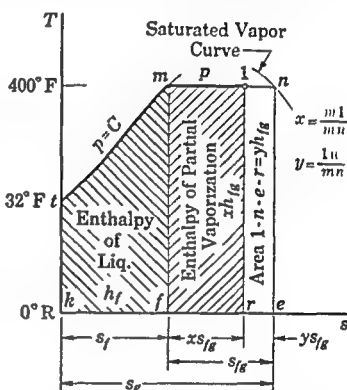


Fig. 106. Properties of Wet Mixture.

mediate range. If the internal energy of a wet mixture is needed, compute h and v from equations (f) and (g), and then $u = h - pv/J$ Btu per lb.

It takes two coordinates to locate a point in a plane. Since, in the wet region, the pressure and temperature both locate the same line, it is necessary to give still another property in order to define the state point. The entropy or the volume will serve, but it is common practice to define the state of the two-phase system by giving either the pressure or temperature and the quality or moisture content.

167. Properties of Superheated Vapors Modern vapor tables usually give the necessary properties of superheated vapors. In Table VIII for steam, we find the values of t , h , and s , the values of h and s being measured above saturated water at 32°F . In order to define the state of a substance in the superheat region, it is necessary to specify two properties. Any two may be used, but the most common practice is to give the pressure and temperature.

At low pressure, when the state of a superheated vapor falls outside of the limits of the superheat table, the vapor may be treated as an ideal gas. For practice in interpolation and to note the differences in values in other tables, the student should solve the following example using the tables in *Problems on Thermodynamics* if available.

Examples (a) What is the internal energy (above saturated liquid at 32°F) of steam at 100 psia and 600°F ?

SOLUTION From Table VIII, we find $h = 1329.1$ and $t = 6.218$

$$u = h - pv/J = 1329.1 - \frac{(100)(144)(6.218)}{778} = 1214 \text{ Btu/lb}$$

(b) What is the entropy of steam at 297 psia and 512°F ?

SOLUTION This problem illustrates the method of making a double interpolation. By comparing the temperature given with the saturation temperature corresponding to 297 lb, we note that the steam is superheated. From more complete tables than Table VIII, we have taken the following values of entropy:

At 500°F		At 520°F	
For $p = 295$	$s = 1.5725$	For $p = 295$	$s = 1.5847$
For $p = 300$	$s = 1.5701$	For $p = 300$	$s = 1.5824$
Difference	$= 0.0024$	Difference	$= 0.0023$
$(25)(0.0024)$	$= 0.00096$	$(25)(0.0026)$	$= 0.0009$
For $p = 297$,	$s = 1.5715$	For $p = 297$	$s = 1.5835$
At $p = 297$ psia			
For $t = 500^\circ$,	$s = 1.5715$		
For $t = 520^\circ$	$s = 1.5835$		
Difference	0.0123		
$(12/50)(0.0123)$	$= 0.0074$		
For $t = 512^\circ$	$s = 1.5789$ Ans		

However, at low temperature and pressure, water acts very much as an incompressible fluid, in which case the points b , c , and B , become quite close together. These three points are defined by an isothermal compression ab , an isentropic compression ac , and a constant volume compression aB . As these points approach each other at low pressures, their properties approach equality. One cannot say exactly where a line should be drawn, but for pedagogical purposes, let us say that for pressures and temperatures of steam below 400 psia and 400°F , use properties of saturated liquid, do not correct for the compressed-liquid effect.

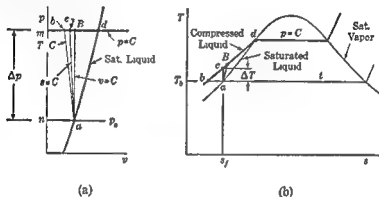


Fig 107 Repeated

If a liquid is pumped reversibly in steady flow to a higher pressure, the work is ($\Delta K = 0$, $\Delta P = 0$)

$$(h) \quad W = -(\Delta h)_s, \quad [\text{EQUATION (32)}]$$

where the subscript s indicates constant entropy. This change can be evaluated accurately with the properties of subcooled liquid. In the absence of known properties, we can obtain a good estimate of equation (h) for liquids which are nearly incompressible by the relation from equation (17), § 42 ($\beta = 0$)

$$(i) \quad \Delta h = \int v dp = \frac{v \Delta p}{\beta}, \quad [\beta = 0]$$

Observe that $v \Delta p$ is the rectangular area $naBm$, Fig 107(a), $v = v_a = v_f$ and $\Delta h = h_B - h_a = h_B - h_f$, where the subscript f indicates properties of saturated liquid. Thus, from equation (i), we have

$$(j) \quad h_B = h_f + \frac{v_f \Delta p}{\beta} = h_f + \frac{v_f (p_{act} - p_s)}{\beta}, \quad [\beta = 0]$$

in which h_f and v_f are taken for saturated liquid at the temperature of the liquid before compression, p_{act} is the actual pressure, and p_s is the saturation pressure. The point B , at a temperature slightly higher than that at a ,

is not very far from c , the state after isentropic compression, and the approximation of equation (j) will usually be better than no allowance for the subcooling effect.

169. Examples: Enthalpy of Compressed Water. (a) Given water at 100°F and 3000 psia. What is its enthalpy?

SOLUTION. From Table IX, we get

$$h_b - h_f = 7.88, \quad \text{or} \quad h_b = 67.97 + 7.88 = 75.85 \text{ Btu/lb.}$$

Note that this is at state b , Fig. 107.

(b) If saturated water at 100°F (before compression) is compressed isentropically to 3000 psia, what is the final enthalpy?

SOLUTION. From the chart in Keenan and Keyes, we read $h_c - h_{fa} = 8.9 \text{ Btu/lb.}$, or

$$h_c = 67.97 + 8.9 = 76.87 \text{ Btu/lb.}$$

See Fig. 107. If you do not have the chart referred to, try the approximation given by equation (j); $v_f = 0.01613$ (Table VI);

$$h_B = 67.97 + \frac{(0.01613)(3000 - 0.949)(144)}{778} = 67.97 + 8.96 = 76.93 \text{ Btu/lb.}$$

This value is in good agreement with the one obtained from the chart, but note that the temperature is low. At high temperatures, water becomes relatively compressible and the agreement between the results would not be so good.

170. Other Vapors. Properties of many other vapors are available in tables and charts (see the *problem book*). Before using an unfamiliar set of tables, study them carefully. There are different arrangements, different nomenclature, and different symbols. They are not all based on the same

Table X. SATURATED AMMONIA

Temp. $^{\circ}\text{F}$	Abs. Pres.	Specific volume		Heat Content (Enthalpy)			Entropy	
	Psi p	Liquid v_f	Vapor v_g	Liquid h_f	Vapor h_g	Latent Heat h_{fg}	Liquid s_f	Vapor s_g
-60	5.55	0.02278	44.73	-21.2	589.6	610.8	-0.0517	1.4769
-40	10.41	0.02322	24.86	0.0	597.6	597.6	0.0000	1.4242
0	30.42	0.02419	9.116	42.9	611.8	568.9	0.0975	1.3352
20	48.21	0.02474	5.910	64.7	617.8	553.1	0.1437	1.2969
40	73.32	0.02533	3.971	86.8	623.0	536.2	0.1885	1.2618
60	107.6	0.02597	2.751	109.2	627.3	518.1	0.2322	1.2294
80	153.0	0.02668	1.955	132.0	630.7	498.7	0.2749	1.1991
90	180.6	0.02707	1.661	143.5	632.0	488.5	0.2958	1.8146
100	211.9	0.02747	1.419	155.2	633.0	477.8	0.3166	1.1705

datum state, which does not matter if one does not become confused with signs. For example, the properties of *refrigerants*, substances which are used in refrigerating cycles, are generally measured from a state of saturated

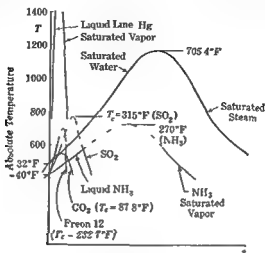


Fig 108 Various Fluids on T_s Plane

liquid at -40°F —as in Table X for ammonia, extracted from *Tables of Thermodynamic Properties of Ammonia*, US Bureau of Standards.* Refrigeration temperatures go below -40°F in some industrial processes in which

case the enthalpy and entropy of saturated liquid are negative. The negative signs mean only that measurement is below the datum. Carry the sign into the basic equations and stick to the rules of algebra.

The saturated liquid and vapor lines for several substances are shown in Fig 108, as plotted for one pound. Notice the variability of the latent heat of evaporation, proportional to the distance between the liquid and vapor lines at a particular temperature. Sulfur dioxide, carbon dioxide, ammonia and freon 12 are refrigerants (H_2O is also used as a refrigerant). Mercury (Hg) is used to generate power in turbines in so-called binary-vapor cycles. Mercury has

a small latent heat per pound, which means that the work which can be obtained per pound between given temperatures is small relative to that

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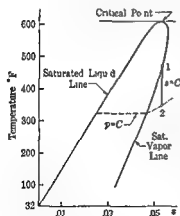


Fig 109 Saturation Curve for Acetic Acid

which can be obtained from steam, but the saturation pressure of mercury is low at high temperatures.

Not all saturated-vapor curves slope downward toward the right. Some show a double curvature, for example, benzene; and several such curves slope downward toward the left as, for example, the saturated-vapor curve of acetic acid (Fig. 109). If a substance with this characteristic undergoes an isentropic expansion, it becomes dryer or more highly superheated, whereas other substances discussed here become wetter or lose superheat.

171. The Ts Diagram Including the Solid Phase. The solid phase is of no concern in power generating cycles, but it is significant in many other ways. If heat is extracted at constant pressure from a superheated vapor

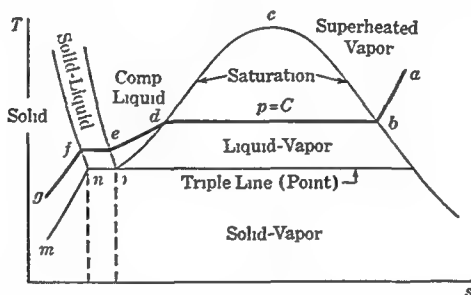


Fig. 110. Ts Diagram with Solid Phase. Simplified and distorted. Notice that the triple point on this plane is a line.

at a , Fig. 110, first the superheat is removed, ab , then the vapor is condensed, bd . After the liquid is cooled below its saturation temperature at d , it is a compressed liquid; and it begins to freeze at this particular pressure when the state point passes e ; solidification is ef (melting is fe). If heat is extracted from the solid at f , Fig. 110, its temperature decreases along some path fg . The triple line (32.02°F for steam) is the temperature at which the solid, liquid, and vapor phases can exist together in equilibrium. Below the triple line, also called the *triple point*, the solid sublimates to the vapor phase. The area under nr is the latent heat of fusion (enthalpy of fusion) at the pressure of the triple point. The area under fe is the heat of fusion at the pressure p .

Any three properties of a substance may be used to construct a solid, or surface in space. A Tps surface is shown in Fig. 111. Notice that what is shown on the Ts plane, Fig. 110, is the projection on that plane of the surface in space (although Figs. 110 and 111 are not drawn to the same scale). A constant pressure line $ab \dots g$ and other points are lettered correspondingly in both illustrations. Table 5 of Keenan and Keyes (71) gives properties of H_2O in the solid-vapor phase.

172. Closure. The purpose of this chapter is to acquaint the reader with the characteristic phenomena of liquids and vapors and the use of the vapor tables. Carefully study available vapor tables. If you should happen to

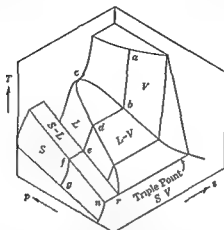


Fig 111 Tps Surface Simplified After (72) The surface, done in detail for a particular substance, is the locus of all equilibrium states

be using two different tables for the same substance, differences in a particular property should not alarm you. However, never mix values from different tables in a particular problem because this practice may lead to a serious error.

You perhaps recall from your study of physics, that the critical temperature, T_c , in Fig 111, is the temperature above which a gas cannot be liquefied. Representative critical temperatures are

Ammonia, 270°F,
Carbon Dioxide, 88°F,
Freon 12, 232°F,
Helium, 9.33°R,
Hydrogen, 59.8°R,

Methane, -116°F,
Nitrogen, 227°R,
Octane, 565°F,
Oxygen, 278°R,
Sulfur dioxide, 315°F

11

PROCESSES OF VAPORS

173. Introduction. The basic energy relations for the processes as defined for perfect gases also hold for vapors. In fact, all previous energy equations in terms of the general symbols W , Q , H , h , U , u , K , P , apply to any substance under the circumstances specified. The equations derived from the assumption of an ideal gas ($pv = RT$, Joule's law, etc.) do not hold. Thus, the reader's problem is largely one of classifying in his own mind the conditions which do and do not apply to vapors and of learning to solve energy equations with the use of vapor tables—which is the purpose of this chapter.

Remember that the areas on the pv plane under the curve of an internally reversible process represent $\int p dv$, even though the integral has not been made, and that this area is the work of a nonflow process. The area behind this same curve, to the p axis, is the $\int v dp$. The areas on the Ts plane under the curve of an internally reversible process represent heat transferred to (state point moving rightward) or from (state point moving leftward) the system. There is no change in the convention of signs previously established. The energy equations $dQ = du + p dv/J$ and $dQ = dh - v dp/J$ may be applied to any internally reversible processes and, for convenient reference, the steady flow equation may be written

$$(9) \quad W = h_1 - h_2 + K_1 - K_2 + Q,$$

in which $K = v^2/(2g_oJ)$, the kinetic energy, and there is no change in potential energy, $\Delta P = 0$.

174. Constant Pressure Process. (See § 67.) To define a process completely, it is necessary to give the location of two points, with a description of the path joining these points, or enough information to find two coordinates of each of these two points. When the defining relation is known (say, pressure constant), this relation is a coordinate of each of the two points

in question. In addition to this defining relation another property at each point is needed. Suppose in this case that point 1 is in the wet region and point 2 in the superheat region. Fig. 112. With the pressure known a point in the wet region is generally defined by giving its quality x or percentage moisture y . (Point 1 could be defined also by giving the entropy or

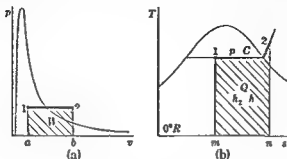


Fig. 112 Reversible Constant Pressure (Isobaric) Process. The area $a-1-2-b$ in (a) represents the work done in a nonflow process. The area $m-1-2-n$ in (b) represents the transferred heat.

the internal energy.) Similarly, point 2 in the superheat region is generally but not necessarily defined by giving its temperature (pressure known).

The work of a reversible nonflow process at $p = C$ is

$$(a) \quad \int p \, dv = p(v_2 - v_1) \text{ ft lb / lb of vapor}$$

where the specific volume v_2 of the superheated vapor is taken directly from the superheat tables. v_1 is the specific volume of the wet vapor

$$v_1 = (v_g - yv_{fg})_1 = (v_f + xv_{fg})_1$$

The heat transferred is

$$Q = \Delta u + \frac{\int p \, dv}{J} = u_2 - u_1 + \frac{p v_2}{J} - \frac{p v_1}{J}$$

[ANY REVERSIBLE PROCESS] [p = C]

$$(b) \quad Q = h_2 - h_1 \text{ Btu/lb}$$

[FLOW OR NONFLOW p = C]

where h_2 is taken from the superheat tables (for Fig. 112) and h_1 is found from $h_1 = (h_f + xv_{fg})_1$ or $(h_g - yv_{fg})_1$. The change of internal energy is found from the enthalpies

$$(c) \quad u_2 - u_1 = \left(h_2 - \frac{p v_2}{J} \right) - \left(h_1 - \frac{p v_1}{J} \right) \text{ Btu/lb}$$

175 Constant Volume Process (See § 65.) Choose point 1 in the superheat region, point 2 in the wet region. Fig. 113. In the solution of problems it is often necessary to use the defining relation of the process in

this case,

$$(d) \quad v_1 = v_2 = (v_g - y_2 v_{fg})_2 = (v_f + x_2 v_{fg})_2 \text{ cu. ft./lb.}$$

From this equation, the quality x_2 may be found, given v_1 . Knowing this quality and an additional property, determine other properties of the vapor

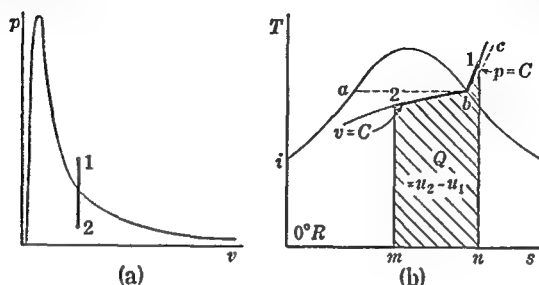


Fig. 113. Reversible Constant Volume (Isometric) Process. The area $m-2-1-n$ represents the transferred heat and also Δu in this process. The dotted line abc represents a constant pressure line.

at 2. Or if x_2 is known, the volume $v_2 = v_1$ can be found, after which it is usually possible to find in the superheat tables other properties at 1.

Since the $\int p dV$ is zero, the energy equation is

$$(e) \quad Q = \Delta u = u_2 - u_1 = \left(h_2 - \frac{p_2 v_2}{J} \right) - \left(h_1 - \frac{p_1 v_1}{J} \right) \text{ Btu/lb.}$$

Some tables give values of the internal energy.

176. Adiabatic and Isentropic Processes. (See § 70.) For the adiabatic process including the isentropic, $Q = 0$, the nonflow energy equation $Q = \Delta u + W$ gives

$$(f) \quad \begin{array}{lll} W = -\Delta u & \text{or} & W = u_1 - u_2 \\ \text{[NONFLOW]} & & \text{[REVERSIBLE]} \end{array} \quad \text{or} \quad \begin{array}{l} W = u_1 - u_2' \\ \text{[IRREVERSIBLE]} \end{array}$$

where the u 's are generally obtained from $u = h - pv/J$ and the states

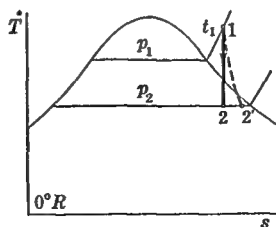


Fig. 114. Isentropic Process.

are as shown in Fig. 114. The steady flow equation with $Q = 0$ and $\Delta P = 0$ becomes

$$(g) \quad W = -(\Delta h + \Delta K) \quad \text{or} \quad W = h_1 - h_2, \quad \text{or} \quad W = h_1 - h_2$$

[STEADY FLOW] [REVERSIBLE, $\Delta K = 0$] [IRREVERSIBLE, $\Delta K = 0$]

An example of steady flow to which this equation applies is a steam turbine if the process is reversible (isentropic),

$$(h) \quad s_1 = s_2 = (s_g - y s_{fg})_1 = (s_f + x s_{fg})_2,$$

[REVERSIBLE NONFLOW OR STEADY FLOW]

in which it is presumed that state 2 is in the wet region, Fig. 114

177 Example: Isentropic Process Three pounds per second of steam expand isentropically from $p_1 = 300$ psia and $t_1 = 700^\circ\text{F}$ to $t_2 = 200^\circ\text{F}$. Find x_2 and the works for steady flow ($\Delta K = 0$) and nonflow processes

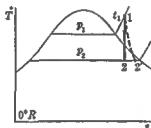


Fig. 114 Repeated.

SOLUTION See Fig. 114. From Table VIII, we find $s_1 = 1.6751$, $h_1 = 1368.3$, and $v_1 = 2.227$. From Table VI, we get $p_1 = 11,526$ psia and

$$\begin{array}{lll} s_{f2} = 0.2938, & h_{f2} = 167.99, & v_{f2} = 0.01663, \\ s_{fg2} = 1.4824, & h_{fg2} = 977.9, & v_{fg2} = 33.62, \\ s_{g2} = 1.7762, & h_{g2} = 1145.9 & v_{g2} = 33.64 \end{array}$$

(One often saves time by writing down all the table values at once.) Using $s_1 = s_2 = (s_g - y s_{fg})_2$ we have

$$1.6751 = 1.7762 - 1.4824 y_2, \text{ whence, } y_2 = 6.82\% \text{ and } x_2 = 93.18\%$$

$$h_2 = (h_g - y h_{fg})_2 = 1145.9 - (0.0682)(977.9) = 1079.2 \text{ Btu/lb}$$

The work of the steady flow isentropic is

$$W = h_1 - h_2 = 1368.3 - 1079.2 = 289.1 \text{ Btu/lb, or } 867.3 \text{ Btu/sec}$$

for 3 lb/sec. Next, find the internal energies at states 1 and 2

$$\begin{aligned} u_1 &= h_1 - \frac{p_1 v_1}{J} = 1368.3 - \frac{(300)(144)(2.227)}{778} = 1244.6 \text{ Btu/lb} \\ v_2 &= v_{g2} - y v_{fg2} = 33.64 - (0.0682)(33.62) = 31.35 \text{ cu ft/lb} \\ u_2 &= 1079.2 - \frac{(11,526)(144)(31.35)}{778} = 1012.3 \text{ Btu/lb} \end{aligned}$$

The nonflow isentropic work is

$$W = u_1 - u_2 = 1244.6 - 1012.3 = 232.3 \text{ Btu/lb.}$$

or 696.9 Btu/sec for 3 lb/sec.

178. Lost Work and Unavailable Energy. As a simple and convenient partial measure of the short comings of an actual work-producing process or cycle, we may use *lost work*, which is defined as the ideal work W of the corresponding process (or cycle) minus the actual work W' of the process (or cycle);

$$(i) \quad \text{Lost work} = W - W'. \\ \text{[POSITIVE WORKS]}$$

For steady flow adiabatic processes 1-2 and 1-2' between pressures p_1 and p_2 , Fig. 115, $W = h_1 - h_2$ and $W' = h_1 - h_{2'}$, for $\Delta K = 0$; then

$$(j) \quad \text{Lost work} = h_1 - h_2 - (h_1 - h_{2'}) \\ = h_{2'} - h_2 \text{ Btu/lb.},$$

which is seen to be represented by the area under the constant pressure line 2'-b-2. Note that the increase of unavailable energy $\Delta E_u = T_o \Delta s$ as previously defined is also a good measure of the effect of irreversibility when compared with the original available energy, but it should not be confused with lost work. Read the caption to Fig. 115.

179. Example: Irreversible Adiabatic Process. Let the data be the same as in § 177 except that an irreversible adiabatic expansion occurs in a steam turbine to state 2' on the saturated vapor line, $x_{2'} = 100\%$. What is the horsepower output? What percentage of the ideal work is lost? See Fig. 116.

SOLUTION. The work of the irreversible expansion ($\Delta K = 0$) is

$$W' = h_1 - h_{2'} = 1368.3 - 1145.9 \\ = 222.4 \text{ Btu/lb.}$$

For a flow of 3 lb./sec. and 42.4 Btu/hp-min., we get

$$hp = \frac{(3)(222.4)(60)}{42.4} = 942 \text{ hp.}$$

The lost work is $W - W' = 289.1 - 222.4 = 66.7$ Btu/lb., which is

$$\frac{66.7}{289.1} = 23.1\%$$

of the ideal work between the given pressures.

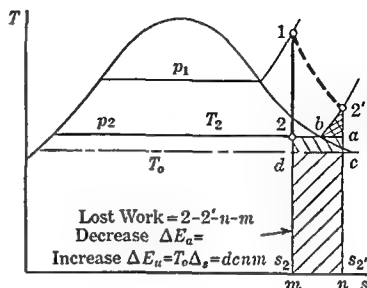


Fig. 115. Lost Work and Unavailable Energy. Irreversibility results in increasing entropy; point 2' is to the right of point 2. If $T_o = T_2$, with 2' as shown, the increase of unavailable energy $T_o \Delta s$ is less than the lost work by the triangular area a-b-2'. If 2' is in the wet region and $T_o = T_2 = T_{2'}$, the increase in unavailable energy is the same as the lost work. If $T_o < T_2$ as shown, the loss of available energy is always less than the lost work.

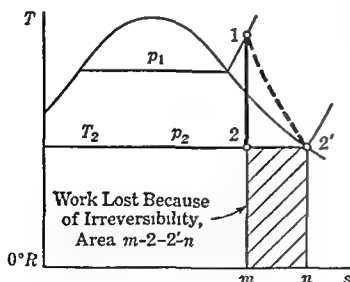


Fig. 116.

main If a sample of this high-pressure steam is throttled to a lower pressure, then, within limitations, it will become superheated, point 2 In the superheated condition, we can measure its pressure and temperature, thus locating precisely point 2 With the pressure and temperature known, the enthalpy h_2 may be obtained from the superheat tables Then

$$(t) \quad h_2 = (h_g - y_1 h_{fg})_1 \text{ Btu/lb},$$

from which $x_1 = 1 - y_1$ may be found by looking up h_g and h_{fg} , corresponding to some known main line pressure p_1 , Fig 119

The drawback in using equation (t) is that the actual temperature and pressure at 2 is nearly always such that a double interpolation in the superheat tables is necessary If the highest degree of accuracy is not required, the computation may be simplified with the use of the specific heat of steam at constant pressure In Fig 119 we note that the enthalpy at 2 is equal to the enthalpy of saturated steam h_g plus the enthalpy to superheat the steam from g to 2 The heat to superheat the steam Q_{sh} may be computed from $Q_{sh} = u c_p \Delta t_{sh}$ because the points g and 2 are on a constant pressure curve For the conditions encountered in a calorimeter c_p for steam may be taken as 0.48 Btu per lb $^{\circ}\text{F}$ Thus for one pound,

$$(u) \quad h_2 = h_g + c_p \Delta t_{sh} = h_g + 0.48 \Delta t_{sh}$$

where Δt_{sh} = degrees of superheat at point 2, Fig 119 and is obtained from

$$\Delta t_{sh} = (\text{actual temperature at 2}) - (\text{saturation temperature corresponding to } p_2)$$

183 Calorimeters In finding the quality of the steam as explained in § 182, we allow a sample of steam to flow into an instrument known as the *throttling calorimeter*, Fig 120 thence to the atmosphere Before temperature readings are taken at the calorimeter the steam should flow long enough to obtain internal equilibrium that is, long enough for all parts to be heated to a temperature which remains constant To maintain adiabatic conditions the instrument should be well insulated and for reliable results, the steam should have at least 10° of superheat in the calorimeter If the steam is initially very wet, it may not become superheated in the calorimeter in which case the calorimeter readings are meaningless, the temperature and pressure being those at saturation This condition is illustrated by process mn , Fig 119 The minimum quality that can be reliably measured by the throttling calorimeter depends upon the initial pressure p_1 (the final

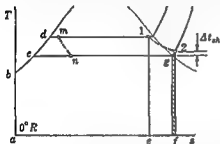
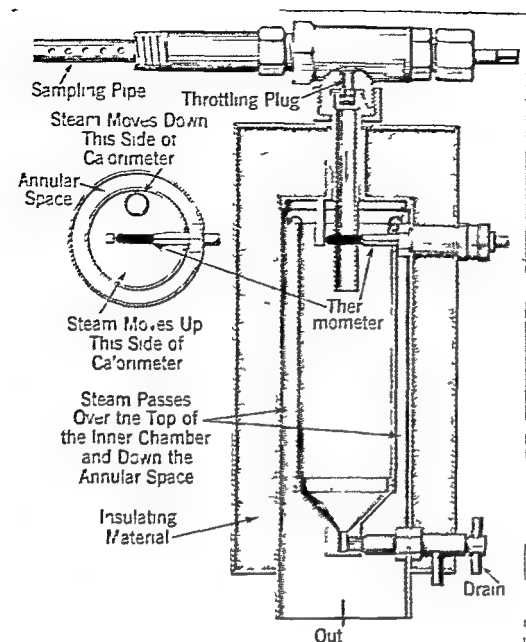
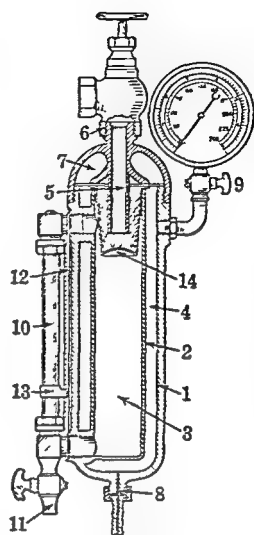


Fig 119 Throttling Process (Irreversible) (Area $a-b-d-1-e$) (area $a-b-c-2-f$) because these areas represent, respectively, h_1 and h_2 (closely)



Courtesy Lewis M. Ellison, Chicago

Fig. 120. Throttling Calorimeter. The sampling pipe, which is in the steam main with holes facing upstream, should be as far as possible downstream from elbows and valves in order for the sample to be representative, preferably in a vertical steam line. The steam flows into the inner chamber. A thermometer gives the temperature. The pressure is virtually atmospheric. Flowing out of the inner chamber at the top, the steam turns downward into the annular space and leaves the instrument at the bottom. The hot steam surrounding the inner chamber (a patented feature) is an aid in insulating against loss by radiation.



Courtesy Emmer and Amend, New York

Fig. 121. Separating Calorimeter. The wet steam, entering at the top through the valve, undergoes a sudden reversal of direction of motion when it strikes the baffle plate 14. This causes the water, which has a greater inertia, to separate from the mixture. The water collects in the inner chamber and can be measured by the gage glass 10. The steam moves upward from the inner chamber 3 and downward in the annular space 4.

pressure p_2 being nearly atmospheric), and varies from about 97% for $p_1 = 50$ psig to 94% for $p_1 = 600$ psig.

If the steam is too wet to be measured by a throttling calorimeter, a separating calorimeter, Fig. 121, or a combination separating and throttling calorimeter may be used. In the separating calorimeter, the moisture is separated from the steam. The amount of collected water is determined. The corresponding mass of dry steam is calculated from flow equations applied to a nozzle (see Chapter 13) or read from a gage; or the steam leaving

the nozzle may be condensed and weighed. The quality is then computed from the formula

$$(v) \quad x = \frac{(\text{mass of dry steam})}{(\text{mass of water}) + (\text{mass of dry steam})}$$

Incomplete separation of water and steam and inaccuracies in the devices and methods used to find the weight of flow of dry steam will result in an incorrect evaluation of the quality.

184 Mollier Diagram Using any two point functions of substances such as temperature and entropy we may construct to scale a diagram containing a series of lines each representing a constant pressure another series where each line represents a particular volume and another series for constant quality etc. On the Ts plane Fig 122 have been plotted several constant pressure constant volume and constant quality (also superheat) lines. Thus a point may be located by any two of these coordinates after which all other properties pictured may be read from the diagram. The degree of accuracy of the reading will depend upon the spacing of the lines and the size of the diagram.

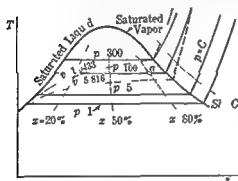


Fig 122 Temperature-Entropy Diagram for Steam. The point *a* for example represents steam at a pressure of 100 psia and a quality of 80%. These two coordinates could be used to locate *a* and then the volume could be determined from the constant volume lines in this case between $v = 1.6433$ and $v = 8.816$. (There are not enough constant volume lines on this figure for accurate interpolation.)

a series of constant temperature lines. The constant temperature lines of course coincide with the constant pressure lines in the wet region (below the saturated vapor line) but bend toward the right away from the constant pressure lines in the superheat region. Your steam tables contain a Mollier chart but one which shows a section similar to that marked off by the dotted lines of Fig 123.

The Mollier chart is most useful in connection with steady flow processes. The constant quality lines are labeled according to their per cent of moisture $y = 1 - x$. The constant pressure lines in Fig 123 are straight in the wet region the break shown being the result of a change of scale for entropy. See the examples in the caption to Fig 123 for the method of using the chart.

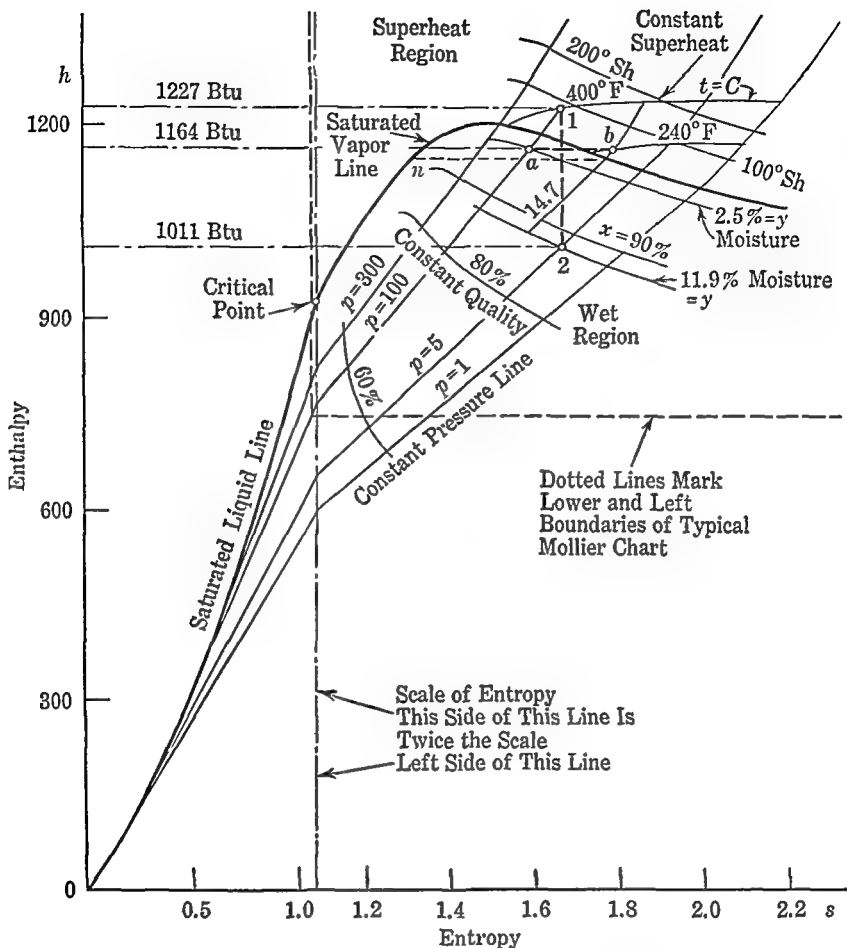


Fig. 123. Enthalpy-Entropy (Mollier) Diagram for Steam. Example. Steam in the main is at $p_a = 100$ psia. The sample in the calorimeter is at 14.7 psia and 240°F . What is the moisture content y_a of the steam in the main?

Solution. From the conditions in the calorimeter, locate point b at the intersection of the curves $p_b = 14.7$ psia and $t_b = 240^\circ\text{F}$. Since the throttling process into the calorimeter is a constant enthalpy process, $h_a = h_b$, move along a horizontal line until the pressure line $p_1 = 100$ is reached, and locate point a . Read the answer $y_a = 2.5\%$. If the initial pressure is too high, $p_1 > p_a \approx 1800$ psia, expansion does not reach superheat at 14.7 psia and a throttling calorimeter could not be used.

Example. Steam enters a turbine at a pressure of 100 psia and 400°F , point 1, and then expands isentropically to a pressure of 5 psia. What work is done if $\Delta K = 0$?

Solution. Using the properties of the entering steam, locate point 1 at the intersection of the curves $p_1 = 100$ psia and $t_1 = 400^\circ\text{F}$. Now follow a constant entropy (vertical) line until the 5 -psia line is reached, and locate point 2. Move to the left ordinate from point 1 and read $h_1 = 1227$ Btu per lb.; move to the left ordinate from point 2 and read $h_2 = 1011$ Btu per lb. Then equation (g), § 176, gives $W = 1227 - 1011 = 216$ Btu/lb. Also, we read the moisture at 2; $y_2 = 11.9\%$.

185 Heat Exchanger Process. The typical heat exchanger is a steady flow device, an open system, involving two substances, one rejecting heat and the other receiving the heat which is being rejected. The system may be either one or both substances. No work as shaft work is done in these devices and usually the change in kinetic energy of either substance is quite small ($\Delta K \approx 0$) and $\Delta P = 0$. For these conditions, the energy equation (9), repeated at the beginning of this chapter, applied to one substance, yields

$$(w) \quad Q = \Delta h = h_2 - h_1 \text{ Btu/lb}$$

which is seen to be the same as for a constant pressure process, equation (b). For a steady flow process as defined, equation (w) applies for either constant pressure (a reversible flow) or for the case where the pressure decreases in the downstream direction (as it actually does—an irreversible flow). Descriptions of several heat exchangers are given in the next chapter, boilers, air heaters, economizers, feedwater heaters, condensers, etc.

186. Output of Boilers. Big hydroelectric power plants notwithstanding, the great preponderance of electric energy used in this country is developed in steam power plants. Therefore, we may appropriately give a few moments attention to boilers which produce the steam for this power. The output of boilers is often expressed as the number of pounds of steam delivered per hour. However, since steam at different boiler pressures and temperatures contains different amounts of energy, the number of pounds of steam does not constitute an absolute unit. It is evident that an absolute unit of capacity must measure the amount of heat transferred to the steam. The ASME recommends a unit of 1000 Btu per hr, called a *kilo Btu* (kB) per hour, or a unit of 1,000,000 Btu per hr called a *mega Btu* (mB) per hour. Let h_1 be the enthalpy of the water as it reaches the boiler (or economizer if one is used), let h_2 be the enthalpy of the steam delivered by the boiler (or superheater if one is used), and let w_{bo} be the pounds of steam delivered by the boiler (or superheater) per hour. Then, the total transferred heat in Btu per hour is $w_{bo}(h_2 - h_1)$, equation (w), and the output of the boiler is

$$(x) \quad \text{Boiler output} = \frac{w_{bo}(h_2 - h_1)}{1000} \text{ kilo Btu/hr}$$

For accurate results, one should consider the enthalpy of the entering water h_1 as being that of compressed liquid (§ 168). The pressure on the water is taken as boiler pressure, yet the temperature is below saturation temperature. If the weight of steam w_{bo} is the maximum amount that the boiler can produce, equation (x) gives the maximum capacity.

A unit of boiler output or boiler capacity that has been commonly used for years is the *boiler horsepower*. Although, at one time, one boiler horsepower produced roughly one horsepower in the prime mover, today, because of the greater efficiency of modern prime movers, there is no relation between

a boiler horsepower and the horsepower of an engine. Since this is true, the name *boiler horsepower* is misleading, it causes much confusion in the mind of the novice, and its use is discouraged. Specifically, one boiler horsepower is the evaporation of 34.5 lb. of water per hour from and at 212°F. By *from and at* is meant from saturated water at 212°F into saturated steam at 212°F. For a saturation temperature of 212°F, the pressure is 14.696 psia. The amount of heat required to evaporate one pound of water from and at 212°F is $h_{fg} = 970.3$ Btu, p. 196; hence, the amount of heat for 34.5 lb. of water is $(34.5)(970.3) = 33,475.3$ Btu. Consequently,

$$\text{One boiler horsepower} = 33,475.3 \text{ Btu/hr.},$$

of heat transferred to water and steam. We see then that the output or capacity of a boiler may be expressed as

$$(y) \quad \text{Boiler horsepower (bo. hp)} = \frac{w_{bo}(h_2 - h_1)}{33,480},$$

where w_{bo} , h_2 , and h_1 are as defined for equation (x). Although the unit *boiler horsepower* is frowned upon, its use is so widespread that the engineer must know its meaning.

187. Closure. The work of this chapter and the preceding one lays the foundation for the study of vapor engines and vapor cycles. In solving problems, the student should aim to achieve facility in the use of the vapor tables and of the charts. On this account, it is advisable to find both vapor-table and chart solutions of problems whenever both methods apply.

Since we have met with a number of adiabatic processes, it may be useful to summarize certain features in tabulated form. The following relations are true for either a perfect gas or a vapor:

Nonflow $(Q = 0)$	$\left\{ \begin{array}{l} \text{reversible} \\ \text{irreversible} \end{array} \right. \left\{ \begin{array}{l} \Delta s = 0, \\ W = -\Delta u, \\ \Delta s > 0, \\ W = -\Delta u, \end{array} \right.$	Steady flow $(Q = 0, \Delta K = 0)$	$\left\{ \begin{array}{l} \text{reversible} \\ \text{irreversible} \\ \text{throttling} \end{array} \right. \left\{ \begin{array}{l} \Delta s = 0, \\ W = -\Delta h, \\ \Delta s > 0, \\ W = -\Delta h, \\ \Delta s > 0 \\ \Delta h = 0 \\ W = 0. \end{array} \right.$
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12

VAPOR CYCLES AND ENGINES

188. Introduction The Carnot cycle, which consists of two isothermals and two isentropics, operates on any substance, hence, the limits it sets are as appropriate to a vapor cycle as to any, and all the conclusions based on the Carnot cycle are valid. Nevertheless, in practice, it is advisable to use as a standard of comparison a cycle which is less perfect, but which accords more closely with the actual events of a particular cycle, just as we used the Otto cycle as an approximation of the events of a gasoline engine and as we used the Brayton cycle as the ideal prototype of the gas turbine.

189. Ideal Cycle, Ideal Engine In one important respect, the cycles for vapors will differ from those for the internal combustion engines, namely all the ideal processes of the cycles for internal combustion engines were conceived of as taking place *within* the engine, whereas in the vapor cycles, the working substance is passed successively through distinct and separate devices, such as a boiler, an engine, a condenser, a pump. Because of this difference, it is often advisable to consider separately the *ideal cycle* and the *ideal engine*.* The *ideal cycle* includes all the processes which occur—those in the boiler, the engine, the condenser etc. The *ideal engine* is concerned only with the processes associated directly with the engine, those of *delivering the substance to the engine, of expansion in the engine, of exhaust from the engine*, including any transfer of heat while the substance is in the engine. The practical reason for this distinction lies in the desire to charge losses to the proper pieces of equipment. For example, a boiler may generate steam at 400 psia pressure and 700°F, but this steam, due to friction of flow and radiation from the steam line, may arrive at the throttle of the prime mover at a pressure of 370 psia and 660°F. In this instance,

* The word *engine*, unless otherwise defined by its context, should be understood to include all appropriate types of prime movers

there is a distinct loss between boiler and engine. The manufacturer of the *engine* would justly object to charging the engine with the higher-pressure, higher-temperature steam of the boiler, since the engine never receives such steam. Thus, in this illustration, the ideal *engine* would have one efficiency based upon the condition of the steam as *delivered to it*, and the ideal *cycle* would have another efficiency based upon the condition of the steam as *delivered by the steam generator*. The other principal difference between the ideal engine and the ideal cycle of a steam-power plant is that the output (work) of the ideal *cycle* is charged with the work of pumping the feedwater into the boiler, because this expenditure of energy necessarily occurs in the performance of the cycle, but the ideal (or actual) *engine* is considered independently of the pump.

190. Rankine Cycle and Engine. The Rankine *cycle*, or complete expansion cycle, was the first accepted standard of comparison for steam power plants and is often so used today. (See Fig. 40, p. 91.) Figure 124 shows

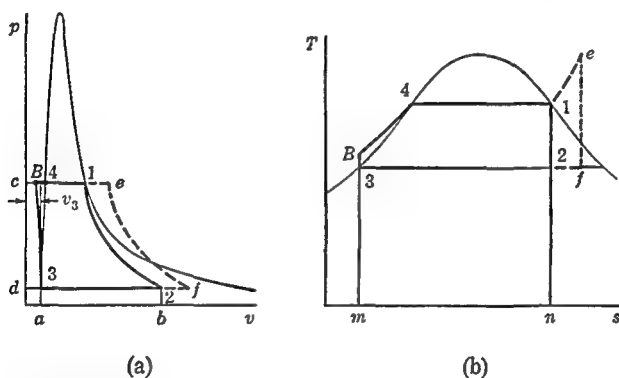


Fig. 124. Rankine Cycle. The volumes of the liquid in (a) and the temperature rise $3B$ in (b) are greatly exaggerated.

the processes on the pv and Ts planes. The steam leaves the boiler in the state 1, moves to the engine without loss, expands isentropically in the ideal engine to point 2, and passes to the condenser (or to the atmosphere). Circulating water condenses the steam to a saturated liquid at 3, from which state it is isentropically pumped into the boiler in a subcooled liquid state B . Note the irreversible process of mixing cold water at temperature t_B with the hot water in the boiler at temperature $t_4 = t_1$. The compressed liquid at B is heated until it becomes saturated at 4, after which it is evaporated to steam at 1, and the cycle begins again. If the steam is superheated before it leaves the steam generator, the corresponding Rankine cycle would be $e-f-3-B-4-e$.

The Rankine* cycle may be analyzed either by considering the cycle as a whole or by studying each of the individual pieces of equipment, as shown in Fig 125. The cycle as shown in Fig 124 has heat added at constant pressure $B-4-1$ (or steady flow, $\Delta K = 0$, § 185),

$$(a) \quad Q_A = h_1 - h_B \text{ Btu/lb,}$$

represented by the area $m-B-4-1-2-n$ on the Ts plane. Verify equation (a) from Fig 125, where in the ideal cycle $h_a = h_1$, that is, *there is neither friction of flow nor heat loss Q in any pipe line of the ideal cycle*. The state point is

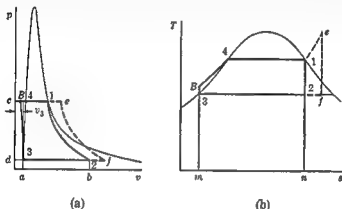


Fig 124 Repeated

moving leftward only from 2 to 3, Fig 124 (b), so the rejected heat as a positive number (constant pressure or steady flow with $\Delta K = 0$) is

$$(b) \quad Q_R = h_2 - h_3 \text{ Btu/lb,}$$

represented by the area $n-2-3-m$, Fig 124 (b). Since $W_{\text{net}} = Q_A - Q_R$, the net work of the cycle is

$$(c) \quad W_{\text{net}} = h_1 - h_B - (h_2 - h_3) = h_1 - h_2 + h_3 - h_B \text{ Btu/lb,}$$

represented by the enclosed areas $1-2-3-B-4$, Fig 124. The value of $h_B = h_2 + W$, [see equation (e) below] is the enthalpy of the liquid after an isentropic compression from a saturated (or near saturated) state 3; for water, it can be found from Keenan and Keyes (71) (see their Fig 3). In

* William John M. Rankine (1820-1872), a professor at Glasgow University, was contemporary with those giants of thermodynamics Joule, Maxwell, Kelvin, Clausius. It was during the lifetime of and largely due to the efforts of these five men that the laws and the science of thermodynamics were formulated and interpreted. Rankine had no small part in these developments. He was a man of versatile genius and a prolific contributor to the engineering and scientific literature of his day. Among his published writings are included an important textbook on mechanics, a civil engineers' manual, and works on water supply, the steam engine, shipbuilding, and many other subjects of a technical nature, not to mention *Songs and Fables*—for he was also a composer of music and a vocalist of good talent. Rankine's modification of Gordon's column formula is in wide use today.

the absence of Keenan and Keyes *Steam Tables*, about the only thing one can do is to estimate $h_B - h_3$ from equation (17) for $T ds = 0$ and $v = C$, as in § 168:

$$(d) \quad h_B - h_3 = \int \frac{v dp}{J} = \frac{v_3(p_B - p_3)}{J} = \frac{v_f(p_B - p_s)}{J} \quad [\text{APPROXIMATE}]$$

as a positive number, where p_B is the pressure on the discharge side of the pump, and in the general form, v_f is the volume of saturated liquid and p_s is the saturation pressure at the temperature of the liquid entering the pump.

Considering the energy diagram of the pump as a system in Fig. 125, we see that if the heat Q_p is negligible, as it usually is and as it is in the ideal case, the ideal work of the pump is

$$(e) \quad W_p = (h_B - h_3)_s \text{ Btu/lb.,}$$

represented (in foot-pound) by the area 3-B-c-d, Fig. 124(a); and equation (d) gives an approximate value of the ideal W'_p . The right-hand side of

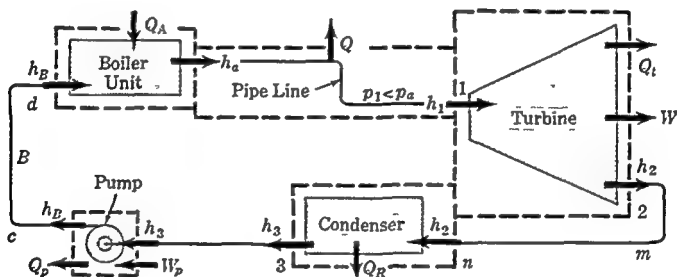


Fig. 125. Energy Diagrams for Steam Power Plant. The radiation losses Q , Q_t and Q_p are usually small. There would be some heat loss from the water line cd , but it would be quite small. The line mn is not really a line at all because the condenser is attached directly below the turbine. In this course, ignore Q 's which are not specified or implied.

equation (d) is the rectangular area whose width is v_3 and whose height is dc in Fig. 124(a). The actual pump work W'_p can be obtained if the efficiency η_p is known:

$$(f) \quad W'_p = \frac{(h_B - h_3)_s}{\eta_p} \text{ Btu/lb.,}$$

where the subscript s is a reminder that, strictly, the change of enthalpy is for constant entropy. At high pressures and temperatures where water is significantly compressible, equation (d) may be seriously in error as a percentage of the true value of $h_B - h_3$, but the error as a percentage of the work of the Rankine cycle will be small.

Since other ideal vapor cycles contain the equivalent of one or more Rankine cycles, it will be advantageous to put equation (c) into a form that will shorten future analyses. First we note from the energy diagram of the

turbine (which in the ideal case of an *isentropic expansion* is the Rankine engine (Fig 125), or from § 176, that the work is

$$(65) \quad W = -\Delta h = h_1 - h_2 \text{ Btu/lb,} \\ \text{[RANKINE ENGINE } \Delta K = 0 \text{ } \Delta s = 0]$$

represented by the area $c-1-2-d$, Fig 124(a), and to a more natural scale by area $b-1-2-a$, Fig 126. When equation (65) is applied to the engine, point 1

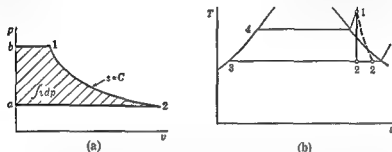


Fig 126 Work of Rankine Engine In (a), area $1-2-a-b = p v_1/J + (u_1 - u_2) - p_2 v_2/J = h_1 - h_2$ Btu per lb. Observe that this area is that of an indicator card for a complete-expansion reciprocating engine without clearance. Only the expansion process appears on the Ts plane, as seen in (b).

is the state at the entrance to the engine and point 2 is the state at the exhaust pressure after an isentropic expansion. Comparison of equations (65) and (c) shows that the work of the cycle W_{net} is equal to the gross work W minus the pump work W_p . This is also revealed by the energy diagram of all the H_2O in the cycle (Fig 40, p 91). Thus, we observe that the work of a Rankine engine ($\Delta K = 0$, $\Delta s = 0$) is the initial minus the final enthalpy, and the work of the Rankine cycle is this decrease of enthalpy minus the pump work

$$(66) \quad W_{net} = W - W_p = h_1 - h_2 - W_p \text{ Btu/lb} \\ \text{[RANKINE CYCLE]}$$

Besides the difference of the pump work another significant difference is as follows

$$\begin{array}{l} \text{Cycle} \left\{ \begin{array}{l} \text{point 1 is state of steam leaving steam generator} \\ \text{point 2 is on a constant entropy line from 1} \end{array} \right. \\ \text{Engine} \left\{ \begin{array}{l} \text{point 1 is state of steam entering engine} \\ \text{point 2 is on a constant entropy line from 1} \end{array} \right. \end{array}$$

At low pressures, the pump work is negligible as compared with the work of the cycle and can often be omitted. But in modern central station plants with the pressures becoming higher and higher, it should be counted. The decision as to whether or not to include W_p in equation (66) is a matter of judgment, a decision which the engineer makes, based on experience. For academic purposes, we may take an arbitrary dividing line. Omit pump

work for pressures at or below 400 psia; include it for pressures above 400 psia. It is not always advisable to consider $\Delta K = 0$ for the turbine (89).

191. Rankine Efficiencies. The thermal efficiency of the Rankine cycle is

$$e = \frac{W_{\text{net}}}{Q_A} = \frac{h_1 - h_2 - (h_B - h_3)}{h_1 - h_B},$$

where the subscripts refer to Fig. 124. Since the pump work $W_p = h_B - h_3$ [equation (e)], we may substitute $h_B = h_3 + W_p$ into the foregoing equation and obtain the thermal efficiency of a Rankine cycle as

$$(67) \quad e = \frac{W_{\text{net}}}{Q_A} = \frac{h_1 - h_2 - W_p}{h_1 - h_3 - W_p}.$$

In setting up the expression for the efficiency of steam engines and turbines, we must decide concerning the quantity of energy E_c that should be charged against the engine. Custom and the ASME Test Code require that the engine be credited with the enthalpy of saturated liquid at the exhaust pressure. It is argued that the water from the condensation of the steam with an appreciable enthalpy may be returned to the boiler without further loss. Moreover, the steam prime mover is not designed to obtain work from liquids, and it would therefore be unfair to charge the enthalpy of the liquid against the engine. This practice of crediting the engine with the enthalpy of saturated liquid at the exhaust condition is followed whether the engine exhausts to a condenser, whence the condensate is returned to the boiler, or whether the engine exhausts to the atmosphere with the consequent actual loss of the enthalpy of the liquid. Therefore, the thermal efficiency of the ideal Rankine engine is

$$(68) \quad e = \frac{W}{E_c} = \frac{h_1 - h_2}{h_1 - h_{f2}},$$

where h_1 is the enthalpy of the throttle steam, h_2 is the enthalpy of the steam leaving the engine, and h_{f2} , which is the same as h_3 in Fig. 124, is the enthalpy of saturated liquid at the exhaust pressure. Comparison of equations (67) and (68) shows that the efficiencies of the ideal cycle and engine are the same whenever the pump work W_p is negligible, since $h_3 = h_{f2}$ —but only then.

192. Mep and Steam Rates. The mean effective pressure of the Rankine engine is the work in foot-pound units divided by the displacement in cubic-

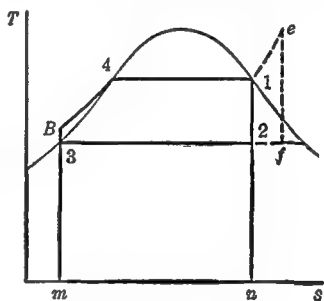


Fig. 124(b). Repeated.

foot units, that is,

$$p_m = \frac{W}{V_D} = \frac{J(h_1 - h_2)}{v_2} \text{ psf,}$$

where v_2 is taken as the effective displacement *per pound of steam* of the Rankine engine without clearance, the volume at state 2, Fig 126

The water rate or *steam rate* is the number of pounds of steam supplied to an engine per unit of work. The unit of work is usually either the horsepower-hour or the kilowatt-hour; therefore the steam rate is expressed as pounds per horsepower-hour or pounds per kilowatt-hour. Let the work W be in Btu per pound, and note that there are 2544 Btu per hp-hr. Then, the steam rate w is

$$(69A) \quad w = \frac{2544}{W} = \frac{(\text{Btu})/(\text{hp hr})}{(\text{Btu})/(\text{lb steam})} = \frac{\text{lb steam}}{\text{hp-hr}},$$

$$(69B) \quad w = \frac{3412}{W} = \frac{\text{lb steam}}{\text{kw-hr}}$$

As we have learned, the work output of a steam engine may be measured at any one of three points (Fig 103, p 185). It follows that for the *actual* engine, there may be stated three steam rates, one each based upon the indicated work W_I , brake work W_B , or combined work W_K . If actual steam rates are known, we may find actual works from them. Thus, let w_i = indicated steam rate in pounds per indicated horsepower-hour, w_b = brake steam rate in pounds per brake horsepower-hour, and w_k = combined steam rate in pounds per kilowatt-hour output of the generator. Then from equation (69), we can have

$$W_I = \frac{2544}{w_i} \text{ Btu/lb,} \quad W_B = \frac{2544}{w_b} \text{ Btu/lb,} \\ W_K = \frac{3412}{w_k} \text{ Btu/lb}$$

It should be remarked that the relative performances of two or more prime movers cannot be determined from the steam rates alone. An engine or turbine operating through a large pressure range may have a relatively small steam rate, while the engine operating through a small pressure range may have a relatively large steam rate, yet the latter engine may more nearly approach perfection, the approach being measured by the engine efficiency. Moreover, the Btu input to the latter engine to produce a unit of work may be less than the Btu input to the engine operating on the large pressure range. Thus, while the steam rate is a convenient number, we must use the thermal and engine efficiencies and heat rates for reliable indices of absolute and comparative performances.

193. Efficiencies in Terms of Steam Rates. The *heat rate* of an engine or power plant is the input of energy per unit of work. For a power plant

as a whole, it is usually measured as the higher heating value of the fuel consumed in Btu per kw-hr. of work (electricity) output.

The energy charged against the engine is $E_c = h_1 - h_{f2}$ Btu per lb. (§ 191). If the engine has a steam rate of w lb. per hp-hr., the input energy chargeable against the engine is

$$(g) \quad \text{Heat rate} = w(h_1 - h_{f2}) \text{ Btu/hp-hr.},$$

where the heat rate may be for the ideal engine (use w), or it may be the indicated heat rate (use w_i and get Btu per ihp-hr.), or the brake heat rate (use w_b and get Btu per bhp-hr.) If the combined steam rate is w_k lb. per kw-hr., this value in equation (g) gives the heat rate in Btu per kw-hr. Since the thermal efficiency of the engine is the work *out* divided by the energy chargeable as the input, and since there are 2544 Btu per hp-hr.,

$$(h) \quad e = \frac{(\text{Btu of work})/(\text{hp-hr.})}{(\text{Btu of chargeable energy})/(\text{hp-hr.})} = \frac{2544}{w(h_1 - h_{f2})},$$

where h_1 is the enthalpy of the throttle steam and h_{f2} is the enthalpy of saturated liquid at the exhaust condition. Now the *ideal engine* corresponding to any actual engine is one wherein (1) the pressure and quality (or superheat) at the beginning of isentropic expansion in the ideal engine (point 1, Fig. 126) is the same as the pressure and quality (or superheat) at the throttle of the real engine, and (2) the pressure at the end of isentropic expansion in the ideal engine is the same as the exhaust pressure of the real engine. It follows that h_1 and h_{f2} in equation (h) are the same for both actual and ideal engines. Therefore, equation (h) will yield the ideal thermal efficiency, the indicated thermal efficiency e_i , or the brake thermal efficiency e_b , depending upon whether the steam rate is, respectively, the ideal w , the indicated w_i , or the brake w_b . Also, if w_k is lb. per kw-hr., the *combined thermal efficiency* is

$$(i) \quad e_k = \frac{3412}{w_k(h_1 - h_{f2})}.$$

The *engine efficiency* η not only is used to measure the approach of the actual engine to the ideal but is also useful in estimating the performance of a newly designed engine where old data on the efficiencies of similar engines are available. We recall (§ 156) the definition of engine efficiency as

$$(60) \quad \eta = \frac{\text{actual work of a system}}{\text{work of the corresponding ideal system}}.$$

Review § 156. Since equation (69) is true, we may note that

$$(j) \quad \eta_i = \frac{W_I}{W} = \frac{w}{w_i}, \quad \eta_b = \frac{W_B}{W} = \frac{w}{w_b}, \quad \eta_k = \frac{W_K}{W} = \frac{w}{w_k}.$$

In each expression, W and w are values for the proper ideal engine. In these ratios, the steam rates of the numerator and denominator must of course be in the same units, either both lb per hp hr or both lb per kw hr. The work quantities may each be in Btu per pound of steam, or any energy unit per unit of time, such as a horsepower.

The *mechanical efficiency* is the ratio of the brake work to the indicated work, or $\eta_m = W_B/W_I = \text{bhp}/\text{ihp}$, etc.

194 Effect of End Conditions on the Thermal Efficiency of Rankine Engine Though there are limitations to this statement, we have found by experience that when certain changes bring about an increase in the efficiency of the ideal cycle, analogous changes in the actual cycle generally increase the actual thermal efficiency. This is one reason for the importance of the ideal cycles. For the Rankine engine, a numerical study of equation (68) will show that the following changes increase the Rankine efficiency:

1 If the condenser temperature is lowered (Fig. 127) the heat rejected will be less the work will be greater and therefore the efficiency will be increased. Nature,

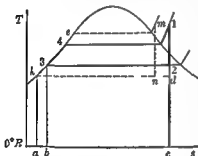


Fig. 127 Effect of Operating Pressures on the Efficiency of the Rankine Cycle. A lowering of the final pressure from 2 to d increases the Rankine work by the amount of the area $2-d-k-3$, reduces the heat rejected from $2-3-b-c$ to $dkac$. For the increased boiler pressure at m , it is assumed that the maximum temperature remains the same, $t_m = t_1$. Subcooling effects are ignored.

greater power production in winter (condenser pressure of 1 in. Hg abs.) than in summer (condenser pressure 2.5 in. Hg abs.)

2 If the boiler pressure is raised, the evaporation line is raised, say from $4-1$ to em (Fig. 127), and the work area $mnke$ is greater than $1-d-k-4$. True, more heat is transferred to the working substance at the higher pressure, but a greater proportion of the heat supplied is converted into work. Accordingly, we find that the efficiency of actual plants using higher pressures is greater than that of plants using lower pressures, which accounts in part for the modern trend toward high pressures.

however, precisely defines the limit of improvement that may be obtained by this means. In order to condense the steam, some cooling medium at a temperature lower than that of the steam in the condenser must be available for carrying away the rejected heat. Since we know from the second law that there will be no net gain if artificial cooling is resorted to, we must be satisfied with a condenser temperature somewhat (say 20°) above that of condensing water from rivers, lakes, etc. The best condensers are so well designed that we may confidently expect no great thermal improvement in this phase of the cycle, but efforts at improvement are always being made by someone. It is interesting to note that condensing plants commonly operate more efficiently in the winter than in the summer because of the lower cooling water temperature in the winter. A particular plant reports a 10%

Mooney (10) has computed Rankine efficiencies for pressures up to 3000 psia with the initial state always on the saturated vapor line (point 1 is saturated steam) and finds that the ideal efficiency reaches a maximum at $p_1 \approx 2500$ psia, when the exhaust pressure is about 2 psia. Above about 2500 psia, the efficiency decreases (when the steam is initially saturated).

3. Although it is not evident from a Ts diagram, the use of superheated steam, for example, cycle $e-f-3-4-e$, Fig. 124(b), improves the thermal efficiency as compared with a cycle with the same pressure limits and with saturated steam at the beginning of expansion. In fact, the improvement in the actual thermal efficiency is usually greater than that in the corresponding ideal cycle. Small particles of water in steam moving at high speed past the blades of the turbine, besides having an eroding effect on the blades, reduce the efficiency of the transformation of energy. Observe from Fig. 127 that the higher the pressure for a given initial temperature, the wetter the steam at the exhaust—point n versus point d . Consequently, at higher steam pressures, superheat becomes more desirable in order to reduce the degree of wetness in the low-pressure stages of the turbine. Superheated steam improves the efficiency of the actual reciprocating steam engine because it reduces the loss from initial condensation. For these reasons, modern central station power plants invariably use superheated steam.

195. Incomplete-Expansion Engine. Since complete expansion is seldom obtained in a reciprocating steam engine, a complete-expansion engine is not the fairest standard of comparison for it. The events for an incomplete-expansion engine without clearance are shown in Fig.

128. The engine draws in steam at constant pressure along $b-1$; the steam expands isentropically to some pressure p_2 ; at 2, the exhaust valve opens, steam flows from the cylinder until the pressure drops to the exhaust pressure 3, after which the steam is pushed from the cylinder along $3-a$. The enclosed area 1-2-3- a - b represents the work of the ideal incomplete-expansion engine. To get the work equation into the most satisfactory final form, it will be advisable to find the area 1-2- F - b (which was found to be $h_1 - h_2$ in Fig. 126 for $Q = 0$) and add to it the area 2-3- a - F , which is $v_2(p_2 - p_3)/J$. Thus we have

$$(k) \quad W = h_1 - h_2 + \frac{v_2(p_2 - p_3)}{J} \text{ Btu/lb.}$$

In finding the thermal efficiency of this ideal engine, we credit the engine with the enthalpy of the liquid at the exhaust pressure (§ 191), and use $E_c = h_1 - h_{f3}$. Therefore

$$(l) \quad e = \frac{h_1 - h_2 + v_2(p_2 - p_3)/J}{h_1 - h_{f3}},$$

where h_{f3} is the enthalpy of *saturated liquid* at the temperature or pressure at

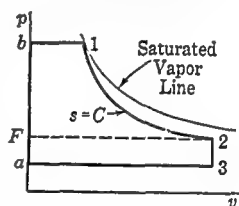


Fig. 128. Diagram for Incomplete-Expansion Engine. This diagram may be thought of as a conventional indicator card for an engine without clearance. Only 1-2 is a thermodynamic process involving constant mass.

point 3, Fig 128 Steam rates and engine efficiencies are found as previously explained Typical values of the *indicated engine efficiency* for 100-hp simple engines are single-valve engine, 56%, Corliss engine, 65%, unafflow engine, 73% These values tend to increase as the size of the engine increases, for example, a 1000 hp unafflow engine may be expected to have an engine efficiency of about 83%

The *mean effective pressure* of the ideal incomplete expansion engine is $p_m = W/V_D$, or

$$p_m = \frac{J(h_1 - h_2) + v_2(p_2 - p_3)}{v_2} \text{ psf,}$$

where both work and displacement are values per pound of steam The volume v_2 , Fig 128, is the maximum required in the cylinder when one pound of steam is admitted

To determine the operating conditions of an ideal engine corresponding to some real engine, let p_1 (ideal engine) be equal to the throttle pressure of the real engine, let the quality (or superheat) at 1 in the ideal engine be equal to the quality (or superheat) at the throttle of the real engine, let p_2 (ideal engine) be equal to the release pressure of the real engine, and let p_3 (ideal engine) be equal to the exhaust pressure of the real engine Of course, the ideal expansion 1-2 is isentropic

196 Example Steam at $p_1 = 160$ psia and $t_1 = 400^\circ\text{F}$ is delivered to the throttle of an engine The steam expands to 30 psia, where release occurs Exhaust is at 15 psia A test of the engine showed a steam consumption of 21.8 lb/ihp-hr and a mechanical efficiency of 85% Find (a) the ideal work and ideal thermal efficiency, (b) the ideal steam rate (c) the brake and indicated works, (d) the brake thermal efficiency, (e) the brake engine efficiency, (f) the mep of the ideal engine and the indicated mep See Fig 129

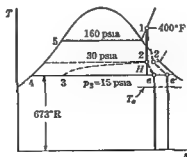


Fig 129 Incomplete Expansion Engine

SOLUTION (a) The student should check all values taken from the steam tables pp 196-199 The properties at 2 are found by using $s_1 = s_2$

$$\begin{aligned}
 s_1 &= (s_g - y s_{fg})_2 \\
 1.5908 &= 1.6993 - 1.3313 y_2 \\
 y_2 &= 8.15\%, \quad x_2 = 91.85\% \\
 h_2 &= 1164.1 - (0.0815)(945.3) = 1087 \text{ Btu/lb.} \\
 v_2 &= 13.746 - (0.0815)(13.729) = 12.61 \text{ cu. ft./lb.}
 \end{aligned}$$

$$\begin{aligned}
 W &= h_1 - h_2 + \frac{v_2}{J} (p_2 - p_1) \\
 &= 1217.6 - 1087 + \frac{12.61}{778} (30 - 15)(144) = 165.8 \text{ Btu/lb.}
 \end{aligned}$$

The ideal thermal efficiency is

$$e = \frac{W}{(h_1 - h_{f2})} = \frac{165.8}{(1217.6 - 181.11)} = 16\%.$$

(b) The ideal steam rate is

$$w = \frac{2544}{W} = \frac{2544}{165.8} = 15.4 \text{ lb./hp-hr.}$$

(c) The indicated work is

$$W_I = \frac{2544}{w_i} = \frac{2544}{21.8} = 116.7 \text{ Btu/lb.}$$

The brake work is

$$W_B = \eta_m W_I = (0.85)(116.7) = 99.2 \text{ Btu/lb.}$$

$$(d) \quad e_b = \frac{W_B}{(h_1 - h_{f2})} = \frac{99.2}{(1217.6 - 181.11)} = 9.59\%.$$

$$(e) \quad \eta_b = \frac{W_B}{W} = \frac{99.2}{165.8} = 60\%.$$

$$(f) \quad \text{Ideal mep} = \frac{\text{ideal work}}{V_D} = \frac{\text{ideal work}}{r_2} = \frac{(165.8)(778)}{(12.61)(144)} = 70.7 \text{ psi.}$$

$$\text{Indicated mep} = \frac{\text{indicated work}}{V_D} = \frac{W_I}{r_2} = \frac{(116.7)(778)}{(12.61)(144)} = 49.9 \text{ psi.}$$

197. Enthalpy of Exhaust Steam. The reciprocating engines may be analyzed as steady flow machines; thus, Fig. 130 may represent any engine, where W and W' are works of the fluid, ideal and actual, and h_2 and h_2' are the enthalpies of the exhaust, ideal and actual. If the process is adiabatic and $\Delta K = 0$, energy balances give

$$(m) \quad h_2 = h_1 - W, \quad [\text{IDEAL ENGINE}]$$

$$(n) \quad h_2' = h_1 - W' \quad [\text{ACTUAL ENGINE, } Q = 0, \Delta K = 0]$$

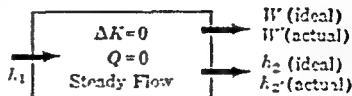


Fig. 130. Enthalpy of Exhaust.

If there is a loss of heat from the engine, Q may be included without difficulty for the actual engine. Equation (m) applies to any ideal engine and may be used to locate state c as pictured in Fig. 129 [h_2 in (m) = h_c].

In equation (n), W' is the work done by the fluid. With certain test data, W' is unknown but may be approximated. For reciprocating steam engines, W' is approximately the *indicated work* W_I . The small difference ($W' - W_I$) is due to radiation (which can be corrected for, if known) and to mechanical frictional effects, especially between the piston and the cylinder. In the turbine, the mechanical losses are small, so that the work of the fluid W' is approximately the same as the *brake work* W_B . The small difference ($W' - W_B$) is largely accounted for in friction at the bearings and packings. Adapting equation (n) to these two machines, we have

$$(70) \quad h_2 \approx h_1 - W_I = h_1 - \frac{2544}{w_1} \text{ Btu/lb}$$

[RECIPROCATING STEAM ENGINE]

$$(71) \quad h_2 \approx h_1 - W_B = h_1 - \frac{2544}{w_2} \text{ Btu/lb}$$

[TURBINE]

From another point of view, and especially in the design procedure, we may use a *turbine efficiency* η_t , which is defined as the actual work of the fluid W' divided by the corresponding Rankine work W . In general, between any two states 1 and 2 on an isentropic line, we have

$$(72A) \quad \eta_t = \frac{h_1 - h_2}{h_1 - h_2'}$$

where state 2' is the actual state after the expansion. Fig. 126(b). From (72A),

$$(72B) \quad h_2 = h_1 - \eta_t(h_1 - h_2') \text{ Btu/lb}$$

The turbine efficiency is an engine efficiency, and is sometimes so called, but it is not precisely either an indicated or brake engine efficiency. If the exhaust steam is superheated, the temperature and pressure readings define the state 2'. If the exhaust steam is wet, then the value of h_2 must be estimated by computation. In well-insulated machines, the radiation loss Q should not exceed 1% of h_1 .

198 Example. For the engine defined in the example of § 196, determine the enthalpy of the exhaust from the ideal engine, and the enthalpy and quality of the exhaust from the actual engine. Also find the lost work of the actual engine (§ 178) and the increase of unavailable energy for a sink temperature of $T_s = 550^\circ\text{R}$.

SOLUTION. From § 196, we take the following data: $h_1 = 1217.6$, $W = 165.8$, $W_I = 116.7$. From Table VII, for $p_s = 15 \text{ psia}$, $h_g = 1150.8$, $h_{fg} = 969.7$, $s_g = 1.7549$, $s_{fg} = 1.4415$, $t_{sat} = 213.03^\circ\text{F}$. See Fig. 129.

Ideal engine, $h_e = h_1 - W = 1217.6 - 165.8 = 1051.8$ Btu/lb.

Actual engine, $h_{e'} = h_1 - W_I = 1217.6 - 116.7 = 1100.9$ Btu/lb.

$$h_{e'} = (h_g - y_{e'} h_{fg})_{e'}$$

$$1100.9 = 1150.8 - 969.7 y_{e'}, \quad \text{or } y_{e'} = 5.15\%, \quad x_{e'} = 94.85\%.$$

$$\text{Lost work} = \text{ideal work} - \text{actual work} = 165.8 - 116.7 = 49.1 \text{ Btu/lb.}$$

Also, the lost work is $h_{e'} - h_e$, Fig. 129. Check this. The increase of unavailable energy during the process 1- e' is $T_o \Delta s$ as usual, where $\Delta s = s_{e'} - s_e$. With the moisture contents at e and e' , the values of s may be computed at those states; whence $T_o \Delta s$ may be computed. However, in this example, *since point e' is in the wet region*, the areas which represent lost work and increase of unavailable energy are each rectangles of width ee' ; therefore ΔE_u and lost work are proportional to the heights of the respective rectangles, that is to the absolute temperatures. The saturation temperature at 15 psia is $213.03^\circ\text{F} \approx 673^\circ\text{R}$. With $T_o = 550^\circ\text{R}$,

$$\begin{aligned} \Delta E_u &= (\text{lost work}) \left(\frac{550}{673} \right) \\ &= \frac{(49.1)(550)}{673} = 40.1 \text{ Btu/lb.} \end{aligned}$$

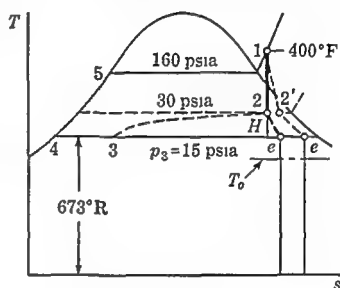


Fig. 129. Repeated.

199. Loss Due to Friction of Flow. Internal friction causes a process to be internally irreversible, and actual motion of a fluid cannot occur without such friction. There is therefore an inevitable loss in any pipe line, a loss which is affected by the magnitude of such variables as the velocity of flow, density, and viscosity of the fluid. If the fluid is superheated steam, the states can be defined by pressure and temperature readings. Then, considering the steam in the pipe line as the system (Fig. 125), we get

$$(o) \quad h_a = Q + h_1,$$

when $\Delta K = 0$. (Note that ΔP , the change of potential energy, may become significant when the fluid is a liquid.) Equation (o) may be used to determine Q when states a and 1 are known. The throttling process is sometimes used to govern steam engines and turbines, in which event the throttling may be large as the governing mechanism acts to reduce the opening of a valve in the steam passageway when the load on the engine is lightened. Because of its destructive effect on available energy, full-throttling governing is used only on small prime movers.

The lost work due to the change of state from a to 1, Fig. 131, is taken as the Rankine work from state a ($= h_a - h_b$) minus the Rankine work from state 1 ($= h_1 - h_2$), or

$$(p) \quad \text{Lost work}_{a-1} = h_a - h_b - h_1 + h_2 \text{ Btu/lb.}$$

If $Q = 0$, as it nearly is in a well insulated pipe, the process approaches an adiabatic throttling process, constant enthalpy, represented in Fig 131 by aA . In this case,

$$(q) \quad \text{Lost work}_{aA} = h_a - h_b - h_A + h_B - h_B - h_b \text{ Btu/lb},$$

since $h_a = h_A$

200 Example Lost Work by Throttling and Heat Loss Leaving a steam generator at $p_a = 1800$ psia and 700°F , steam enters a turbine at $p_1 = 1400$ psia and 640°F Fig 131. The condenser pressure $p_2 = p_b = 1$ psia. (a) Determine the lost work for $T_2 = 561.7^\circ\text{R}$. (b) Determine the lost work if the throttling is adiabatic, process aA , Fig 131

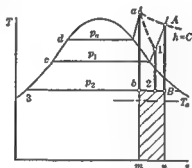


Fig 131 Losses in Pipe Line The only event occurring in the engine (turbine) which is shown on the Ts plane is the expansion process, aA represents the throttling in the pipe line, $Q = 0$, $a1$ represents throttling plus a heat loss Q . The effects are very much exaggerated in order to separate the points a , 1 , and A for clarity

SOLUTION (a) From Tables V II and VIII pp 196-199 we find

$p_a = 1800$ psia	$p_1 = 1400$ psia	$p_2 = 1$ psia
$t_a = 700^\circ\text{F}$	$t_1 = 640^\circ\text{F}$	$t_2 = 101.74^\circ\text{F}$
$h_a = 1260.3$	$h_1 = 1240.4$	$s_g = 1.9782$
$s_a = 1.4044$	$s_1 = 1.4079$	$s_{fg} = 1.8456$
		$h_g = 1106.0$
		$h_{fg} = 1036.3$

To obtain the properties at b Fig 131, we have

$$s_a = (s_g - y_h s_{fg})_h$$

$$1.4044 = 1.9782 - 1.8456 y_h \quad \text{or} \quad y_h = 31.1\%$$

$$h_b = 1106 - (0.311)(1036.3) = 784 \text{ Btu/lb}$$

To check into the situation, use $h_a = 1260.3$ and determine the state at A after an adiabatic throttling. In Table V III note that for $h_A = 1260.3$ and $p_A = 1400$ psia the temperature is 660°F . Since the given temperature is 640°F , some heat has

been lost and the situation in this problem corresponds to a-1 and 1-2, Fig. 131. Thus, at state 2, we have

$$s_1 = s_2 = 1.4079 = 1.9782 - 1.8456y_2, \quad \text{or} \quad y_2 = 30.9\%.$$

$$h_2 = 1106 - (0.309)(1036.3) = 786 \text{ Btu/lb.}$$

The Rankine works are

$$W_{ab} = h_a - h_b = 1260.3 - 784 = 476.3 \text{ Btu/lb.}$$

$$W_{1-2} = h_1 - h_2 = 1240.4 - 786 = 454.4 \text{ Btu/lb.}$$

$$\text{Lost work}_{a-1} = 476.3 - 454.4 = 21.9 \text{ Btu/lb.}$$

(b) The lost work for an adiabatic throttling process aA , Fig. 131, is $h_B - h_b$. A quick check in the steam tables establishes the fact that state B is in the wet region so that $h_B - h_b$ is represented by the rectangular area $mbBn$. Therefore

$$h_B - h_b = T_b(s_B - s_b) = T_b(s_A - s_a) = 561.7(1.4258 - 1.4044) = 12 \text{ Btu/lb.,}$$

the lost work for adiabatic throttling. The data in this example were chosen for convenience in the use of Table VIII and are not representative of ordinary operating conditions.

The Rankine and incomplete-expansion cycles are no longer important standards of comparison for large central station power plants. They are important, however, for many small plants in which reheating and regenerative feedwater heating are not used. Moreover, the Rankine cycle is basic in that all modern cycles may be considered as being made up of two or more Rankine cycles. In the various ideal cycles, all processes are considered as internally reversible and/also as adiabatic except where there is an intentional transfer of heat, as in the boiler and condenser.

201. Modern Cycles. Some designers consider the maximum permissible moisture while the steam is in the turbine blades as 12%, because the water particles erode the blades, resulting in expensive maintenance. At a particular upper temperature, the moisture in the steam after an isentropic process (or other adiabatic work process) increases as the initial pressure increases (Fig. 133). The highest temperature which can be used is limited, because metals become weaker as the temperature increases above a certain value (about 600–700°F for ordinary steel). High temperatures of 1000°F or somewhat more have been common practice (in large installations) for several years. Some designs are currently for 1150°F (5); where the extra cost is justified, temperatures as high as 1400°F are being used (7). Given a particular temperature ceiling, a point is reached where the steam in the low-pressure stages becomes too wet. To avoid this condition, we may *resuperheat* the steam after it has partly expanded through the turbine, a

step which also improves the thermal efficiency (§ 194) The corresponding cycle is called the *reheat cycle*

The following cycles represent modern modifications of the simple Rankine cycle for the purposes of improving the efficiency and avoiding excessive moisture in the low pressure regions Notice that in each case the relation between the engine and cycle works is cycle work plus pump work is equal to engine work and that the energy chargeable against the engine to obtain its thermal efficiency is the heat added in the corresponding ideal cycle plus the pump work of the cycle (§ 191) See reference 159 for more detail

Many engineering problems on steam are satisfactorily solved with the use of the Mollier diagram, which is generally a great timesaver The student should solve all problems suitable to the diagram by both steam tables and diagram in order to develop facility in the use of each

202 The Ideal Reheat Cycle Figure 132 is a diagrammatic layout showing the principal features of the reheat cycle and Fig 133 shows the cycle on the Ts plane Compare these illustrations carefully, and before proceeding with the study note the locations of points 1, 2, 3, 4, 5 and B , which have the same meanings on the two diagrams

Heating occurs in the boiler and superheater along $B-6-1$ Fig 133 After the isentropic expansion 1-2 the steam is led from the turbine and is reheated 2-3 After another isentropic expansion 3-4 the steam is condensed 4-5 At 5 the saturated liquid is pumped into the boiler as a compressed liquid condition B As we learned in § 190 the work of a Rankine engine minus the total pump work is the net work of the cycle See Fig 132 We shall use this approach to the cycle analyses of this

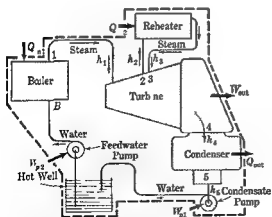


Fig 132 Diagrammatic Layout for Reheating Plant The system is all the H_2O , $W_{out} - W_p = Q_n - Q_{out}$, $W_{net} = \Sigma Q$

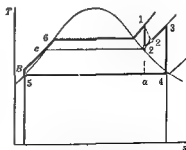


Fig 133 Reheat Cycle Moisture at 4 is less than at a , pump work $5-B$ considerably magnified

chapter, although of course $W_{\text{net}} = \Sigma Q$. The gross work, as indicated in Fig. 133 for the equivalent of two Rankine engines 1-2 and 3-4, is

$$(r) \quad W = h_1 - h_2 + h_3 - h_4 \text{ Btu/lb.,} \\ \text{[ENGINE]}$$

the gross work for the ideal cycle as shown on Fig. 133. The ideal cycle net work is

$$(73) \quad W_{\text{net}} = h_1 - h_2 + h_3 - h_4 - W_p \text{ Btu/lb.,} \\ \text{[CYCLE]}$$

where (Fig. 132) $W_p = W_{p1} + W_{p2} = h_B - h_5$, the total pump work. The heat added (Fig. 133) is

$$(s) \quad Q_A = h_1 - h_B + h_3 - h_2 \\ Q_A = h_1 - h_5 + h_3 - h_2 - W_p \text{ Btu/lb.,}$$

where $h_B = h_5 + W_p$. Thus, the thermal efficiency of the ideal cycle is

$$(74) \quad e = \frac{W_{\text{net}}}{Q_A} = \frac{h_1 - h_2 + h_3 - h_4 - W_p}{h_1 - h_5 + h_3 - h_2 - W_p}$$

In the absence of Keenan and Keyes *Steam Tables*, the pump work of the ideal cycle may be estimated (see § 190) as

$$(t) \quad W_p = h_B - h_5 \approx \frac{v_5(p_1 - p_5)}{J} \text{ Btu/lb.}$$

Rankine efficiency should be 3% or better for $p_1 = 1300$ psig or more, but may be less if the reheating pressure is taken too low. In fact, at unreasonably and unlikely low reheat pressures, the reheat efficiency is less than the Rankine efficiency (5).

203. Regeneration. We observed in the Rankine cycle, and it is true in the reheat cycle just described, that cold subcooled water is plunged into the hot water of the boiler, there to mix irreversibly with the hot water. If some way could be devised whereby the water could be heated to boiler temperature by a reversible interchange of heat within the system, as in the Ericsson cycle (§ 90), this irreversible mixing would be prevented, and the resulting ideal cycle may have an efficiency as high as that of the Carnot cycle.

A plan by which this heating of the water may be accomplished is shown

diagrammatically in Fig 134, in which the turbine rotor is surrounded by a hollow casing. Let the steam at the beginning of expansion be saturated (point 1, Fig 135). The feedwater from the condenser is to be pumped through the hollow part of the turbine casing (Fig 134), back to the boiler.

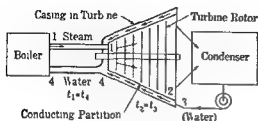


Fig 134 Regenerative Cycle Diagrammatic. The working substance is the system.

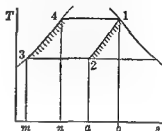


Fig 135 Perfect Regenerative Heating. Pump work effects ignored. Since curve 1-2 is parallel to 4-3, the changes of entropy ab and mn are equal.

The condensate enters the hollow part in condition 3, Fig 135. On the inside of the casing is the steam just about to be exhausted in condition 2. Now, as the condensate or feedwater moves along the casing toward the high-pressure end of the turbine, it picks up heat from the steam on the other side through a perfectly conducting partition. Considering the history of a drop of water, we may imagine this drop picking up heat at such a rate that it is always at the temperature of the steam opposite it on the other side of the casing. This is the reversible manner of transferring heat. Thus one pound of water will be heated gradually along 3-4 and at the same time one pound of the steam in the turbine will lose gradually a like amount of heat during expansion 1-2. Such heating is called *regenerative heating*. The water now enters the boiler in a saturated condition 4. Since the heat rejected during 1-2 (area 1- b - a -2) is equal to the heat supplied during 3-4 (area m -3-4- n), and since this transfer of heat is simply an interchange *within* the system, these heat quantities do not enter into the expressions for work and efficiency. Thus, the net work is ΣQ , where the heat quantities are those transferred to or from an *external source*. We may write

$$W_{net} = \Sigma Q = T_1(s_1 - s_4) - T_2(s_2 - s_3).$$

$$Q_A = T_1(s_1 - s_4)$$

Since $s_2 - s_3$ is equal to $s_1 - s_4$ (the curves 1-2 and 4-3 are parallel), the efficiency is the same as for the Carnot cycle,

$$e = \frac{T_1 - T_2}{T_1}.$$

The lesson from the Carnot cycle, Chapter 8, is that to obtain the best efficiency, heat must be added at the highest temperature. If all the heat cannot be added at the highest temperature, any step that increases the average temperature at which the system receives heat will result in improvement. That is what actual regenerative feedwater heating does—increases the average temperature at which heat is received from an external source.

204. Regenerative Cycle. Even if the foregoing regenerative cycle could be practically approximated, it is unlikely that it would be used because of the low quality of the steam during the latter stages of the expansion near point 2, Fig. 135. The same effect of regeneration could be obtained by *bleeding* or *extracting* small quantities of steam at various points during the expansion, utilizing as fully as possible the energy of the *bled steam* rather than absorbing the required energy from all the steam. In this way, the major portion of the steam, that part which continues to expand and to do work, is not subjected to excessive condensation. Figure 136 shows diagrammatically a regenerative cycle with three open heaters (O.H.). Figure

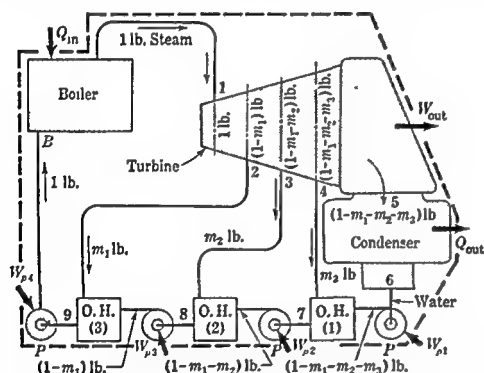


Fig. 136. Diagrammatic Layout for Regenerative Heating.

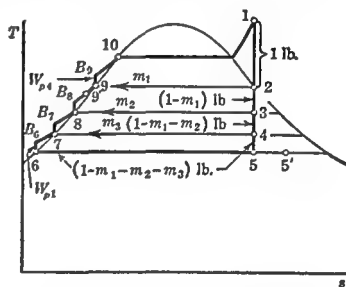


Fig. 137. Regenerative Cycle. The pump works (6-B₁, etc.) are considerably magnified. Mass of water heated: from 6 to 7 is $(1 - m_1 - m_2 - m_3)$ lb.; from 7 to 8 is $(1 - m_1 - m_2)$ lb.; from 8 to 9 is $(1 - m_1)$ lb.; from 9 to 10 is 1 lb.

137 is a representation on the Ts plane of this same cycle. To simplify the analysis, consider one pound of steam, which enters the turbine in state 1, having arrived there without loss from the boiler. After an expansion to state 2, m_1 lb. (a fractional part of 1 lb.) is bled for feedwater heating. The remainder of the steam $(1 - m_1)$ lb. continues to expand in the turbine to state 3, where another quantity of steam m_2 lb. is bled for feedwater heating. The remainder, $(1 - m_1 - m_2)$ lb., continues isentropic expansion. At state 4, m_3 lb. is bled for heating, so that there is left in the turbine $(1 - m_1 - m_2 - m_3)$ lb., which expands further and passes to the condenser. This weight of steam is condensed and the condensate is pumped to heater (1),

6 to B_6 , where it is heated from B_6 to 7, Fig 137, by mixing with the m_3 lb of steam bled for the purpose and condensed from 4 to 7. Since the heater is assumed to be an open heater, the m_3 lb of condensed steam mixes with the $(1 - m_1 - m_2 - m_3)$ lb of condensate from the condenser, so that the amount of water pumped, 7 to B_7 , to heater (2) is $(1 - m_1 - m_2)$ lb. In heater (2), this water is heated from condition B_7 to condition 8, Fig 137, by the condensation of m_2 lb from 3 to 8. Leaving heater (2) is $(1 - m_1)$ lb of water, which is pumped 8 to B_8 and heated from B_8 to 9, Fig 137, by the condensation of m_1 lb from 2 to 9 in heater (3). The one pound of water which now leaves heater (3) is pumped 9 to B_9 into the boiler.

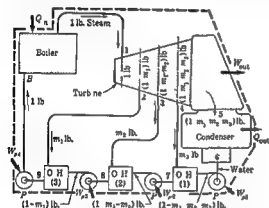


Fig 136 Repeated

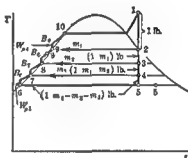


Fig 137 Repeated

First we shall find equations from which the fractional amounts of steam bled, m_1 , m_2 , m_3 , may be found. In so doing, we consider only the law of conservation of energy and we neglect the fact that the water enters the heater slightly subcooled, that is, we shall consider 6 and B_6 , 7 and B_7 , etc., as coincident points. For each heater, the heat added to the water is equal to the heat given up by the condensation of the bled steam. Thus we have

Heat to water = heat from steam,

- (u) Heater (1), $(1 - m_1 - m_2 - m_3)(h_7 - h_6) = m_3(h_4 - h_7),$
 (v) Heater (2), $(1 - m_1 - m_2)(h_8 - h_7) = m_2(h_3 - h_8),$
 (w) Heater (3), $(1 - m_1)(h_9 - h_8) = m_1(h_2 - h_9),$

where, by way of explanation, $h_2 - h_9$ is the heat given up by the condensation of one pound of steam from 2 to 9 and m_1 is the mass of steam condensed, $h_9 - h_8 \approx h_9 - h_{B8} = h_9 - h_8 - W_{p2}$ is the heat required to heat one pound of water from 8 to 9 and $(1 - m_1)$ is the mass of water involved. Having found the various specific enthalpies at 2, 3, 4, 5, 6, 7, 8, and 9, we may use equations (u) and (v) and (w) in this order to find m_1 , m_2 , and m_3 .

Next observe that this regenerative cycle may be conceived of as being made up of a series of Rankine cycles, 1-2-9-10, 2-3-8-9, 3-4-7-8, and 4-5-6-7.

As we have learned, it is easy to write directly the expression for the gross or engine work for each of these imaginary cycles, but we should note that the mass of steam involved in each of the imaginary cycles is not one pound. Referring to Fig. 137 for the mass involved in each case, we find that the work is

$$(x) \quad W_{\text{net}} = h_1 - h_2 + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4) \\ + (1 - m_1 - m_2 - m_3)(h_4 - h_5) \text{ Btu/lb. throttle steam,}$$

which is the work of the *regenerative engine* with three stages of regenerative heating. The *net* work of the cycle becomes

$$(75) \quad W_{\text{net}} = h_1 - h_2 + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4) \\ + (1 - m_1 - m_2 - m_3)(h_4 - h_5) - \Sigma W_p \text{ Btu/lb. throttle steam.}$$

The heat supplied from an *external source* to the ideal cycle is $Q_A = h_1 - h_{B9}$ where $h_{B9} = h_9 + W_{p4}$, Figs. 136 and 137. Notice that W_{p4} represents the pump work of the fourth feedwater pump. Thus we have

$$Q_A = h_1 - h_9 - W_{p4} \text{ Btu/lb.}$$

From 6 to 9, there is an *internal interchange* of energy which is not part of Q_A , the heat supplied from an external source. The thermal efficiency of the ideal cycle is the ideal work from equation (75) divided by Q_A ; thus we have

$$(76) \quad e = \frac{W \text{ [from eq. (75)]}}{h_1 - h_9 - W_{p4}}.$$

The total pump work ΣW_p may be found from the following if Keenan and Keyes *Steam Tables* are available:

$$(y) \quad \Sigma W_p = (1 - m_1 - m_2 - m_3)(h_{B6} - h_6) + (1 - m_1 - m_2)(h_{B7} - h_7) \\ + (1 - m_1)(h_{B8} - h_8) + h_{B9} - h_9 \text{ Btu/lb.}$$

of steam reaching the throttle. The individual pump works can be approximated by a constant volume compression as before. For example, the pump work from p_9 to p_1 is

$$(z) \quad W_{p4} = h_{B9} - h_9 \approx \frac{v_{f9}(p_1 - p_9)}{J} \text{ Btu/lb.}$$

Moreover, a satisfactory approximation for the total pump work ΣW_p , since the total of this energy is not large, is that for one pound of saturated liquid pumped at constant volume from p_6 to p_1 ; thus,

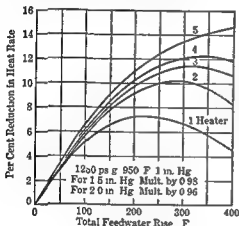
$$(a) \quad \Sigma W_p \approx \frac{v_{f6}(p_1 - p_6)}{J} \text{ Btu/lb.}$$

At every extraction point, Fig. 137, there should be a break in the expansion line and the saturated-vapor line (since it is plotted for one pound) to account for the departing steam. However, these breaks are not shown,

because they generally confuse rather than elucidate. The simplest plan then is to sketch the diagram as though one pound of steam were involved throughout, and indicate upon this diagram the weights involved between the various points as in Fig 137. Although steam departs at various points during the expansion the remaining steam expands isentropically. Consequently, we find the quality at each point on the expansion line just as though one pound expanded all the way, the specific entropy is constant (Fig 137)

$$s_1 = s_2 = s_3 = s_4 = s_5$$

If a point is in the wet region as point 4, $s_4 = (s_g - y_4 s_{fg})_4$. The student should observe carefully the method of analysis of this regenerative cycle in order that he may be able to set up the proper equations for any number of stages without further aid.

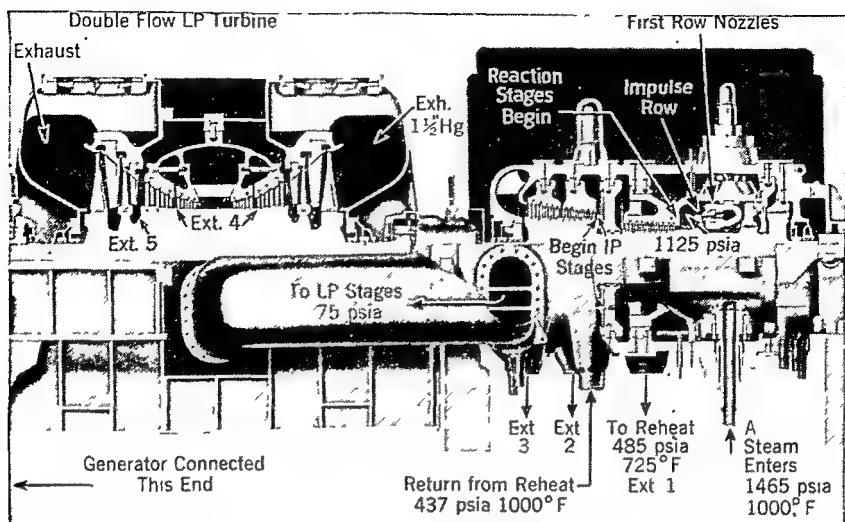


Courtesy Babcock & Wilcox Co. New York

Fig 138 Saving in Heat Rate from Regenerative Heating These curves are for the operating conditions shown. They indicate that for a particular number of heaters the saving is a maximum at a certain temperature rise of the feedwater. For example, using 4 heaters give a maximum saving of about 12.3% of the corresponding Rankine cycle when the feedwater increases 340°F.

205 Ideal Reheat-Regenerative Cycle In order to gain the advantage of drier steam in the low pressure stages of the turbine and to gain the significant increase in thermal efficiency that accompanies regenerative feedwater heating, reheat and regeneration are often used together. For illustration we have chosen a three-stage regenerative cycle with reheat occurring at the first bleeding point. A diagrammatic layout is shown in Fig 140 and the Ts diagram in Fig 141. The various points of the two figures have corresponding numbers. Let one pound of throttle steam be the

The cycle just described would be termed an ideal regenerative cycle with three stages of feedwater heating. Actual cycles in use have from one to nine stages. The number of stages used is purely an economic matter. The use of one stage results in a decided improvement in the thermal efficiency (Fig 138) and would justify a relatively large expenditure for equipment. The increment improvement due to the addition of a second heater is not so great as that due to the first heater, with further diminishing returns as additional heaters are added. Four stages of feedwater heating become profitable for pressures of 600 psia. The economic number of stages increases as the initial pressure and temperature increase. See Fig 138.



Courtesy Allis-Chalmers, Milwaukee, Wisc.

Fig. 139. Large Steam Turbine, Tandem Compound. A 100,000-kw. unit. There are five entry pipes, one of which is shown at A, individually controlled by the governor, each leading to a nozzle chest or bank of nozzles. As the load increases, the governor opens additional entry pipes for a greater flow of steam. The first stage is a single impulse; others are reaction stages. By IP is meant intermediate pressure; LP means low pressure; Ext. stands for extraction; Exh. stands for exhaust. The one reheat occurs just before the IP section, at which section steam is also extracted for feedwater heating. All five extraction points are designated, but only the locations for stages 4 and 5. The LP unit is double flow. The final blades are 23 in. long. Notice the pressures and temperatures at the various sections.

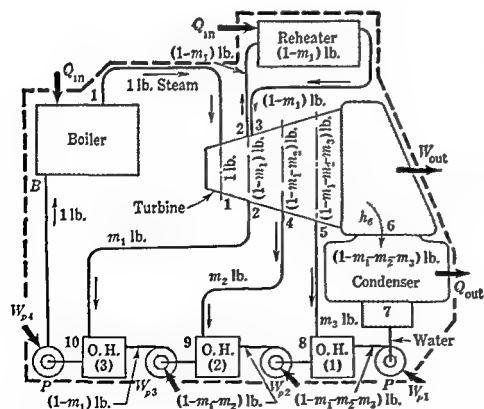


Fig. 140. Diagrammatic Layout for Reheat-Regenerative Plant. The working substance is the system.

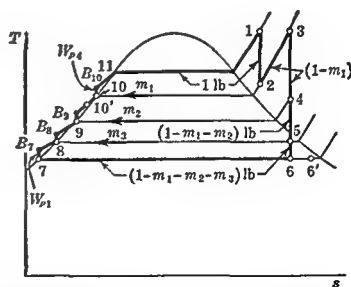


Fig. 141. Reheat-Regenerative Cycle. The pump works (7-B₁, etc.) are considerably magnified. Mass of water heated: from 7 to 8 is $(1 - m_1 - m_2 - m_3)$ lb.; from 8 to 9 is $(1 - m_1 - m_2)$ lb.; from 9 to 10 is $(1 - m_1)$ lb.; from 10 to 11 is 1 lb.

unit of mass. At point 2, m_1 lb passes to the open heater (3) and the remainder $(1 - m_1)$ lb passes to the reheater. Steam from the reheater returns to the turbine in condition 3, expands to 4, where m_2 lb of steam is bled for heater (2). From point 4, the remaining steam $(1 - m_1 - m_2)$ lb expands to point 5, where m_3 lb is bled. Finally, $(1 - m_1 - m_2 - m_3)$ lb completes the expansion and is condensed to a saturated liquid at 7. The regenerative heating has already been described. For each heater only enough steam is bled to heat the water to the temperature of the steam at the bleeding point. The heat added to the condensate in the first heater

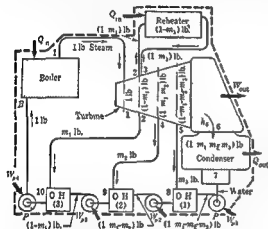


Fig 140 Repeated

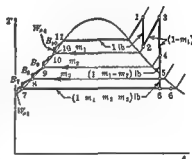


Fig 141 Repeated

(1) is $(h_8 - h_{B7})$ Btu per lb of condensate, but the approximation $h_8 - h_7$ is probably satisfactory. Thus, equating energy quantities, we have as before (Fig 141),

Heat from steam = heat to water

$$m_3(h_5 - h_8) = (1 - m_1 - m_2 - m_3)(h_8 - h_7),$$

$$m_2(h_4 - h_9) = (1 - m_1 - m_3)(h_9 - h_8),$$

$$m_1(h_3 - h_{10}) = (1 - m_1)(h_{10} - h_7),$$

from which values of m_1 , m_2 , and m_3 for the ideal cycle are obtained. As before, we imagine this cycle to be composed of a series of Rankine cycles, 1-2-10-11, 3-4-9-10, 5-6-8-9, and 7-8-6-7-8, Fig 141. Since the net work of the ideal cycle is equal to the gross work W minus the total pump work ΣW_p , we get

$$(77) \quad W_{net} = h_1 - h_2 + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5) \\ + (1 - m_1 - m_2 - m_3)(h_5 - h_6) - \Sigma W_p \text{ Btu/lb throttle steam}$$

The total pump work ΣW_p is given by equation (y) of § 204 [but see equation (a)] for the three-stage heating. The heat supplied to the cycle is

$$(78) \quad Q_A = h_1 - h_{10} + (1 - m_1)(h_3 - h_7) - W_{p1} \text{ Btu/lb throttle steam}$$

In equation (78), the term $(1 - m_1)(h_3 - h_2)$ represents heat supplied in the reheater. The thermal efficiency of this cycle is evidently

$$(b) \quad e = \frac{W \text{ [from eq. (77)]}}{Q_A \text{ [from eq. (78)]}}$$

If a reader *understands* this analysis, he will be able to obtain proper equations for any combination of reheaters and regenerative water heaters. A large turbine designed for one reheat and five extractions is shown in Fig. 139.

206. Converting Mass into Energy. We have previously mentioned that, according to Einstein's equation $E = mc^2$, the amount of energy equivalent to a mass of one pound is 39×10^{12} Btu, a misleading number because at this time we do not know how to convert and control more than about 0.1% of a fissionable mass into energy, much less any other mass. If the process of using this energy from mass is via thermal processes or heat engines, the second law introduces certain additional limitations. Moreover, there is perhaps some law not yet formulated concerning the *availability* of mass for conversion into energy. The first observed fission of an atomic nucleus was in 1939. As a matter of interest, we may inquire briefly into the mechanism by which power is obtained from fission of the atoms, and start by a review of some definitions.

An *atom* is the smallest unit of a particular chemical element. An *element* is a basic kind of matter (such as oxygen and uranium, of which 100 have been found in nature or have been artificially created). The current picture of the atom, which is substantiated by imposing experimental evidence, is that it has a nucleus about which *electrons* (negatively charged particles) move, much as planets move about a sun. The electrons have a relatively small mass, so that practically all the mass of an atom is in its nucleus. The nucleus is composed of *protons*, which are positively charged particles, and *neutrons*, which are neutral particles, both of virtually the same mass. A normal atom of a particular element has a certain number of negatively charged electrons, which is the *atomic number* Z , moving about a nucleus in which there is the same number of protons (positively charged); so the atom is neutral. The atomic number identifies the element. The isotopes of a particular element have the same number of electrons and protons (the same atomic number), but they differ in mass because they have different numbers of neutrons. The total number of protons and neutrons in the nucleus is called the *mass number* A , which is the integer nearest the atomic weight. Thus, the atomic weight of the most common isotope of uranium is 238.07, and its mass number A is therefore 238, which is the total number of protons and neutrons in its nucleus. Uranium 235, the kind which fissions, has 235 protons and neutrons in its nucleus, but the same number of protons (and electrons) as U 238. To identify a particular atom, the following symbolization is used:

$$(c) \quad {}_Z X^A \begin{cases} X = \text{chemical symbol of substance,} \\ A = \text{mass number (of atom),} \\ Z = \text{atomic number} \end{cases}$$

You recall that the *atomic weight* of oxygen is arbitrarily taken as 16 by the chemist and since oxygen has three isotopes the value of 16 is the average value for oxygen as it is normally found. Since the physicist must distinguish between the isotopes, he has assigned the mass of 16 to the lightest and most abundant (about 99.8%) isotope (${}_8\text{O}^{16}$), which results in *atomic weights* of the other two isotopes of oxygen as 17.0045 (${}_8\text{O}^{17}$) and 18.0049 (${}_8\text{O}^{18}$), and an average atomic weight somewhat greater than 16 (16.0044). Thus the scale of relative atomic weights as used by the nuclear physicist is slightly different from the established chemical scale. A few values of atomic numbers Z and mass numbers A are

	Z	A		Z	A
Americium (Am)	95	243	Barium (Ba)	56	137
Boron (B)	5	10	Carbon (C)	6	12
Cesium (Cs)	55	133	Hydrogen (H)	1	1
Krypton (Kr)	36	84	Lithium (Li)	3	7
Nitrogen (N)	7	14	Oxygen (O)	8	16
Plutonium (Pu)	94	239	Rubidium (Rb)	37	85
Sodium (Na)	11	23	Uranium (U)	92	238

By way of illustration we see from this tabulation and equation (c) that the physicist's designation of uranium 238 is ${}_{92}\text{U}^{238}$ uranium 235 would be ${}_{92}\text{U}^{235}$.

We obtain energy from a chemical reaction (combustion) as a result of a change to a new molecular structure. The mass equivalent of the energy released (heat of combustion) is so small as to go undetected. Nuclear energy is obtained (or consumed) when there are changes in the structure of the atom's nucleus. In particular we obtain energy by virtue of the splitting of an atom of a certain substance into two atoms which are two different substances called *fission fragments*. (Energy is also obtainable from certain of the light atoms by a fusion process as in the hydrogen bomb.) There are only three atoms whose nuclei we are now able to split with a resulting advantageous output of energy: *uranium 235* the rare isotope occurring in nature; *uranium 233*, which is an isotope formed when neutrons are captured by thorium 232; and *plutonium 239* which is formed when an atom of uranium 238 captures a neutron. When one of these atoms splits it changes into two atoms of different elements, and in order to produce energy, the stored energy (or mass) of the *fission fragments* must be less than the stored energy (or mass) of the original atom. Now we are using the words *mass* and *energy* in the same sense. In other words, the mass which "disappears" on fission reappears as energy, in accordance with Einstein's equation

The unit of energy used in nuclear physics is the electron volt (ev) which

165 are very rarely formed, the most likely ratio of the atomic numbers of the fragments being approximately 3 to 2 (85)

A schematic picture of how a chain reaction can occur is shown in Fig 142 If the first neutron n_1 fissions the nucleus N_1 , there results the two high-energy fragments F and say the two neutrons n_2 and n_3 Possibly each of these neutrons fissions other nuclei N_2 and N_3 , which in this illustration produces five more neutrons Then each of these five neutrons are available though not so likely, to fission five more nuclei At any rate, there is a possible multiplication effect If on average more than one of the released neutrons fissions a nucleus, something will eventually burn up, or there will

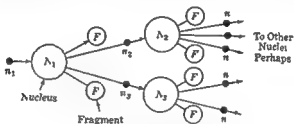


Fig 142 Chain Reaction Explosive

be an explosion when the multiplication rate is fast enough For a steady chain reaction the neutrons from one split nucleus sooner or later split one other nucleus which is the condition to be maintained in a reactor for a power plant in a steady state If less than an average of one released neutron per fission splits another nucleus the reaction rate decreases To have a sustaining reaction there is also the matter of a *critical mass*, which we shall not go into except to say that if the mass is too small so many of the emitted neutrons will escape before they find a nucleus to split that the average of future fissions becomes less than one fission per fission

207 The Reactor. The present plan of obtaining power from atomic energy is to have a reactor in which the chain reaction proceeds to provide means of using the released energy for heating a working substance, and then to use the working substance in a *heat engine* There are a number of different reactors including the pressurized water type, the boiling water type, the fast breeder, and the homogeneous type but space does not permit complete descriptions here The basic components of any power reactor include

1 **Core** A core of fissionable material in an amount greater than the critical mass is necessary

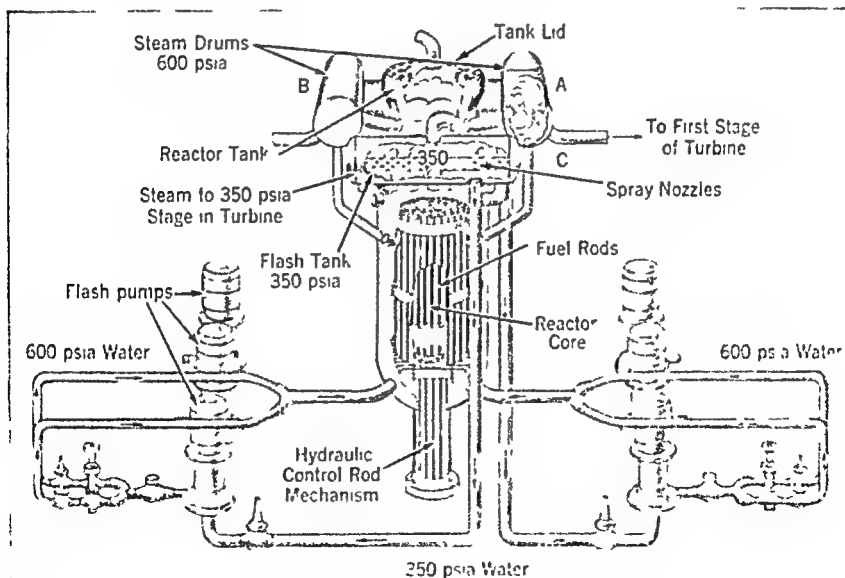
2 **Control rods** Control rods are made of a material with a high affinity for neutrons When these rods are fully inserted they absorb a large proportion of the neutrons from fissioning and the rate of fissioning is reduced to a low value When the rods are entirely withdrawn the rate of fissioning accelerates The control rods can be automatically moved in and out to maintain the fissioning at the desired rate Materials used include cadmium, boron and hafnium

3. **Coolant.** A coolant is used to pick up and carry away the energy released by the reaction.

4. **Moderator.** There may or may not be a *moderator*, which is any substance used for the purpose of slowing the neutrons to thermal energies. By *thermal energy* is meant energies of the order of those possessed by gaseous molecules (0.025 ev), in contrast with the relatively very high energy (speed) which a neutron possesses on emission (2 mev). These very high-speed neutrons are not so likely to cause fission as the slow neutrons; thus, to use as many neutrons as possible for fission, we arrange to have them traverse a moderator (carbon or water, for example) where they gradually lose energy, reaching a range of kinetic energy called thermal energy. A reactor which contains a moderator is called a *thermal reactor*; one which does not is called a *fast reactor*, because it depends upon the high-speed neutrons to cause fission.

5. **Shielding.** Shielding is used to protect life against the lethal radiation of gamma rays and beta particles (electrons). There are, of course, many other items involved, such as pumps, heat exchangers, and instruments, which are not concerned with this discussion.

A boiling reactor is shown in Fig. 143, in which the water within the reactor serves as both a moderator and a coolant. The pressures shown are typical



Courtesy General Electric Co., Schenectady

Fig. 143. Boiling Reactor. The fuel is encased in "rods" (long slender cylinders) in this reactor, but sometimes it is incorporated in a circulating liquid. These rods are supported in hexagonal zirconium coolant channels, which form a honeycomb pattern. The feature of this reactor is that the coolant channels extend above the reactor core. The resulting chimney effect improves circulation (and heat transfer). For about two-thirds of the length of the core, the water is subcooled (no steam bubbles), the consequence being that the reaction is not impeded by steam bubbles.

but illustrative only, and pressure and temperature drops are ignored in the following discussion. The primary steam moves into the steam drums *A* and *B* at 600 psia, thence to the turbine, preferably via a superheating furnace (fired by conventional fuels), because turbines have a better efficiency when they are operated on superheated steam (§ 308). In the upper part of the reactor, the water will be saturated at the temperature of 486.21°F (Table VII), corresponding to 600 psia. If some of this water undergoes a throttling process (§ 182) to, say, a pressure of 350 psia, the result is a mixture of a little steam and much water, $h_{f600} = (h_f + xh_{fg})_{350}$. This process is said to *flash* steam, hence, the name *flash tank*. The saturated steam at 350 psia goes to the turbine, entering it at the stage where the pressure is 350, expanding to condenser pressure. The 350 psia water at a saturation temperature of 431.72°F is pumped back into the reactor.

The reason for the extra paraphernalia of the flash tank, etc., lies in the fact that steam among the fuel rods reduces the transfer of energy because a particular volume of steam picks up less energy than the same volume of water. Hence, not so much steam can be generated if any of the space about the fuel rods is occupied by steam. By throttling saturated water, which the reactor produces efficiently, additional steam, albeit at a lower pressure, is obtained. (The throttling process is destructive of available energy and perhaps someone can devise a way to avoid it.)

In practice, the plan is to provide most of the turbine governor adjustment on the 350-psia steam. Then, if there is a demand for more power, the governor allows more steam to enter the turbine from the flash tank which reduces the pressure in this tank somewhat, which results in a greater flow of water into the flash tank to provide the additional steam needed for the greater load. In the meantime, the flow of 600 psia steam remains more or less constant. With a steady flow of the 600 psia steam, additional steam is *not* generated in the core, which action, if it happened, would reduce the output of the reactor just at a time when the load increases (88). Another mitigating influence is that the flash pump, in maintaining the water level in the flash tanks, pumps more subcooled water into the reactor, this reduces the number of steam bubbles within the core, resulting in an increase of output of the reactor. There are several other types of reactors.

The amount of electricity generated from nuclear reactors is expected to increase rapidly over the next several years. One prediction is that by 1980 about half of the newly installed electric power will be obtained with nuclear steam generators.

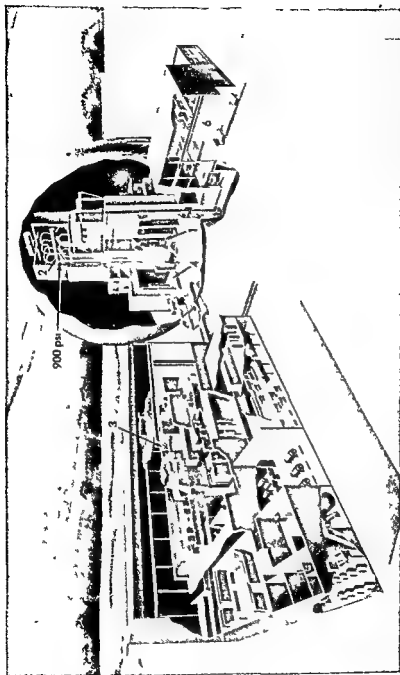
208 Closure. There are so many ways in which various ideas may be combined for the purpose of producing power that there is no alternative but to understand the basic principles. In this connection, a review of the discussion of the second law would be appropriate, notice the example of § 200.

The reader may have noticed that the power cycle with the nuclear reactor

utilized saturated steam and also that superheated steam is advantageous thermodynamically and practically. Because of the advantages of superheated steam, it is reasonable to suppose that land-based nuclear power plants will use furnace superheaters if it continues to be infeasible to superheat with nuclear energy. In other cases, in order to protect the turbine blades from erosion by the entrained water particles moving at high speed, the blades are surfaced with a very hard alloy, such as stellite. This shielding of the blades with hard material is also practiced now for those blades in the low-pressure stages subjected to wet steam.

The advantages of high temperature have been emphasized. The highest steam temperature used in commercial power plants at this time is probably 1150°F. As an example of current ideas, the new Philadelphia Electric plant generates 275,000 kw, using steam at 5000 psi and, eventually, 1200°F. There are nine stages of regenerative heating (feedwater temperature of 565°F) and two reheats, each to 1050°F.

At least in the large units operating on modern cycles, there is an optimum initial pressure at which the heat rate is a minimum for a particular size of unit. This pressure for a 100,000-kw turbo-generator, one reheat, is about 2800 psia (103). The reason for this phenomenon is not theoretical but is associated with the low initial volume at high pressures. The volume of flow can be too small for good efficiency. So, as the initial pressure is increased, a point is reached at which the theoretical gain in efficiency from higher pressure is more than offset by the loss of turbine efficiency from low volume of flow. For a further reduction of heat rate at pressures above the critical pressure of steam (3206 psia), the units must be quite large in order to obtain adequate flow volumes. A 200,000-kw unit with two reheats does not show a minimum heat rate for pressures up to 5500 psia (103).



Courtesy General Electric Co., Schenectady

Nuclear Power Station Capacity, 180,000 kw, 1 is boiling water reactor, 2 is steam drum, 3 is first stage of turbine, secondary steam generators 4 produce 600 psia steam for admission to turbine at a lower stage, 5 is the condenser, 6 is storage for spent fuel, to be processed later

13

NOZZLES

209. Introduction. The manner in which work originates in a gas or vapor turbine is: first the expansible fluid expands through a nozzle, during which process some of the fluid's initial energy is converted into kinetic energy; then the issuing jet of high-velocity fluid passes across turbine blades which have been designed to change the momentum of the stream. The consequence of the change of momentum is a force (Newton) which does work turning the turbine shaft. This chapter covers the flow of expansible fluids through nozzles, omitting the flow of such fluids through flow measuring devices, such as venturi meters.* Since the approach is from the point of view of the energy relations, the reader should be informed that the general subject of *fluid mechanics* says much more about the phenomena of flow. Moreover, this discussion does not cover the flow of nonexpansible fluids (liquids).

Since we shall be dealing with both gases and vapors, this chapter provides an excellent opportunity to clarify in your mind the differences in the methods of handling these substances.

210. Flow through Nozzles. A nozzle is a device (open system) used for the purpose of guiding the expansion of a substance to a state where the kinetic energy of the substance is relatively large. There are two types, *convergent-divergent nozzles* (Fig. 144) and *convergent nozzles* (Fig. 145). They may have any cross-sectional shape to suit the application. Sometimes the section is circular. The elements of the surface of the divergent section (Fig. 144) are generally straight, for convenience in manufacture. The smallest cross-sectional area of a convergent-divergent nozzle is called the *throat* (Fig. 144). The customary manner of proportioning a convergent-divergent nozzle is to determine the throat area, as explained later,

* See reference (159) for a thermodynamic treatment of this kind of flow.

to provide a well rounded entrance, to choose a nozzle length L , Fig 144, such that the flare of the sides of the divergent section is within good limits in accordance with experience. With respect to the flare, too large an angle θ , Fig 144, above about $12-15^\circ$, results in excessive turbulence and consequent irreversibility. The nozzle will be excessively long if θ is too small, say, less than about 6° .

A convergent nozzle with a short section of parallel elements Fig 145(b), tends to discharge a better formed jet than that in Fig 145(a). Convergent nozzles as used in turbines are commonly foil nozzles, that is they are formed by stationary blades which are small foils.

For any nozzle, the shaft work is zero $\dot{W} = 0$. The length of time that a particular mass of substance is within the nozzle is a small fraction of a

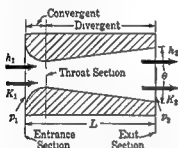


Fig 144 Convergent Divergent Nozzle. The pressure drop per unit of length is not constant. (See § 216.)

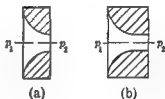


Fig 145 Convergent Nozzles

second, therefore for practical purposes the flow is adiabatic. The only pertinent energies are therefore the internal energy and flow work (enthalpy) and the kinetic energy K , as shown in the energy diagram of the system in Fig 144. Letting $K = v^2/(2g_oJ)$, we have for steady flow, $dK = -dh$, or

$$(a) \quad \frac{v_1^2}{2g_oJ} + h_1 = \frac{v_2^2}{2g_oJ} + h_2 \text{ Btu/lb,}$$

$$[\text{ANY FLUID } Q = 0, \dot{W} = 0]$$

applicable to ideal or actual expansions when state 2 is the ideal or actual state at the exit section. The initial velocity v_1 is usually difficult to determine and sometimes it is negligible ($K_1 \approx 0$), in which case,

$$(b) \quad v_2 = [2g_oJ(h_1 - h_2)]^{1/2} = 223.8(h_1 - h_2)^{1/2},$$

$$[\text{GAS OR VAPOR } v_1 \text{ NEGLIGIBLE}]$$

applicable to any expansible fluid, where $2g_oJ \approx 50,000$ and $(2g_oJ)^{1/2} \approx 223.8$. These constants are convenient ones to memorize for current use. As necessary, we use the conservation of mass equation (4), § 23,

$$\frac{A_1 v_1}{v_1} = \frac{A_2 v_2}{v_2},$$

from which

$$(c) \quad v_1 = \left(\frac{A_2 v_1}{A_1 v_2} \right) v_2 \text{ fps,}$$

in which A sq. ft. is the cross-sectional area and v cu. ft. is the specific volume. Substituting the value of v_1 from equation (c) into equation (a) and solving for v_2 , we get

$$(79) \quad v_2 = \left[\frac{2gJ(h_1 - h_2)}{1 - [A_2 v_1 / (A_1 v_2)]^2} \right]^{1/2} \text{ fps,}$$

[GAS OR VAPOR, IDEAL OR ACTUAL FLOW]

where the states 1 and 2 are at any two sections 1 and 2, wherever taken. Comparing equations (79) and (b), we see that the factor

$$(d) \quad I = \left[\frac{1}{1 - \left(\frac{A_2 v_1}{A_1 v_2} \right)^2} \right]^{1/2} = \left\{ \frac{1}{1 - \left[\frac{A_2}{A_1} \left(\frac{p_2}{p_1} \right)^{1/k} \right]^2} \right\}^{1/2} = \left[\frac{1}{1 - \left(\frac{A_2 \rho_2}{A_1 \rho_1} \right)^2} \right]^{1/2}$$

corrects for the effect of initial velocity and it may be kept in mind as a correction factor applicable to equations obtained later; ρ lb. per cu. ft. is the density. For circular channels, it may be convenient to note that $A_2/A_1 = (D_2/D_1)^2$.

211. Stagnation Properties. Stagnation properties, which will be useful in this chapter, are those which a moving stream of fluid would have if it were brought to rest by an isentropic compression. If a moving stream with a kinetic energy K Btu per lb. and enthalpy h Btu per lb. is brought to rest isentropically, the resulting kinetic energy is zero and the enthalpy is h_o . At the entrance to the nozzle (Fig. 144), $h_o = h_1 + K_1$. Thus, we see that h_o includes the effect of initial velocity. Refer to § 140, which deals with stagnation enthalpy relative to a body moving in a fluid at rest.

If the substance is a flowing vapor for which vapor tables are available, and if the state where the enthalpy and kinetic energy are h and K is defined (say, pressure and temperature in the superheated region), the properties in the stagnation state are easily found from

$$h_o = h + K \quad \text{or} \quad h_o - h = K \quad \text{and} \quad s_o = s,$$

which locate the state in the tables. If the substance is an ideal gas, the property relations of an isentropic process apply; repeated for convenience from § 70,

$$p_o v_o^k = p v^k, \quad \frac{T}{T_o} = \left(\frac{p}{p_o} \right)^{(k-1)/k} = \left(\frac{v_o}{v} \right)^{k-1}.$$

The ideal gas equation of state applies, $p_o v_o = RT_o$, where T_o is the stagnation temperature; p , v , and T in the foregoing equations are values at any other state for which the entropy is the same and in which the kinetic

energy is K . Since $\Delta h = c_p \Delta T$ for an ideal gas, we have

$$(e) \quad c_p(T_* - T) = K \quad \text{or} \quad T_* = T + \frac{v^2}{2g_c J c_p}$$

In terms of stagnation enthalpy, the energy equation for a steady flow nozzle is obtained from equation (a) as

$$(80) \quad h_o = h_2 + K_2, \quad \text{or} \quad v_2 = 223.8(h_o - h_2)^{1/2},$$

[GAS OR VAPOR, $s = C$ or $s \neq C$ $Q = 0$]

where $h_o = h_1 + v_1^2/(2g_c J)$. Equation (80) is the same as equation (79) when the initial velocity is significant and it is the same as equation (b) when v_1 is negligible.

212. Mass of Flow through Nozzle. The mass rate of flow is computed from equation (4),

$$(4) \quad w = \frac{Av}{v} \text{ lb/sec},$$

where v fps, obtained from equation (80), and v cu ft per lb are values at a section whose area is A sq ft. This equation may be applied at any section of the nozzle, but it is most often applied to the throat and exit sections. The method of finding the volume $v = v_2$ at any section depends upon whether the substance is an ideal gas or a vapor. For an ideal gas in an isentropic expansion,

$$(f) \quad v_2 = \left(\frac{p_o}{p_2}\right)^{1/k} v_o = \left(\frac{p_o}{p_2}\right)^{1/k} \left(\frac{RT_o}{p_o}\right),$$

where p_1 and v_1 may be used in place of p_o and v_o , because

$$p_o v_o^k = p_1 v_1^k = p_2 v_2^k$$

213 Example. Air Nozzle. At the rate of 5 lb/sec, air at 200 psia and 60°F enters a nozzle with negligible velocity and expands isentropically to 15 psia. Determine the final velocity, specific volume, and the area of the exit section.

SOLUTION The final temperature is

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (520) \left(\frac{15}{200}\right)^{0.4/1.4} = 248^\circ\text{R}$$

Using $\Delta h = c_p \Delta T$, we have from equation (80),

$$v_2 = 223.8 \sqrt{(0.24)(520 - 248)} = 1810 \text{ fps}$$

The specific volumes are

$$v_1 = \frac{RT_1}{p_1} = \frac{(53.3)(520)}{(200)(144)} = 0.962 \text{ cu ft/lb},$$

$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{1/k} = (0.962) \left(\frac{200}{15}\right)^{1/1.4} = 6.1 \text{ cu ft/lb}$$

From equation (4), we get for $w = 5$ lb./sec.,

$$A_2 = \frac{wv_2}{v_2} = \frac{(5)(6.1)}{1810} = 0.01685 \text{ sq. ft., or } 2.425 \text{ sq. in.}$$

214. Equilibrium Flow of a Vapor. When a vapor starts its expansion in a nozzle from the superheat region, as at 1, Fig. 146, and follows an isentropic path 1- a -2, it should start condensing at a where the process line crosses the saturated vapor line, and it should become progressively wetter as the expansion proceeds to 2. If this should be true (which it is not—§ 221), the vapor-liquid mixture would at all times be an equilibrium mixture and the expansion would be an equilibrium expansion. Hence, for internal *equilibrium conditions*,

$$\begin{aligned} s_1 &= s_2 = (s_g - y_2 s_{fg})_2, \\ h_2 &= (h_g - y_2 h_{fg})_2 \quad \text{and} \quad v_2 = (v_g - y_2 v_{fg})_2, \end{aligned}$$

where s_1 may be s_o if stagnation properties are known. These equations, together with equations (4) and (80), give the solution to problems in which equilibrium conditions are assumed.

215. Example: Steam Nozzle, Equilibrium Flow. Steam enters a nozzle with negligible velocity at 160 psia and 400°F, flowing at the rate of 10 lb./sec. If it expands isentropically to 10 psia, what should be the area of the exit section (process 1-2, Fig. 146).

SOLUTION. From the steam tables,

$$\begin{aligned} h_1 &= 1217.6 & s_{g2} &= 1.7876 & h_{fg2} &= 982.1 \\ s_1 &= 1.5908 & s_{fg2} &= 1.5011 & v_{g2} &= 38.42 \\ v_1 &= 3.008 & h_{g2} &= 1143.3 & v_{fg2} &= 38.4 \end{aligned}$$

$$\begin{aligned} s_1 &= s_2 = 1.5908 = 1.7876 - 1.5041y_2, & y_2 &= 13.05\%. \\ h_2 &= 1143.3 - (0.1305)(982.1) = 1015.1 \text{ Btu/lb.} \\ v_2 &= 38.42 - (0.1305)(38.4) = 33.4 \text{ cu. ft./lb.} \\ v_2 &= 223.8(h_1 - h_2)^{1/2} = 223.8(1217.6 - 1015.1)^{1/2} = 3180 \text{ fps.} \\ A_2 &= \frac{wv_2}{v_2} = \frac{(10)(33.4)(144)}{3180} = 15.1 \text{ sq. in.} \end{aligned}$$

216. Variation of Velocity, Specific Volume, and Area of Section of a Nozzle. It will be revealing to investigate the manner in which the velocity, specific volume, and section area vary as the pressure drops through a nozzle, an investigation which may be made for either a gas or vapor. The method of procedure for a gas is as shown in detail in § 213, those calculations being repeated for several different p_2 's. The answers are tabulated below Fig. 147, in which the values are plotted.

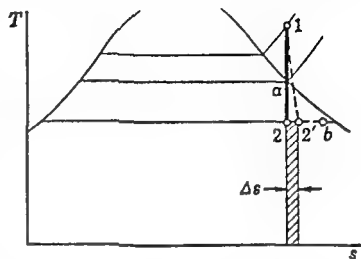


Fig. 146. Equilibrium Expansion.

The most notable feature, true for vapor as well as gas nozzles, is that the area decreases to a minimum, already named the *throat* and then increases, that is, the nozzle *must* be converging diverging. Since w is the same for all sections, the A/w curve would look like the A curve, Fig 147, then since

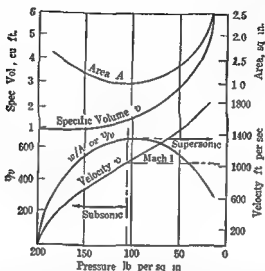


Fig 147 Specific Volume Area and v/v Plotted Against Pressure Vapors and other gases flowing through nozzles have attributes quite similar to these curves plotted for air. Observe that the velocity may be very large the maximum in this case being 1810 fps ~ 1259 mph. It is apparent that the nozzle for this expansion must have a convergent portion followed by a divergent portion. This illustration is plotted for a uniform pressure drop (abscissa). However, the pressure drop in the nozzle is not constant per unit length of nozzle. There is no reason for it to be.

p_1 Psia	v_1 Cu ft/lb	v_2 Fps	A_1 Sq in	v_2/v_1 w/A_1^*
200	0.962			
175	1.059	483	1.578	486
150	1.18	700	1.212	593
125	1.346	887	1.091	659
106	1.515	1020	1.070	673
100	1.58	1000	1.072	671
50	2.59	1431	1.302	553
25	4.24	1675	1.821	395
10	6.1	1810	2.425	297

* A_1 in sq ft

$A/w = v/v$ the $w/A = v/v$ curve naturally moves to a maximum and then down. The reason that these curves move as they do is that in the early part of the expansion the velocity increases at a greater rate than the specific volume but later the specific volume increases at the more rapid rate. Study the curves.

Note that it is *necessary* for the area to increase after the throat section if the expansion (volume increase) is to continue. In the tabulation of Fig. 147, the minimum area is 1.07 sq. in. at $p_2 = 106$ psia. If this nozzle, cut off at the throat section, should discharge into a passage of 1.07-sq. in. area, Fig. 148, there would be no further expansion, and in the absence of friction and heat the air would continue to move in this passage at a pressure of 106 psia and $v = 1020$ fps for an indefinite distance. If the pressure p_2 at the end of the pipe B , Fig. 148, is less than 106 psia, there is a sudden, uncontrolled expansion and the air leaves the pipe in a chaotic condition, the degree of chaos increasing as the pressure drop increases. If the pressure p_2 were 100 psia, that is, any pressure a little lower than 106, much of the acquired kinetic energy could be converted into work in turbine blades. On the other hand, if the pressure p_2 were 15 psia, the confusion in the stream would be so great that much of the acquired kinetic energy would be lost in internal friction before it could do work on turbine blades. When the expansion is guided, as suggested by the dotted lines at B , not only is the kinetic energy acquired at C conserved, but additional kinetic energy is also generated during the further expansion, all of which is available for work on turbine blades.

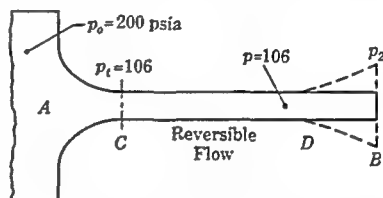


Fig. 148. The properties of the fluid do not change from, say, C to D .

Now suppose that the pressure p_2 is greater than 106 psia, say, 175 psia. Then in the absence of friction, this pressure would build up in the pipe line theoretically to the section C ; that is, p_t would be 175 psia. If this should be true, the velocity acquired at C would be 483 fps (instead of 1020 fps) as shown in the tabulation under Fig. 147, but to obtain the same flow w , the area at C would have to be increased to 1.578 sq. in. See also § 223 and Fig. 151.

217. Critical Pressure in a Nozzle. Since the pressure at C , Fig. 148, will not go below $p_t = 106$ psia, called the *critical pressure*, the conditions at the throat deserve further study. Let us find an equation for w in terms of the properties of an ideal gas and study the result. First, we have the following transformations:

$$\begin{aligned}
 v_2 &= 223.8(h_o - h_2)^{1/2} = 223.8[c_p(T_o - T_2)]^{1/2} \\
 &= 223.8 \left[c_p T_o \left(1 - \frac{T_2}{T_o} \right) \right]^{1/2}, & [\text{IDEAL GAS}] \\
 (g) \quad v_2 &= 223.8 \left\{ c_p T_o \left[1 - \left(\frac{p_2}{p_o} \right)^{(k-1)/k} \right] \right\}^{1/2} \text{ fps,} \\
 &[\text{IDEAL GAS, } s = C]
 \end{aligned}$$

where $223.8 = \sqrt{2g\alpha J}$. Using this equation and $v_2 = (RT_0/p_0)(p_0/p_2)^{1/k}$ from equation (f) in $w = Av/v$, we find

$$\begin{aligned} w &= \frac{223.8 A_2}{(p_0/p_2)^{1/k} (RT_0/p_0)} \left\{ c_p T_0 \left[1 - \left(\frac{p_2}{p_0} \right)^{(k-1)/k} \right] \right\}^{1/2}, \\ (h) \quad w &= 223.8 \frac{A_2 p_0}{R} \left\{ \frac{c_p}{T_0} \left[\left(\frac{p_2}{p_0} \right)^{2/k} - \left(\frac{p_2}{p_0} \right)^{(k+1)/k} \right] \right\}^{1/2} \text{ lb/sec}, \\ &\quad [\text{IDEAL GAS, } s = C] \end{aligned}$$

which is the mass of ideal gas flowing isentropically across a nozzle section of area A_2 where the pressure is p_2 . If w is plotted against p_2/p_0 with all other terms constant, the curve *Oba*, Fig. 149, is obtained. Thus, for a particular nozzle with A_2 constant, the value of w from equation (h) has a maximum value at b , a value which occurs when

$$\left(\frac{p_2}{p_0} \right)^{2/k} - \left(\frac{p_2}{p_0} \right)^{(k+1)/k}$$

is a maximum, and the pressure p_2 , which makes this term a maximum, can be found by equating its differential with respect to p_2 to zero, thus,

$$\begin{aligned} \frac{d[(p_2/p_0)^{2/k} - (p_2/p_0)^{(k+1)/k}]}{dp_2} \\ = \left(\frac{1}{p_0^{2/k}} \right) \left(\frac{2}{k} \right) (p_2^{(2-k)/k}) - \left(\frac{1}{p_0^{(k+1)/k}} \right) \left(\frac{k+1}{k} \right) (p_2^{1/k}) = 0, \end{aligned}$$

from which

$$(81) \quad \frac{p_2}{p_0} = \frac{p_t}{p_0} = \left(\frac{2}{k+1} \right)^{k/(k-1)} \quad \text{or} \quad p_t = p_0 \left(\frac{2}{k+1} \right)^{k/(k-1)}$$

Since $T_t/T_0 = (p_t/p_0)^{(k-1)/k}$, equation (81) yields

$$(i) \quad T_t = T_0 \left(\frac{2}{k+1} \right) \quad [s = C \text{ ONLY}]$$

The pressure at the throat p_t is called the *critical* pressure* and can be predicted for an ideal gas in an ideal expansion (and quite closely for any gaseous expansion) by equation (81), the ratio p_t/p_0 is the *critical pressure ratio*, T_t in equation (i) is the temperature at the throat after an isentropic expansion. If one solves for A_2 from equation (h) and considers the result, he concludes that A varies with the pressure ratio function in the brackets. The differential of the bracketed terms equated to zero yields the condition of minimum area, which is the same as equation (81). In other words, the section of the critical pressure and the section of minimum area coincide—which is what you had no doubt already concluded. This being so the mass of flow is governed by the area of the throat (and also the initial state),

* The word *critical* is overworked

the size of the sections of the nozzle subsequent to the throat are irrelevant to the mass flowing.

As was pointed out in connection with Fig. 148, the pressure will not fall below p_t in a nozzle that has no divergent section; therefore, the dotted portion of the curve bo of Fig. 149 is a physical impossibility; the flow rate does not fall off as the pressure goes below the critical pressure p_t , but remains constant, and is represented by curve abc , Fig. 149. Summarizing, we may say

1. The flow is less than the maximum attainable if the final pressure is greater than the critical pressure p_t , and
2. The flow is the maximum for all final pressures equal to or less than p_t .

The final state defines the area of the final section. If, in the tabulation of Fig. 147, the final pressure is 25 psia, the area of the exit section should be 1.821 sq. in., as seen in the tabulation; if the final pressure is 50 psia, the exit area should be 1.302 sq. in. In both cases, the discharged mass would be the same.

Because the use of equations (g) and (h) conceals some thermodynamic fundamentals, the student is advised to avoid them except for special studies, such as in this article, and except for the purpose of deriving special forms of equations for repeated applications to particular situations.

218. Values of Critical Pressures. Observing that the curves for A and U/v , Fig. 147, do not change abruptly at the critical pressure, we correctly conclude that the throat area is not sensitive to the true value of the critical pressure; that is, k in equation (81) may vary somewhat from its true value without significant effect on the nozzle design. For this reason, equation (81), which strictly applies to an isentropic expansion, is commonly used for computing critical pressures. The actual expansion is practically adiabatic, and because a correctly designed and well-made nozzle is highly efficient, the expansion is only slightly irreversible. For ideal gases with $k = 1.4$ (which means diatomic gases at temperatures not too far from atmospheric), the critical pressure is

$$(j) \quad p_t = p_o \left(\frac{2}{k+1} \right)^{1/(k-1)} = p_o \left(\frac{2}{2.4} \right)^{1.4/0.4} \approx 0.53p_o. \quad [\text{DIATOMIC}]$$

The critical pressure of air for an initial stagnation pressure of 200 psia is $(0.53)(200) = 106$ psia (see Fig. 147). For $k = 1$, the critical pressure ratio

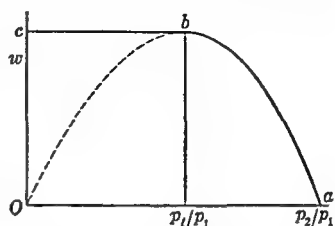


Fig. 149. Discharge from Nozzle. The curve Oba is for a constant area A_2 of the exit section as obtained from equation (h). Curve abc represents the actual discharge, which reaches a maximum at the critical pressure ratio p_t/p_1 and then remains there along bc . Curve abc represents equation (h) when A_2 is allowed to vary so as always to correspond to p_2 .

is 0.606, for $k = 1.67$, $p_t/p_o = 0.487$. Note the relatively small variation. Other values in common use are

Superheated steam,	$k = 1.3$,	$p_t = 0.545p_o$,
Supersaturated steam,	$k = 1.3$	$p_t = 0.545p_o$,
[MODERATE PRESSURES]		
Wet steam	$k = 1.13$,	$p_t = 0.58p_o$

Keenan and Keyes gives values of k for superheated steam

219 Throat Velocity If equation (g) is applied to the throat section, $p_2 = p_t$, we obtain an expression for the throat velocity. Take these steps

- 1 Substitute p_t/p_o from (81) into equation (h) and simplify,
- 2 Substitute $\sqrt{2g_oJ}$ for 223.8
- 3 Substitute for $c_p = Rk/[J(k-1)]$, which is equation (26), § 61, and simplify,
- 4 Substitute for T_o its value from equation (i), $T_o = (k+1)T_t/2$, and simplify to find

$$(k) \quad v_t = (g_o k R T_o)^{1/2} = (g_o k p_o \rho_o)^{1/2} = \left(\frac{g_o k p_t}{\rho_t} \right)^{1/2} \text{ fps,}$$

where $\rho_o = 1/v_o$, lb per cu ft is the density and $p_o v_o = RT_o$. The velocity given by equation (k) is the *sonic velocity* or the *acoustic velocity* (Mach 1), which is the velocity of sound in a gas whose specific gas constant is R ft-lb per lb-°R, whose $c_p/c_v = k$, and whose temperature is T_o . Perhaps you recall from your physics that Laplace found that the velocity of sound in a gas is $v_s = (g_o \beta v)^{1/2}$ where β is the isentropic bulk modulus and v is the specific volume of the gas. By definition, the *bulk modulus* is the pressure (stress) change dp divided by the accompanying volume change per unit volume (strain) dv/v , or

$$(l) \quad \beta = - \frac{dp}{dv/v},$$

where the negative sign recognizes that the volume decreases as the pressure increases. To get dp/dv for an isentropic process of an ideal gas differentiate $pv^k = C$ and find $-dp/(dv/v) = pk = \beta$. Then substitute pk for β in

$$v_s = (g_o \beta v)^{1/2} = (g_o k p v)^{1/2},$$

the velocity of sound in a gas. This expression is seen to be the same as the second form in equation (k). It is often convenient to express velocities in terms of Mach numbers M , which is the ratio of the actual velocity of the substance (or of a body in the substance) divided by the velocity of sound in the substance, that is,

$$(m) \quad M = \frac{v}{v_s} = \frac{v}{(g_o k p v)^{1/2}}$$

Mach numbers greater than unity are *supersonic velocities*; Mach numbers less than unity are *subsonic velocities*. We note that in a convergent nozzle, the velocity of the substance is always subsonic; in a convergent-divergent nozzle, the velocity is $M < 1$ up to the throat, Mach 1 at the throat, and $M > 1$ in the divergent part.

220. Example: Throat Size. What should be the throat area for the steam flow defined in the example of § 215; that is, $p_1 = 160$ psia, $t_1 = 400^\circ\text{F}$, $p_2 = 10$ psia, 10 lb./sec., isentropic process?

SOLUTION. The throat pressure for superheated steam (from § 218) is

$$p_t = (0.545)(160) = 87 \text{ psia (nearly).}$$

Taking values from the steam tables for 87 psia, we find

$$s_1 = s_t = 1.5908 = 1.6139 - 1.153y_t, \quad \text{or} \quad y_t = 1.99\%.$$

$$h_t = 1184.6 - (0.0199)(896.5) = 1166.8 \text{ Btu/lb.}$$

$$v_t = 223.8(1217.6 - 1166.8)^{1/2} = 1592 \text{ fps.}$$

$$v_t = 5.055 - (0.0199)(5.037) = 4.95 \text{ cu. ft./lb.}$$

$$A_t = \frac{wv_t}{v_t} = \frac{(10)(4.95)(144)}{1592} = 4.47 \text{ sq. in.,}$$

which is the necessary throat area to pass 10 lb./sec. of steam in an ideal expansion.

221. Supersaturated Flow. As previously mentioned, the time that it takes for a particular molecule of substance to pass through a nozzle is

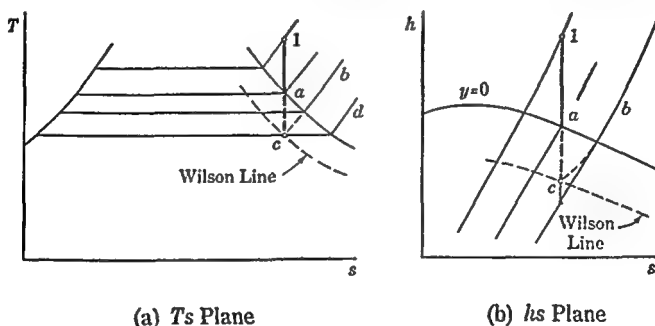


Fig. 150. Ideal Supersaturated Flow.

short (of the order of 0.001 sec.). Therefore it should not be too surprising that condensation does not start at a , Fig. 150, where the expansion line $1-a-c$ crosses the saturated vapor line. Instead, the steam remains a vapor until some point c is reached when condensation suddenly occurs. Such steam as that between a and c is called *supersaturated steam* and the state of this steam is spoken of as a *metastable state*, meaning that it would take more than a very small action to cause the steam to achieve internal equi-

librium states along ac —say, an action as big as a spray of fine water drops into the stream ahead of point a (not that this result is desired). The actual pressure at the supersaturated state c is p_b , Fig 150, but the saturation pressure corresponding to the temperature t_c is p_s [Fig 150(a)] which is the pressure that would have existed at c if an equilibrium expansion had occurred to t_c . The ratio of these pressures, p_b/p_s , is called the *degree of supersaturation* or *supersaturation ratio*.

A number of studies have been made as to the limit of supersaturation states, a limit named the *Wilson line* (Fig 150). Binnie and Woods (101) located the Wilson line through the points $y = 3.5\%$, $p \approx 10$ psia, and $y = 2.6\%$, $p = 5$ psia. Yellott and Holland locate the Wilson line at

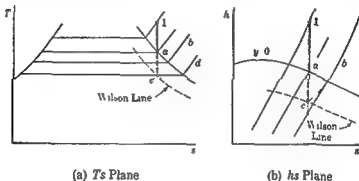


Fig 150 Repeated

approximately 60 Btu below the saturated vapor line on the Mollier diagram, that is ac , Fig 150 (b) is equal to about 60 Btu. As you would expect, this so-called *line* is actually a region, condensation does not begin at a precisely predictable point.

Supersaturated steam is denser than steam in equilibrium at the same pressure. The velocity of supersaturated steam is less than the velocity of steam in equilibrium, both having expanded from the same initial condition. Since the density increases by a larger percentage than the velocity decreases there will be a greater discharge ($w = 1/v \cdot A \cdot v$) when the steam is supersaturated than when it is in equilibrium. Because of this phenomenon the discharge coefficients are sometimes greater than unity, that is, the actual discharge is sometimes greater than that computed for an ideal equilibrium flow (§ 235).

The ideal expansion of supersaturated steam is found to be closely defined by $pr^k = C$, where $k = 1.3$ in the region of moderate pressures. From the energy equation (17), § 42, $T ds = dh - v dp/J$ for a reversible process and for $ds = 0$, we have $dh = v dp/J$, and the integration of $v dp$ can be made for a known pr relation. This integral with $pr^k = C$ has been used at various places in this text: equation (38) and below, § 70, equation (t), § 73, equation (f), § 113, etc. Since $dK = -dh$ from equation (a), we have

$dK = -v dp$ in foot-pound units and

$$\Delta K = - \int_0^2 v dp = \frac{k p_o v_o}{k-1} \left[1 - \left(\frac{p_2}{p_o} \right)^{(k-1)/k} \right]$$

or

$$(82) \quad v_2 = \left\{ \frac{2 g_o p_o v_o k}{k-1} \left[1 - \left(\frac{p_2}{p_o} \right)^{(k-1)/k} \right] \right\}^{1/2} \text{ fps,}$$

[SUPERSATURATED STEAM, SUPERHEATED STEAM, IDEAL GAS]

the velocity of supersaturated steam (or of an ideal gas) after an isentropic expansion when the integration is made from the stagnation state, $K_2 = \Delta K = v_2^2/2g_o$ ft.-lb. per lb. If we use p_1 and v_1 in equation (82) in place of p_o and v_o , then the correction factor I for initial velocity, [equation (d)] should be used when the initial velocity is significant. Equation (82) applies to any reversible expansion where $pv^k = C$ (or $pv^n = C$), which of course includes the ideal gas. It would be easy to show that equations (82) and (g) are identical. Equation (82) may also be applied to superheated steam over a moderate drop in pressure when the corresponding average value of k is used. Supersaturation is likely to persist to beyond the throat even though the steam initially has very little or even no superheat.

The procedure in solving problems for an ideal expansion of supersaturated steam would involve computing

$$v_2 = v_1 \left(\frac{p_1}{p_2} \right)^{1/k} \quad \text{or} \quad v_2 = v_o \left(\frac{p_o}{p_2} \right)^{1/k},$$

where v_1 (or v_o) is taken from the steam tables for p_1 and t_1 (or p_o and t_o), computing v_2 from equation (82), and then using $w = Av/v$ lb. per sec. The throat pressure in a convergent-divergent nozzle is as given in § 218.

Our knowledge of the flow of steam in converging-diverging nozzles is not all to be desired, and space is short here. For the purposes of this course, you may assume that the mixture of water and vapor leaving such a nozzle is in an equilibrium state (if it is more than 60 Btu below the 100% quality line on the Mollier diagram, § 223).

222. Example: Supersaturated Flow. On the basis of supersaturation, find the throat area of a nozzle for the conditions given in §§ 215 and 220; $p_1 = 160$ psia, $t_1 = 400^\circ\text{F}$, $p_2 = 10$ psia, 10 lb./sec., isentropic process, $v_1 \approx 0$.

SOLUTION. From §§ 215 and 220, we take the data: $v_1 = 3.008$ cu. ft., $p_t = 87$ psia. Then

$$v_t = v_1 \left(\frac{p_1}{p_t} \right)^{1/k} = 3.008 \left(\frac{160}{87} \right)^{1/1.3} = 4.81 \text{ cu. ft.,}$$

which is observed to be smaller than for the equilibrium flow (4.95 cu. ft. in § 220) even though the steam is barely across the saturated vapor line. From equation (82), we get

$$v_t = \left\{ \frac{(2)(32.2)(160 \times 144)(3.008)(1.3)}{1.3-1} \left[1 - \left(\frac{87}{160} \right)^{0.713} \right] \right\}^{1/2},$$

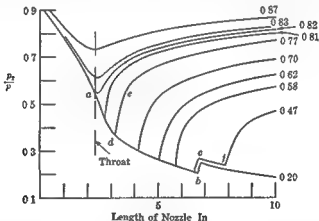
or $v_t = 1590$ fps For 10 lb per sec ,

$$A_t = \frac{wv_t}{v_t} = \frac{(10)(4.81)(144)}{1590} = 4.36 \text{ sq in}$$

This is a better answer than the one in § 220 (4.47 sq in) The difference between the answers would have been greater if the expansion had been farther below the saturated vapor line As a matter of interest we might note that if this much area were needed in a small turbine it would be divided among several nozzles

223 Overexpansion and Underexpansion If the exit section of the nozzle is too large *overexpansion* occurs, that is, the substance expands to some pressure below the pressure in the discharge region and then rises to the discharge pressure If the exit section is too small *underexpansion* occurs, that is the pressure at the exit section is greater than that in the discharge region and as a result there is a free and turbulent expansion after the substance leaves the nozzle Although either overexpansion or underexpansion results in a loss of available energy such operating conditions cannot be avoided at times because of the necessity for varying the output the steam turbine is not always producing the same power the turbo jet engine is not always exerting the same thrust

The curves in Fig 151 are for a convergent divergent nozzle 10 in long and show what happens to the pressure in a steam nozzle for various back



After Binnie and Woods (101)

Fig 151 Effect of Varying Discharge Pressure

pressures The nozzle is designed for a $p_2 = 0.20p_1$ (closely) The throat pressure is nearly $0.55p_1$ Notice that when the back pressure is $p_2 = 0.81p_1$, there is no effect on the throat pressure and therefore no effect on the amount of flow The same amount of steam is discharged from this nozzle for $p_2 = 0.81p_1$ as for $p_2 = 0.20p_1$ (The difference between this case and that discussed in connection with Fig 148 is that there is a diverging channel

after the throat in this instance, thus permitting further expansion.) The curve for 0.82 indicates that the back-pressure effect has just reached the throat at a . For $p_2 = 0.83p_0$, the flow through the nozzle is definitely less. For all final pressures greater than $0.82p_0$, the pressure falls to a minimum near the throat and then rises to the discharge pressure, as for $p_2 = 0.87p_0$. The break bc in the curve, Fig. 151, is where condensation occurs in the supersaturated steam at b . This phenomenon is seen to be accompanied by a small pressure jump. Notice that even when the back pressure is $0.47p_0$ (nearly as much as the throat pressure), the steam expands (to f) in accordance with the increasing area of the channel through as much as 80% of the length of the nozzle. Where the back pressure is much greater than the design exit pressure, there is a shock wave somewhere beyond the throat, where the velocity is supersonic before the shock and is subsonic afterwards, as at de , Fig. 151; and as the substance passes through the wave, there is a sudden pressure rise, as seen in the various curves of Fig. 151.

224. Losses in Nozzles. The *nozzle efficiency* e_n is defined as the actual kinetic energy of unit mass at discharge divided by the ideal kinetic energy per unit mass, each measured to the same final pressure. Let $2'$ be the actual final state, Figs. 146 and 152; then

$$(n) \quad e_n = \frac{K_{2'}}{K_2} = \frac{h_0 - h_{2'}}{(h_0 - h_2)_s} = \frac{v_{2'}^2/(2g_0J)}{(h_1 - h_2)_s + K_1},$$

where the subscript s is a reminder of constant entropy; $K_1 = v_1^2/(2g_0J)$. In a convergent-divergent nozzle, K_1 is nearly always relatively negligible and can be omitted; this is also true when there is a substantial expansion in

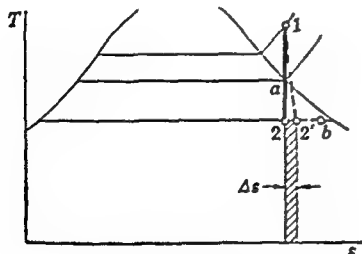


Fig. 146. Repeated.

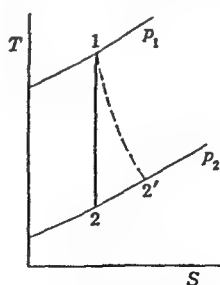


Fig. 152. Gas Expansion. If $v_1 = 0$, state 1 is the same as the stagnation state.

a convergent nozzle. If the nozzle *fits* the expansion, the efficiencies are high, ranging from some 92% for long convergent-divergent nozzles to 98-99% for convergent nozzles. The efficiency of convergent-divergent nozzles *up to the throat* is similarly some 98-99% for well-made ones.

The *nozzle coefficient* or *velocity coefficient* η_n is the ratio of the actual velocity divided by the ideal velocity,

The shaded area in Fig. 146 represents the lost work due to the irreversibilities in the nozzle. In either Fig. 146 or 152, the lost work is $h_{2'} - h_2$. Irreversibility is present not only because of the normal fluid friction on the sides of the nozzle and on itself, but also in the case of steam because of the sudden change from supersaturation to equilibrium, and if water particles are present, because of the different velocities of vapor and water. The vapor overtakes and impinges upon slower water particles. These impacts are irreversible. In Fig. 146, the area under $2'-b$ represents the further loss from turbulence as the steam passes over the blades; some point b represents the condition of the steam leaving the turbine stage.

225. Coefficient of Discharge. The ideal rate of flow is related to the actual rate of flow through a nozzle (or other flow device, such as an orifice) by a *coefficient of discharge* η_a , defined as

$$(s) \quad \eta_a = \frac{\text{actual mass of flow per unit time}}{\text{ideal mass of flow per unit time}}$$

The value of this coefficient can be determined by test on a particular device. Also, it can be estimated, sometimes with considerable accuracy, from past data on similar devices.

In the case of the flow of supersaturated steam through nozzles, there is some confusion as to whether to compute the ideal flow [the denominator of equation (s)] on the basis of equilibrium flow or on the basis of supersaturated flow. Hence, one should always specify which basis is used.

226. Estimated Actual Volume. For most practical purposes in adiabatic expansions, it can be assumed that the actual volume at some pressure p_2 is equal to the ideal volume after an isentropic expansion, unless the efficiency during the expansion is much lower than it normally is in a nozzle. Perhaps in converging-diverging nozzles, the use of the calculated "actual" volume at the exit may be worth while. At any rate, methods of computing $v_{2'}$ are given below. This volume is needed in order to be consistent in the continuity equation, because in $w' = A_2 v_{2'}/v_{2'}$, the values must be simultaneous ones.

(a) *Ideal Gas.* If the substance is an ideal gas, equation (n) for the efficiency may be written

$$e_n = \frac{h_o - h_{2'}}{h_o - h_2} = \frac{c_p(T_o - T_{2'})}{c_p(T_o - T_2)},$$

from which $T_{2'}$ can be computed for a known or assumed efficiency. Then, given $p_{2'} = p_2$, we get $v_{2'} = RT_{2'}/p_2$. Or, if the exponent in $pv^m = C$ for an irreversible adiabatic is known, $v_{2'} = v_1(p_1/p_2)^{1/m}$.

(b) **Equilibrium Flow** If a vapor expands through a nozzle in equilibrium, the efficiency equation (n),

$$e_n = \frac{h_0 - h_2}{h_0 - h_2^*}$$

can again be used to compute h_2 at any desired point 2'. Now with h_2 and $p_2 = p_2^*$, we may determine all properties at 2' from the vapor tables. If point 2' is in the wet region, the wetness is computed from

$$h_2 = (h_g - y_2 h_{fg})_2, \quad \text{then} \quad v_2 = v_g - y_2 v_{fg} \text{ cu ft/lb}$$

(c) **Supersaturated Flow.** The easiest way to compute the actual volume during a supersaturation flow is first to find the value of m from equation (r) and then determine $v_2 = v_2(p_2/p_1)^{1/m}$. This is the same thing as computing v_2 from $e_n(p_2 v_2 - p_1 v_1) = p_2 v_2 - p_1 v_1$.

227 Example Compute the actual flow through the nozzle of § 222, supersaturated conditions, if the efficiency up to the throat is 98%. What is the coefficient of discharge? The data are $p_1 = 160$ psia, $t_1 = 400^\circ\text{F}$, $p_2 = 10$ psia, initial velocity negligible.

SOLUTION From § 222, we have $p_t = 87$ psia, $v_t = 3.008$ cu ft/lb, $v_1 = 1590$ fps and $A_t = 4.36$ sq in. Letting $k = 1.3$, and using equation (r), we get

$$e_n \left[1 - \left(\frac{p_2}{p_1} \right)^{(k-1)/k} \right] = 1 - \left(\frac{p_2}{p_1} \right)^{(m-1)/m}$$

$$0.98 \left[1 - (87/160)^{0.3/1.3} \right] = 1 - (87/160)^{(m-1)/m}$$

The value of the bracket on the left was found in § 222 as 0.131. Simplifying and using the log-log scales on your slide rule find $(m-1)/m = 0.226$ and $m = 1.29$. Then

$$v_1' = v_1 \left(\frac{p_1}{p_t} \right)^{1/m} = 3.008 \left(\frac{160}{87} \right)^{1/1.29} = 4.82 \text{ cu ft/lb}$$

$$v_1' = \eta_n v_1 = (0.98)(1590) = 1570 \text{ fps}$$

$$w' = \frac{A_t v_1'}{v_1} = \frac{(4.36)(1570)}{(144)(4.82)} = 9.88 \text{ lb/sec}$$

This mass will pass across the exit section of the nozzle in accordance with the general indications of Fig. 151. The coefficient of discharge based on supersaturation is $\eta_d = 9.88/10$, or 98.8%. (The computations for a problem like this should be made with logarithms and a calculating machine.)

228 Diffuser. A diffuser receives a fluid at a (relatively) high velocity and discharges it at a higher pressure and lower velocity—the reverse of the nozzle process. That is, some of the initial kinetic energy, which is 100% available, is used as the work of compression. The same conditions, $Q = 0$ and $W = 0$, are applied to diffusers, Fig. 153, to get the same energy relation, equation (a),

$$\frac{v_1^2}{2gJ} + h_1 = \frac{v_2^2}{2gJ} + h_2 \text{ Btu/lb,}$$

except that often the final velocity v_2 (and K_2) is negligible. Letting $K_2 = 0$ and integrating $\int v dp = J(h_2 - h_1)$ for $pv^k = C$ as before (Fig. 18), we get

$$(t) \quad v_1^2 = \frac{2g_0 k}{k-1} (p_0 v_0 - p_1 v_1),$$

where p_0 is the stagnation pressure, which is virtually equal to the discharge pressure when the final velocity is relatively small. The manipulation of this equation is left to the reader inasmuch as the forms which it may take will parallel those for a nozzle. We are able to maintain a fine control over the expansion process in a nozzle, with the result that the efficiency is high (see Chapter 6). But if the initial velocity is high, the efficiency of a diffuser will be less than that of a similar nozzle, because of the greater turbulence in the diffuser. The venturi is quite efficient because the pressure change is small. Let the critical pressure ratio be p_t/p_1 , defined by equation (81); then by way of comparison:

Nozzle

$$\text{If } \frac{p_2}{p_1} < \frac{p_t}{p_1},$$

$v_2 >$ sonic velocity, converging-diverging nozzle.

$$\text{If } \frac{p_2}{p_1} > \frac{p_t}{p_1},$$

$v_2 <$ sonic velocity, converging nozzle.

Diffuser

$$\text{If } \frac{p_2}{p_1} > \frac{p_t}{p_1} \quad \left(\frac{p_1}{p_2} < \frac{p_1}{p_t} \right),$$

$v_1 >$ sonic velocity, converging-diverging diffuser.

$$\text{If } \frac{p_2}{p_1} < \frac{p_t}{p_1},$$

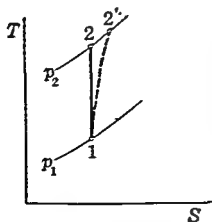
$v_1 <$ sonic velocity, diverging diffuser.

In each instance, p_1 is the initial pressure and p_2 is the final pressure. Since the actual process is an irreversible adiabatic, an appropriate value of m in $pv^m = C$ can be found after the manner explained for the nozzle. It is convenient to use a diffuser efficiency e_d , (Fig. 153),



(u)

$$e_d = \frac{(h_2 - h_1)_s}{h_{2'} - h_1},$$



in which, strictly, states 2 and 2' are stagnation states, ideal and actual. Thus, the initial velocity necessary to compress to a state 2' is

$$v_1' = [2g_0 J (h_{2'} - h_1)]^{1/2} = \left[2g_0 J \left(\frac{h_2 - h_1}{e_d} \right) \right]^{1/2} \text{ fps,}$$

Fig. 153. Diffuser. where 1-2 is an isentropic path.

229. Ejector. The diffuser is also an integral part of the *injector* or *ejector*. These two terms are applied to essentially the same instrument. The operation of an injector or ejector is as follows: Steam enters the usual

type of divergent nozzle at A , Fig 154 At B , the pressure is low and the velocity is high The low pressure in the region B causes the gas or liquid to be sucked into the device, and the momentum of the fast-moving jet of steam carries the substance into the diffuser (through C) where the velocity decreases and the pressure increases until the exit section D is reached

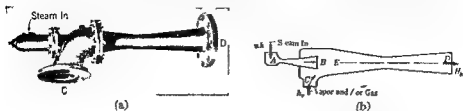


Fig 154 Ejector. AB is a steam nozzle, BD is the diffuser Substance being pumped enters at C The injector, used as a feedwater pump, is similar to this device, and it pumps the water to a higher pressure at D than exists at A

Ejectors are commonly used to rid evacuated spaces (steam condensers, for example) of noncondensable gases such as air, and also as pumps, called *jet pumps*, in vacuum refrigerating systems A single-stage ejector can maintain a vacuum of some 26 in Hg, two-stage, up to about 29.3 in Hg, three stage, up to 29.9 in Hg, and a four-stage ejector can maintain absolute pressures of the order of 1 mm Hg Since the thermal efficiency of the ejector is low, rotary or reciprocating pumps are generally preferred except where large volumes of gases or vapors are to be handled

230. Closure One of the most common errors made by students is in the units of A and p , which should be square feet and pounds per square foot, respectively, unless conversion constants are used or unless the units cancel, as in a ratio (p_2/p_1)

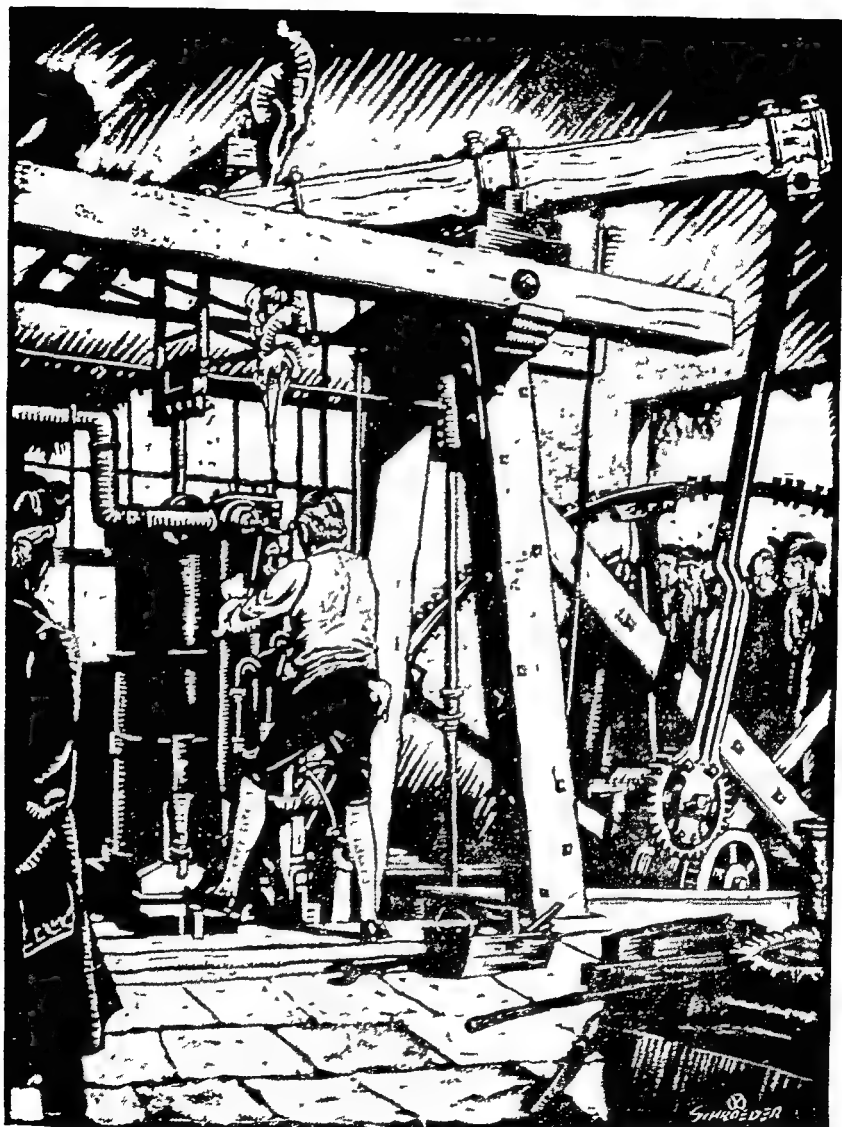
Maybe the reader has wondered about "the velocity" of a stream flowing in a channel, inasmuch as there are different velocities at different points. Thus, we have been obviously thinking of the velocity as a mean or average value, otherwise computations of w from $w = A\bar{v}/v$ are meaningless

Looking through the literature, you will find many different equations which purport to give the mass flow through a nozzle under certain circumstances Nearly all the useful ones can be derived from equation (h), p 262, with possibly the inclusion of a coefficient of discharge If you like mathematical problems, it will be fun to simplify equation (h) for particular conditions For example assume that *maximum flow* occurs, that is, that p_2 is the throat pressure as defined by equation (81) and show that

$$w_{\max} = A p_0 \left(\frac{2}{k+1} \right)^{1/(k-1)} \left[\frac{2g_0 k}{RT_0(k+1)} \right]^{1/2} \text{ lb/sec}$$

for any ideal gas, or that for air,

$$w_{\max} = \frac{0.532 A p_0}{T_0^{1/2}} \text{ lb/sec}$$



Courtesy Johnson Publishing Co , Cleveland

James Watt and His Steam Engine.

internally reversible and it approaches external reversibility as the temperature differences $T_2 - T_o$ and $T_r - T_1$ approach zero.

Since the refrigeration of this cycle is the heat added Q_A , we find

$$(b) \quad \text{Refrigeration} = Q_A = T_1(s_a - s_d).$$

The work is ΣQ or $Q_A - Q_R$, and will be a negative number if expressed in its fundamental form, because $Q_R > Q_A$ in a reversed cycle. The negative sign also shows that net work is done *on* the substance. Observing these facts and remembering that the signs given to energy changes are arbitrary, we may as well write the expression for the work as a positive number and avoid the inconvenience of a negative sign. Accordingly, we use

$$W = Q_R - Q_A$$

and find

$$(c) \quad \text{Work} = W = T_2(s_b - s_c) - T_1(s_a - s_d) = (T_2 - T_1)(s_a - s_d),$$

where $s_b - s_c$ is seen to be equal to $s_a - s_d$. Then the coefficient of performance COP from equations (b) and (c) is

$$(d) \quad \gamma = \frac{\text{refrigeration}}{\text{work}} = \frac{T_1}{T_2 - T_1}.$$

[CARNOT OR OTHER REVERSIBLE CYCLE]

From a previous discussion of the Carnot cycle, we realize that the coefficient of performance of a refrigerating cycle as expressed by equation (d) is the highest possible coefficient for all cycles operating between the temperature limits of T_1 and T_2 . In this sense, equation (d) gives us a standard of comparison; yet, as in power cycles, we find it desirable in practice to set up other ideal cycles more nearly approaching the actual events.

233. Conclusions from the Carnot Cycle. There are certain general conclusions which can be drawn from the Carnot cycle.

1. It is desirable that the work to activate the cycle be a minimum, since it must come from an outside source and be paid for.

2. The work will be reduced as the temperature T_2 , at which heat is discharged, is *lowered*. It follows that we should take all practical steps to keep this temperature down. The lowest temperature T_o , Fig. 155, that is attainable by a natural coolant, such as the atmosphere or the water from a nearby lake or river, is the most economical. There is thus a limit to improvement of a refrigerating cycle at this point.

3. The work will be reduced as the temperature T_1 , Fig. 155, is *increased*. Thus, it is economical of energy to carry on the desired refrigeration at as high a temperature as possible. There are limits here too. To freeze water, temperatures below 32°F

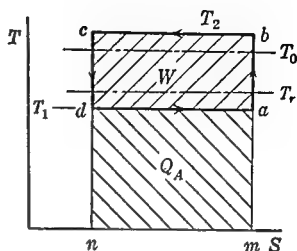


Fig. 155. Reversed Carnot Cycle.

are essential. To cool air for some purposes, much higher temperatures may be used.

4 For particular temperature limits, the heat exchanges should take place at constant temperature for the maximum possible effect. This condition is approachable in the event of vapor refrigeration, but not when a gas is the refrigerant.

234. Definitions and Ratings. The Btu is too small a unit to be convenient in rating commercial refrigerating plants. The larger unit used in refrigerating practice is roughly defined as the number of Btu required to freeze one ton of water at 32°F into ice at 32°F. The heat of fusion of ice is very nearly 144 Btu per lb; therefore, to freeze 2000 lb there must be abstracted $(2000)(144) = 288,000$ Btu. This precise unit, 288,000 Btu, is the definition of a *standard ton* of refrigeration.

To specify *capacity* we must know how long it takes to perform a particular amount of refrigeration. One *standard commercial ton* of refrigeration is defined as 288,000 Btu absorbed at a uniform rate during 24 hr. When the engineer speaks of a *ton* of refrigeration, he generally means a standard commercial ton. Thus, on an hourly basis, a ton of refrigeration is

$$\frac{288\,000}{24} = 12,000 \text{ Btu/hr, or } \frac{12,000}{60} = 200 \text{ Btu/min}$$

If we say that a plant is a 10-ton plant, we are stating the commercial rating which is the same thing as saying that the refrigeration may be at the rate of $(10)(200) = 2000$ Btu per min.

A common method of expressing the actual efficiency of a compression system (§ 235) is to give the horsepower used per commercial ton of refrigeration. This number, which is not a dimensionless number, is obtained of course by test of the actual machine. A relation between the horsepower per ton and the coefficient of performance is easily obtained from the definition of the coefficient of performance.

Let the refrigeration be given as N tons = $200N$ Btu per min. Let the horsepower required for this N tons of refrigeration be represented by hp , then, the corresponding work is $42.4 \text{ hp Btu per min}$ ($42.4 = 23\frac{1}{6}$). Thus, the coefficient of performance COP is

$$(e) \quad \text{COP} = \gamma = \frac{\text{refrigeration}}{\text{work}} = \frac{200N}{42.4hp}$$

from which $W = (\text{refrigeration})/\gamma$ and the horsepower per ton of refrigeration is

$$(f) \quad \frac{hp}{N} = \frac{200}{42.4\gamma} = \frac{4.72}{\gamma}$$

This expression may be used for either actual or ideal cycles.

235 Vapor Compression System. The most common method of securing refrigeration is by a *vapor-compression system*, so called because the vapor

(working substance) is carried from the low-temperature region to the high-temperature region by compressing it. The devices necessary to carry out the vapor cycle are simple and are represented diagrammatically in Fig. 156. (See also Figs. 160 and 161.) In the ideal case, all flow is of course without friction, except flow through the expansion valve, and all processes except those in the condenser and evaporator (cold room) are adiabatic. Figure 157 shows the reversed idealized vapor cycle on the T s plane with numbers corresponding to those on Fig. 156. Starting at state 1, the vaporous refrigerant enters the compressor, which may be either a centrifugal or a reciprocating machine. The refrigerant is at a low temperature as it enters the compressor, but inasmuch as the compression is actually nearly adiabatic 1-2', Fig. 157, and ideally isentropic 1-2, it leaves the

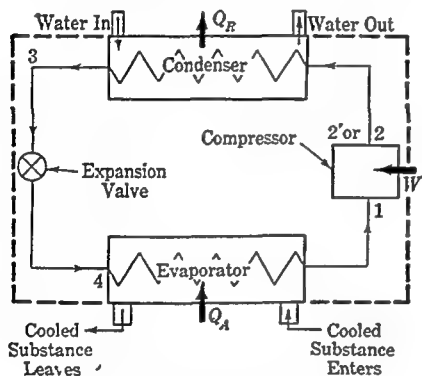


Fig. 156. Compression System for Vapors.

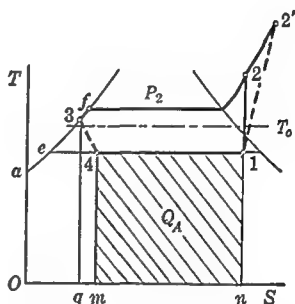


Fig. 157. Refrigeration Cycle.

compressor at some pressure p_2 , such that the saturation temperature corresponding to this pressure is above the normal sink temperature T_0 . This relation of temperature must exist in order that the substance may reject heat to the natural receiver. Leaving the compressor in condition 2 (or 2'), the vapor enters coils in the condenser. As a rule, the heat of condensation is carried away by circulating water, as in the steam condenser, but sometimes circulating air absorbs the rejected heat Q_R . The condenser removes the superheat (if any), the latent heat of vaporization, and generally subcools the liquid a small amount, say from f to 3, Fig. 157. In state 3 as it leaves the condenser, the liquid enters an *expansion valve*, which is a throttling valve separating the region of higher pressure from that of lower pressure. This valve is used to regulate, either manually or automatically, the flow of refrigerant. For the throttling process 3-4 through the expansion valve, $h_3 = h_4$. In order that h_4 , at the lower pressure, be equal to h_3 at the higher pressure, a small amount of the liquid must vaporize. At 4 then, there is a very wet mixture. The wet mixture enters the evaporator

in state 4, absorbing heat Q_A from (doing refrigeration on) the surroundings, process 4-1. The surroundings may be a cold room, as the inside of the household refrigerator, or another substance. In commercial ice manufacture, cold brine circulates about the cans containing water, taking heat from the water for the purpose of freezing it. The brine then flows through the evaporator, where it is cooled again, thence to return to pick up more heat from the water in the cans. When air is to be cooled, it generally circulates directly about the evaporator coils.

The energy diagram of Fig. 156 shows that $Q_A - Q_R = W$, as usual. If the system is operating in a steady state, the process in the condenser and in the evaporator is a steady flow process in which $\Delta K = 0$, $W = 0$, and

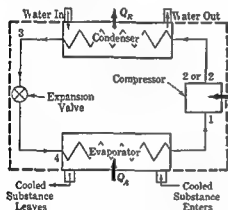


Fig 156 Repeated

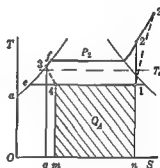


Fig 157 Repeated

$Q = \Delta h$ [§ 67 equation (1)] Thus either for constant pressure as in Fig. 157 or for steady flow ($h_2 = h_1$),

$$(g) \quad Q_A = h_1 - h_4 = h_1 - h_2 \text{ Btu/lb} \quad [\text{REFRIGERATION}]$$

The heat rejected carried away by the cooling water is

$$(h) \quad Q_R = h_2 - h_1 \text{ Btu/lb}$$

As a positive number, the work is

$$(i) \quad W = Q_R - Q_A = h_2 - h_1 - (h_1 - h_2) = h_2 - h_1 \text{ Btu/lb}$$

Now $h_2 - h_1$ is observed to be the difference of the enthalpies at the ends of the isentropic compression ($s_1 = s_2$). From our study of the Rankine cycle, we are reminded that this energy is represented closely by the area 1-2-3-e-1, Fig. 157. The area m-4-1-n represents the refrigeration. The area n-2-f-3-q represents the heat rejected in the condenser. The value of h_1 is taken as that for saturated liquid at temperature t_1 , the small amount of subcooling being always quite negligible. The coefficient of performance of

the ideal vapor cycle for refrigeration is

$$(j) \quad \gamma = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1}.$$

Since the actual compression process is irreversible (with increasing entropy) to some final state 2', Fig. 157, we have

$$(k) \quad W' = h_2' - h_1 - Q \text{ Btu/lb.,}$$

from equation (9A) § 28, for $\Delta K = 0$, with the signs of the terms on the right-hand side changed in order to give a positive sign for W' (in this chapter only). If the compression is adiabatic, $Q = 0$. The *compression efficiency* is $\eta = W/W'$.

236. Displacement of the Compressor. For a particular refrigerating capacity, the size of the compressor depends upon the number of pounds of refrigerant that must be circulated per unit of time to obtain the desired refrigerating effect, and upon the specific volume of the substance at the intake pressure of the compressor. Suppose the plant is to have a capacity of N tons; then the rate of refrigeration is $200N$ Btu per min. Now if the refrigeration per pound of refrigerant is $h_1 - h_4$ Btu, Fig. 157, then the mass of refrigerant circulated is

$$(l) \quad w = \frac{\text{Btu/min.}}{\text{Btu/lb.}} = \frac{200N}{h_1 - h_4} \text{ lb./min.}$$

The condition of the refrigerant at 1, Fig. 157, is known or assumed, so that the specific volume v_1 may be determined. Evidently (w lb./min.) times (v_1 cu. ft./lb.) is the needed displacement volume V_D cu. ft. per min. for 100% volumetric efficiency. Review § 117 again. If the volumetric efficiency is η_v , we have

$$(m) \quad V_D = \frac{wv_1}{\eta_v} = \frac{v_1}{\eta_v} \left(\frac{200N}{h_1 - h_4} \right) \text{ cfm.}$$

Practical values of the volumetric efficiency should usually fall within the range 65–85%. The factors affecting volumetric efficiency in vapor compressors are much the same as those discussed in § 117. Also, the types of compressors used are reciprocating or rotary. Most of the discussion of Chapter 7 applies to refrigerant compressors, except as an ideal gas is postulated.

237. Example. An ammonia compressor receives wet vapor at 10°F and compresses it to a saturated state at 190 psia. The temperature at the expansion valve is 85°F. The compressor is double acting, 12x14 in., it runs at 200 rpm, and its volumetric efficiency at normal operating conditions is 78%. Let the compression efficiency be $\eta = 80\%$. For the ideal cycle, determine (a) the coefficient of performance γ , (b) the tons of refrigeration N , and (c) the horsepower per ton. On the basis of the actual fluid work, determine (d) γ , (e) N , (f) hp/N , and (g) the temperature at 2', Fig. 158, if the actual compression is adiabatic.

SOLUTION From the NH_3 tables in the back of this book or the complete tables of the U S Bureau of Standards, Bulletin No 42, we find for state 1 at 10°F , state 2 at 190 psia, and state 3 at 86°F

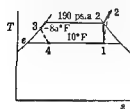


Fig 158

$$\begin{array}{lll} h_{f1} = 53.8 & s_{g1} = 1.3157 & h_{g2} = h_2 = 632.4 \\ h_{f2} = 561.1 & v_{f1} = 0.02446 & s_{g2} = s_2 = 1.1802 \\ s_{f1} = 0.1208 & v_{g1} = 7.304 & h_3 = h_4 = 137.8 \end{array}$$

From $s_1 = s_2$ we get

$$\begin{aligned} 1.1802 &= 0.1208 + 1.1949x_1, \quad \text{or} \quad x_1 = 88.7\% \\ h_1 &= 53.8 + (0.887)(561.1) = 551.5 \text{ Btu/lb} \\ v_1 &= 0.0244 + (0.887)(7.28) = 6.48 \text{ cu ft/lb} \end{aligned}$$

(a) For $W = h_2 - h_1 = 632.4 - 551.5 = 80.9$ Btu/lb and the refrigeration $Q_A = h_1 - h_4 = 551.5 - 137.8 = 413.7$ Btu/lb, we have

$$\gamma = \frac{Q_A}{W} = \frac{413.7}{80.9} = 5.11$$

(b) The displacement in cfm is, for 2π diagrams/min in a double-acting compressor,

$$V_D = \frac{\pi D^2}{4} L(2\pi) = \frac{\pi(14)(14)(2 \times 200)}{(4)(1728)} = 366 \text{ cfm}$$

For $\eta_v = 78\%$ the volume V'_1 drawn in per minute is $V'_1 = (0.78)(366)$ cfm, and for $v_1 = 6.48$ cu ft/lb, the mass of refrigerant circulated per minute is

$$w = \frac{V'_1}{v_1} = \frac{(0.78)(366)}{6.48} = 44.1 \text{ lb/min}$$

Thus, the total refrigeration is $(44.1)(413.7)$ Btu/min. Dividing this value by 200 Btu/min ton gives

$$T = \frac{(44.1)(413.7)}{200} = 91.2 \text{ tons}$$

(c) The horsepower for 44.1 lb/min of refrigerant is

$$hp = \frac{(44.1)(80.9)}{42.4} = 84.2 \text{ hp}$$

and the horsepower per ton is $84.2/91.2 = 0.942$

(d) The actual work of the fluid is $W' = W/\eta = 80.9/0.80 = 101.1$ Btu/lb, and for other conditions the same,

$$\gamma' = \frac{Q_A}{W'} = \frac{413.7}{101.1} = 4.08$$

(e) If the actual rate of circulation of refrigerant remains the same as in the ideal cycle (the horsepower of the motor must be greater than before), the tons of refrigerant are the same as before, 91.2 tons

(f) The actual fluid horsepower is the ideal hp divided by the compression efficiency, or $hp' = 84.2/0.80 = 105$; thus

$$\frac{hp'}{\text{ton}} = \frac{105}{91.2} = 1.152.$$

Including mechanical and other losses, the actual bhp per ton will be somewhat greater than this value.

(g) For an adiabatic steady flow compression 1-2', the work as a positive number is $h_{2'} - h_1 = (h_2 - h_1)/0.80$, or

$$h_{2'} = h_1 + \frac{h_2 - h_1}{0.80} = 551.5 + \frac{80.9}{0.80} = 652.6 \text{ Btu/lb.}$$

In the complete tables at 190 psia and $h = 652.6$, we find the temperature at 2' as $t_{2'} = 122.4^\circ\text{F}$.

238. Refrigerants. Table XI shows a comparison of some characteristics of popular refrigerants. Some of the desirable qualities of a refrigerant are:

1. Refrigerants are preferably nontoxic, so that in case of leakage no one is in danger of injury. This attribute is of paramount importance in air conditioning systems and home refrigerators, for example, but toxic refrigerants are tolerable in commercial installations where reasonable precautions are taken. The Carrenes, Freons (trade names) and carbon dioxide are not toxic, but the others in Table XI are in more or less degree. Methyl chloride is not only toxic, but also is practically odorless; where this refrigerant is used, a warning agent (acrolein) which is irritating to the eyes and nose should be added.

2. Refrigerants should be economical, both in initial cost and in maintenance. Maintenance problems include: controlling leakage (there is less trouble with leakage of large molecules than of small ones); providing adequate lubrication (the refrigerant should not react with the oil to destroy its lubricating qualities); and avoiding corrosion (the refrigerant should not corrode the materials which it contacts). Also the refrigerant should be readily available for recharging the system when necessary.

3. Refrigerants should be nonflammable. A number of hydrocarbons have been and are used as refrigerants, examples of which in Table XI are butane and propane. These and others (ammonia, methyl chloride, etc.) constitute a fire and explosion hazard. The other refrigerants in Table XI are nonflammable.

4. Refrigerants should have a high latent heat at the evaporator temperature (see Fig. 108, p. 206) and a low specific volume. The type and size of compressor is a function of these physical traits. If the latent heat is high (see NH_3 , Table XI), much refrigeration is done by each pound of substance circulated; if in addition the specific volume is low, the volume of substance to be circulated and therefore the size of compressor and passageways are small. Notice that when the ideal displacement volume V_D is small, reciprocating compressors are feasible; when V_D is large (Carrene 1, F 11, and F 113), centrifugal compressors (which can be run at high speed) become necessary. To some extent, however, the total amount of refrigeration to be done may affect the choice of the type of compressor to be

used Centrifugal compressors are likely to be inefficient in the small sizes. If, for example, Carrene 1 were compressed in a reciprocating machine the compressor would have to be $74 \frac{5}{3} 44 = 21.6$ times larger than an NH_3 compressor for the same refrigerating capacity.

Table XI. CHARACTERISTICS OF REFRIGERANTS

The pressures (Pres) are saturation pressures in psia, Q_A , w , and V_D correspond to the case where the fluid leaves the evaporator as saturated vapor (state 1, Fig. 258, is on the saturated vapor line), and for the liquid entering the throttling valve at 86°F , which is a typical condenser temperature. The displacement is for 100% volumetric efficiency, the condenser and evaporator temperatures are 5°F and 86°F , respectively. Under "Type of Compressor". Rec = reciprocating, Rot = rotary, Cen = centrifugal.

NOTES. (a) An azeotropic mixture of Freon 12 and $\text{C}_2\text{H}_4\text{F}_2$; (b) Dichloromethane; also called methylene chloride; (c) Trichloromonofluoromethane; (d) Dichlorodifluoromethane; (e) Monochlorodifluoromethane; (f) Trichlorotrifluoroethane; (g) Dichlorotetrafluoroethane.

	Formula	Pres at 5°F	Pres at 86°F	Q_A Btu/lb	w lb/min-ton	V_D cfm/ton	Type of Compressor
Ammonia	NH_3	34.3	169.2	474.4	0.42	3.44	Rec
Butane	C_4H_{10}	8.2	41.6	123.5	1.62	16.2	Rec, Rot
Carbon dioxide	CO_2	332.2	1045.7	56.7	3.53	0.94	Recip
Carrene 1 (b)	CH_2Cl_2	1.16	10.6	134.1	1.49	74.5	Rot, Cen
Carrene 7 (a)	(a)	31.1	128.1	59.8	3.34	6.2	Rec, Rot
Freon 11 (c)	CCl_3F	2.03	18.3	67.5	2.96	36.4	Rot, Cen
Freon 12 (d)	CCl_2F_2	26.5	107.9	51.1	3.92	5.8	Rec, Rot
Freon 22 (e)	CHClF_2	43.0	174.5	69.3	2.89	3.59	Rec
Freon 113 (f)	$\text{C}_2\text{Cl}_3\text{F}_3$	0.98	7.9	53.67	3.72	100.5	Cen
Freon 114 (g)	$\text{C}_2\text{Cl}_2\text{F}_4$	6.8	36.7	43.1	4.64	19.0	Rot
Methyl chloride	CH_3Cl	21.2	94.7	150.3	1.33	5.95	Rec, Rot
Sulfur dioxide	SO_2	11.8	66.4	141.4	1.41	9.08	Rec, Rot
Propane	C_3H_8	41.9	156.2	123	1.63	4.1	Rec

5. Refrigerants should have low saturation pressures at normal operating temperatures. The cost of design, manufacture, and operation is involved. Very high pressures, as in the case of CO_2 , mean heavy parts and thick-walled pipes (A compensating factor in the case of CO_2 is the small V_D required per ton of refrigeration.) Also, it is preferable that the saturation pressure at the evaporator temperature be slightly above atmospheric pressure, in order to be sure of no leakage of air into the system. Carrene 1 and Freon 113 have saturation pressures below atmospheric even at the condenser temperature of 86°F . Since an evaporator temperature of 5°F is relatively low for many purposes of refrigeration, a higher evaporator temperature may be permissible and result in some of the refrigerants

being at or above atmospheric pressure in the evaporator; for example, butane is approximately at atmospheric pressure when the saturation temperature is at 30°F, Freon 114 at 39°F, and sulfur dioxide at 14°F. Also important is a low *pressure ratio* p_2/p_1 , because of the effect of this ratio on the work necessary for compression.

6. While the foregoing attributes are perhaps the most significant, there are miscellaneous other characteristics which are desirable (124): good thermal conductivity for rapid heat transfer, wetting ability, inertness (the refrigerant should not react in any way with the materials it touches), stability (the refrigerant should not break down into different matter of smaller molecules), low viscosity (for ease of movement), high critical temperature, and a high dielectric strength (in hermetically sealed units where the refrigerant contacts the motors). Also, it stands to reason that the refrigerant should not solidify at any temperature in its cycle.

The traditional refrigerants are still widely used: *ammonia* in industrial and commercial refrigeration; carbon dioxide, because of its low critical temperature of 87.8°F, is used principally in the northern latitudes and for low temperature refrigeration (solid CO₂, known as dry ice, is widely used for cooling purposes); sulfur dioxide is extensively used in industrial applications because of its advantageous thermodynamic traits, but it is toxic.

The "newer" refrigerants (for example, the Freons, often designated by the letter F and a number, as F 12) appeared when physical chemists found that chlorine or fluorine atoms could be substituted for hydrogen atoms in certain hydrocarbon molecules (the methane and ethane series). When substances evolved which were neither flammable nor toxic and had saturation temperatures and pressures suitable for refrigerants, they were found to be more desirable in many situations than the traditional refrigerants. They are extensively used and can be manufactured with nearly any desired property, though not necessarily with all the properties desired. For household refrigerators and air conditioning, F 11, F 12, F 114, and Carrene 7 are common.

A significant commercial advantage of the line of newer refrigerants is that a manufacturer may use the same compressor for different capacities by changing the refrigerant and installing a different size of motor. For example, for a particular displacement, a greater refrigerating effect is obtained from F 22 than from F 12; greater power is also required for the F 22.

239. Absorption Systems of Refrigeration. Absorption systems are characterized by the fact that the refrigerant is absorbed by an absorbent on the low-pressure side of the system and is given up on the high-pressure side. The advantage derived from these additional processes is that the work supplied to the system needs to be only enough to pump a liquid from the low-pressure to the high-pressure region of the system. We have had occasion to note several times before that the work to pump a particular mass of gas or vapor is much larger than the work to pump a like amount of liquid.

depicted in Fig 160. For the larger temperature differences between the condenser and evaporator, a regenerative process may be worth the money. In operation, the first expansion valve lowers the pressure part way, e to r . The saturated vapor, indicated by the quality at r , Fig 160(b), mixes with the superheated vapor b (or b') resulting in the condition c at entry to the second stage of compression. Compression cd (or cd') then occurs, whence the refrigerant, $1 + m$ lb, enters the condenser. While the m lb of vapor leaves the flash chamber, the 1 lb of remaining liquid moves through the second expansion valve h_2 and enters the evaporator where the refrigeration occurs. Thus, in the diagram of Fig 160, it is intended that calculations be based on one pound of refrigerant (per any unit of time) doing refrigeration. Actual computations will show that it is more efficient than would be

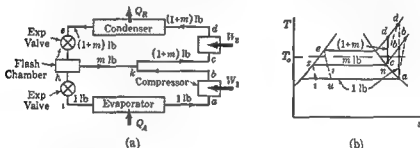


Fig 160 Two Stage Compression with Regenerative Intercooling. The discharge from the low pressure stage at b may be circulated through the flash chamber with a resulting lower temperature at c .

the case for single stage compression from a to the pressure line p_d and throttling from e to u . An energy balance for the mixing operation at k , Fig 160(a), is

$$(n) \quad h_b + mh_n = (1 + m)h_c$$

In the flash chamber, Fig 160(a),

$$(o) \quad (1 + m)h_c = h_d + mh_n$$

Also, the refrigeration Q_A and the total work W are

$$(p) \quad Q_A = h_a - h, \text{ Btu/lb,}$$

$$(q) \quad W = W_1 + W_2 = h_b - h_a + (1 + m)(h_d - h_c) \text{ Btu/lb,}$$

both per pound in the evaporator.

If the cold body is to be maintained at a very low temperature, it may be advisable to use two or more separate refrigerating systems with different refrigerants, as suggested by Fig 161, where the intermediate heat exchanger acts as both a condenser and an evaporator. If w_1 is the mass of refrigerant

circulated per unit of time in the low-temperature cycle (this is determined by the amount of refrigeration required), then the amount of refrigerant

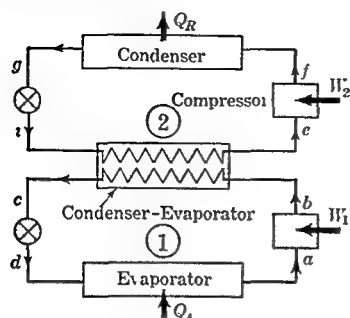
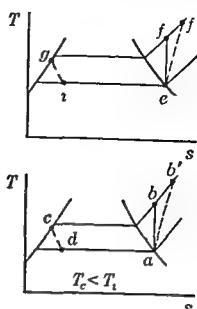


Fig. 161. Cascade Refrigerating System.



w_2 for the other cycle must be such as to make a heat balance in the condenser-evaporator;

$$(r) \quad w_1(h_b - h_c) = w_2(h_e - h_i).$$

In this system, called the *cascade system*, the number of stages may be more than two. The successive refrigerants should be such that the triple-point temperature of the "higher" one is lower than the critical temperature of the "lower" refrigerant.

241. Liquefying Gases. In order for a gas to exist as liquid, its temperature must be reduced below its so-called critical temperature, some values of which are given in § 172, p. 208. For example, nitrogen must be at or below a temperature of -233°F . There are various means used to attain temperatures of this order, one of which is to use the cascade idea (Fig. 161). One cascade system designed to produce liquid N_2 is made up of four refrigerating systems in series, the refrigerants being NH_3 , C_2H_4 , CH_4 , and N_2 , in that order. Similar arrangements are used to liquefy other gases. One method of producing oxygen is to liquefy air and then separate the oxygen by fractional distillation.

Some of the "permanent" gases can be liquefied with equipment simpler than the cascade refrigerating system of four refrigerants. The simplest method of doing this is the Linde process, diagrammed in Fig. 162. Gases

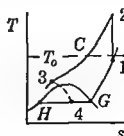
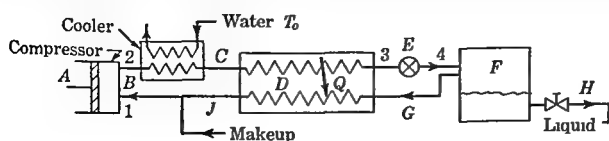


Fig. 162. Linde System of Gas Liquefaction.

which solidify at a temperature higher than the desired liquid temperature are removed from the make-up, for example, cleaning the air of CO_2 and H_2O before it is added to the liquefying system. The compressor A delivers air for example, at pressures of 100 to 200 atm. If the discharge temperature at B is greater than that of the naturally available sink, the air passes through a cooler BC , which lowers its temperature to approximately that of the environment. Then it passes CE through a regenerative heat exchanger D , wherein its temperature is lowered to some value in the vicinity of its critical temperature to about -221°F for air. The expansion valve E throttles the gas to atmospheric pressure $\Delta h = 0$, after which some of it is liquid and some is vapor. The liquid about 10% of the fluid entering F after steady state has been attained is drawn off at H as desired and the vapor passes GJ back through the heat exchanger acting as the heat receiver for the heat given off from C to E . The vapor at J , together with

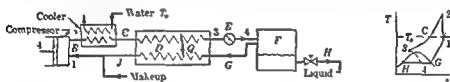


Fig 162 Repeated

the make-up re-enters the compressor. If instead of an expansion valve E an engine is used and work is produced more liquid per unit of work expended on the system is produced. At the low temperatures involved lubrication difficulties practically preclude a reciprocating engine but a turbine may be used. Only a part of the flow is likely to be expanded through an engine and this part at its low temperature is used for a further regenerative cooling of the part which goes directly to the expansion valve. The reader who has followed this text from the beginning should have little trouble in writing equations for mass balances and energy balances from Fig 162 as desired. Notice the process 1-2-3-4 on the Ts plane.

In order for a gas to become cooler on passing through an expansion valve, the value of its Joule-Thomson coefficients $(dT/dp)_h$ must be positive (See footnote on p 62). For this to be so the temperature of the gas must be below its maximum inversion temperature which is roughly a reduced temperature of 5 that is the inversion temperature is approximately five times the critical temperature (T_c). The inversion temperature is that temperature above which the Joule-Thomson coefficient is negative below which it is positive. Usually the value of the inversion temperature presents no problem because it is high enough to be 'out of the way'. However it must be considered in liquefying hydrogen inversion temperature of 351°R , and helium, whose inversion temperature is 42.5°R . Throttling of

these gases at normal temperatures results in a temperature increase. To liquefy helium, we cool it below 42.5°R with liquid hydrogen, which has been obtained by cooling H_2 below about 351°R with liquid nitrogen.

242. Reversed Cycle for Heating. When the reversed cycle is used for heating, it is popularly called a *heat pump* (all reversed cycles are heat pumps). As usual, the Carnot cycle furnishes a simple means of understanding the flow of energy and provides the highest conceivable standard of performance. In Fig. 163, consider the working substance in some condition c , a few degrees above room temperature, and let it expand isentropically to a temperature t_d , below outside atmospheric temperature. The working substance may now be led through outside coils and heat will flow from the outside air (or earth or water) into the substance, process da . In condition a , the substance is compressed to a temperature t_b , above room temperature,

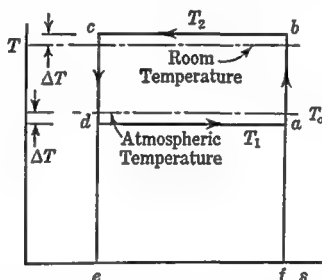


Fig. 163. Heating Cycle. This cycle approaches external reversibility as the ΔT 's approach zero.

and led into inside coils; so heat now flows from the substance into the room, process bc . For the heating cycle, we have

$$Q_A = T_1(s_a - s_d), \text{ heat taken from the sink;}$$

$$Q_R = T_2(s_a - s_d), \text{ heat rejected to the room;}$$

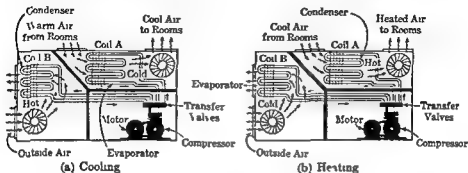
$$W = (T_2 - T_1)(s_a - s_d), \text{ work for the cycle;}$$

$$(s) \quad \gamma = \frac{\text{output}}{\text{input}} = \frac{\text{heating effect}}{\text{work}} = \frac{T_2}{T_2 - T_1},$$

the highest possible coefficient for the specified temperatures. If a coefficient of performance is $\gamma = 5$, and if the work is 100 Btu, the heat rejected to the room will be 500 Btu. We see that this system provides a means of delivering an amount of heat much in excess of the heat equivalent of the work expended. Note that the coefficient of performance of a reversed cycle depends upon whether the objective (output) is cooling or heating.

There are several commercial machines available which are designed to operate as air conditioners both summer and winter—cooling the inside air during the summer and heating it during the winter, by merely throwing a

switch. See Fig 164. During the cooling season, the outside coil *B* acts as the condenser, receiving the discharge from the compressor. The coil *A* is the evaporator and it cools the house air which flows over it. For heating, the coils *A* and *B* swap functions, the coil *A* becomes the condenser, adding heat to the house air which circulates over it, the coil *B* is the evaporator, picking up heat from the outside.



Courtesy General Electric Schenectady, N. Y.

Fig 164 Heat Pump for Heating and Cooling

It is not often that the reversed-cycle machine is an economic means of heating alone, but where refrigeration is needed for other purposes such as air conditioning, it will be easier to justify its use. Favorable factors include low rates for electricity, a mild climate, the availability of relatively warm water as a source of heat, and, in a negative way, expensive heating fuels. The unit operates more efficiently if the summer condenser, coil *B*, is cooled by water. In small installations, the extra cost of water may more than offset the loss of efficiency which goes along with air cooling.

243 Closure As usual in chapters dealing with a specialty, only the basic elements of refrigeration are given here. One must refer to books and current literature devoted to the subject for more detail. [See references (114-126)]

15

GASEOUS MIXTURES

244. Introduction. Since you have already worked many problems with air as the working substance and since air is a mixture of gases, you already know how to handle mixtures of ideal gases—given the appropriate characteristics (R , c_p , c_v , etc.) of the mixture. Review §§ 43 and 56 now, noting Avogadro's law, the universal gas constant, the meaning of a mol, etc.

The symbol M represents molecular weight; thus, M_x is the molecular weight of gas X . A mol of a substance is M lb. or M gm. of the substance. These mols are different and to distinguish between them, we say pound mol (lb. mol) and gram mol (gm. mol or g-mol). However, we shall use only the pound mol in this book and will therefore speak of *mols* without the defining adjective. It will be helpful now to think of the units of M as

$$(a) \quad M \rightarrow \text{lb./mol}; \quad N \text{ mols have a mass of } NM \text{ lb.},$$

where N is the number of mols of substance with molecular weight M .

A mixture of gases is considered to have an *equivalent* or *apparent molecular weight*; thus, air is said to have a molecular weight of 28.97. If the equivalent molecular weight of a mixture of gases is known, the specific gas constant R is found from

$$(b) \quad R = \frac{1545}{M},$$

where $1545 = \bar{R}$ ft.-lb. per mol-°R is the universal gas constant.

245. Characteristic Equation of Ideal Gas in Terms of Mols. Recall that at a specified pressure and temperature, the volume of one mol of one ideal gas is the same as the volume of one mol of *any other* ideal gas, § 56. From $p v = R T$, we get $p M v = M R T = \bar{R} T$, where M is the molecular

weight of the gas or mixture of gases The volume of one mol is $v = V/n$ and if there are n mols of gas the ideal gas equation becomes

$$(c) \quad \underset{[1 \text{ mol}]}{p} = \underset{[1 \text{ mol}]}{\frac{RT}{v}} \quad \text{or} \quad pV = nRT = 1545nT$$

where V is the total volume p is in psf and T in $^{\circ}\text{R}$ Equation (c) is applied to a single component or to a mixture of gases Applied to component x we have $p_x V_m = N_x RT_m$ where p_x is the partial pressure of the gas x , N_x is the number of mols of x , V_m and T_m apply to the mixture and the component

246 Mixtures of Gases The individual gases or vapors in a mixture are called *constituents* or *components* The description of the mixture is given by a *volumetric analysis* or by a *gravimetric analysis* Sometimes one analysis is useful sometimes the other hence we must be able to convert from one to another with ease

The *volumetric analysis* expresses the amounts of the components in the mixture by the percentages of the total volume which each component would occupy if the various gases were placed in separate compartments at

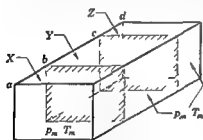


Fig 165 Total Volume is V_m
When $V_m = 1$ cu ft. the volume of
gas X is B_x of gas Y B_y of gas Z B_z

the pressure p_m and temperature T_m of the mixture That is in Fig 165 imagine gases x , y and z separated by partitions at b and c each one at pressure p_m and temperature T_m Thus each one occupies a certain percentage or fraction of the entire volume represented by B_x , B_y and B_z (the symbol B suggesting bulk) Now if the partitions are removed and the gases mix we say that the volumetric percentages of the gases x , y and z are B_x , B_y and B_z

It is important to note that if the vessel in Fig 165 contains one mol of the total mixture the mols of gases x , y and z are B_x , B_y and B_z That is the volumetric fraction B_x is the number of mols of gas x in a mixture which totals one mol $B_x + B_y + B_z = 1$ The *volumetric percentage* B_x is the number of mols of x in a mixture which totals 100 mols

$$B_x + B_y + B_z = 100$$

when all gases are at the same temperature In terms of the actual known mols of each gas n_x , n_y and n_z we write

$$n_m = n_x + n_y + n_z$$

where N_m is the total number of mols in the mixture. Notice that

$$(d) \quad B_x = \frac{N_x}{N_m}, \quad B_y = \frac{N_y}{N_m}, \quad \text{and} \quad B_z = \frac{N_z}{N_m}$$

are the percentages by volume of the constituents X , Y , and Z , respectively. The volumetric percentage is also defined by

$$B_x = \frac{\text{cu. ft. of gas } X \text{ at } p_m \text{ and } T_m}{\text{cu. ft. of mixture at } p_m \text{ and } T_m},$$

and so on for gases Y and Z .

The *gravimetric analysis* describes the mixture by giving the percentages by *weight* (mass) of each constituent. For example, the gravimetric percentage or fraction of gas X is

$$G_x = \frac{\text{mass of gas } X}{\text{total mass of the mixture}}.$$

For a mixture of gases X , Y , and Z , the gravimetric analysis is $G_x\%$ of X , $G_y\%$ of Y , and $G_z\%$ of Z , each expressed as a percentage of the mass; G also stands for the fractional part of the mass.

247. Dalton's Law of Partial Pressures. It was John Dalton (1766–1844) who first stated that the total pressure p_m exerted by a mixture of gases (or vapors) is the sum of the pressures which each gas (or vapor) would exert were it to occupy the vessel alone. The pressure which one gas in a mixture exerts is called its *partial pressure*. Thus, if p_x , p_y , and p_z represent respectively the partial pressures of the mixed gases X , Y , and Z , Dalton's law states

$$(83) \quad \begin{aligned} p_m &= p_x + p_y + p_z + \cdots \\ [T_m &= T_x = T_y = T_z, \quad V_m = V_x = V_y = V_z] \end{aligned}$$

The characteristic equation (c) of an ideal gas in terms of the number of mols is $pV = 1545NT$. Applying this equation to a component X and to the mixture, we get

$$(e) \quad p_x V_m = 1545 N_x T_m \quad \text{and} \quad p_m V_m = 1545 N_m T_m.$$

By division with equations (e), we have

$$\frac{p_x V_m}{p_m V_m} = \frac{1545 N_x T_m}{1545 N_m T_m} \quad \text{or} \quad \frac{p_x}{p_m} = \frac{N_x}{N_m} = B_x.$$

In the foregoing, we used the following principles: If the mixture is in internal thermal equilibrium *all gases are at the same temperature* T_m ; *each gas occupies the same volume* V_m ; we recognized that the number of mols N_x of X divided by the total number of mols N_m is the volumetric percentage; and

we have arrived at the quite useful relation that the volumetric fraction B is equal to the ratio of the partial pressure divided by the total pressure, or

$$(84) \quad p_x = B_x p_m, \quad p_y = B_y p_m, \quad p_z = B_z p_m,$$

true for ideal gases and useful for actual gases

248. Molecular Weight and Gas Constant for Mixture Given N_x or B_x mols of X , N_y or B_y mols of Y , and N_z or B_z mols of Z , the corresponding masses are

(1)	(2)
$N_x M_x$ lb	$B_x M_x$ lb /mol mixture
$N_y M_y$ lb	$B_y M_y$ lb /mol mixture
$N_z M_z$ lb	$B_z M_z$ lb /mol mixture
<hr/> Total = ΣNM lb	<hr/> ΣBM lb /mol mixture (M_m)

If $B_x + B_y + B_z = 1$ in column (2), there is one mol of mixture and the mass in pounds per mol is the molecular weight [equation (a)], thus, $\Sigma BM = M_m$, the equivalent molecular weight of the mixture. By similar reasoning, there are $N_m = N_x + N_y + N_z$ mols in column (1), and $M_m = \Sigma NM / N_m$, or

$$(f) \quad M_m = B_x M_x + B_y M_y + B_z M_z = \Sigma BM \text{ lb /mol},$$

$$(g) \quad M_m = \frac{N_x M_x + N_y M_y + N_z M_z}{N_m} = \frac{\Sigma NM}{N_m} \text{ lb /mol},$$

where, in ΣBM and ΣNM , B and N apply to the individual gases. Also, equations (65) and (66), we see that $\Sigma BM = 1/(\Sigma G/M)$, hence

$$(h) \quad M_m = \frac{1}{\Sigma G/M} \quad \text{and} \quad R_m = \frac{1545}{M_m} \text{ ft lb /lb } ^\circ\text{R}$$

249 Example A mixture of 64 lb of O_2 , 28 lb of N_2 , and 16 lb of H_2 is at 140°F and 15.45 psia. What volume does the mixture occupy?

SOLUTION The easiest way to find the volume (if one does not know the standard mol volume) is to determine first the number of mols of mixture thus

$$N_o = \frac{64 \text{ lb}}{32 \text{ lb /mol}} = 2 \text{ mol } O_2, \quad N_N = \frac{28 \text{ lb}}{28 \text{ lb /mol}} = 1 \text{ mol } N_2$$

$$N_H = \frac{16 \text{ lb}}{2 \text{ lb /mol}} = 8 \text{ mol } H_2,$$

or total number of mols is 11

$$V_m = \frac{1545 N_m T_m}{p_m} = \frac{(1545)(11)(600)}{(15.45)(144)} = 4170 \text{ cu ft}$$

250. Energy Changes of a Mixture of Gases. If we take the heat transferred to or from one pound of a mixture $dQ = c_m dT$ as the sum of the transferred heats for the components, whose masses are G_x , G_y , and G_z lb. per lb. of mixture, we have

$$c_m dT = G_x c_x dT + G_y c_y dT + G_z c_z dT,$$

or

$$(85) \quad c_m = G_x c_x + G_y c_y + G_z c_z = \Sigma G c,$$

where c_x , c_y , and c_z Btu per lb. are instantaneous or average specific heats of gases X , Y , and Z , respectively, for some particular process, and C_m is the specific heat of the mixture for the same process. Equation (85) is also true for variable specific heats.

The *molal* or *molar* specific heat C_m Btu per mol of a mixture is the sum of the products of the individual molal specific heats and the corresponding fractional part of the mol;

$$(i) \quad C_m = B_x C_x + B_y C_y + B_z C_z = \Sigma B C \text{ Btu/mol},$$

where C_m is the molal specific heat of the mixture for a particular process when C_x , C_y , and C_z (etc.) are the molal specific heats of the constituents for the same process (C_x is the specific heat of a mol of gas X , and B_x is the fractional part of the mol of gas X , for example). Also $C_m = M_m c_m$.

It is logical to expect that the total internal energy of a mixture of gases is the sum of the internal energies of the components, energy being a scalar quantity;

$$(j) \quad u_m = G_x u_x + G_y u_y + G_z u_z = \Sigma G u \text{ Btu/lb.}$$

And using a bar over the symbol to distinguish values for a mol, we have

$$(k) \quad \bar{u}_m = B_x \bar{u}_x + B_y \bar{u}_y + B_z \bar{u}_z = \Sigma B \bar{u} \text{ Btu/mol}$$

of mixture. It is sometimes convenient to work with a known total number of mols N ; thus

$$(l) \quad N_m \bar{u}_m = N_x \bar{u}_x + N_y \bar{u}_y + N_z \bar{u}_z = \Sigma N \bar{u},$$

where \bar{u} is the internal energy per mol, $N_m = N_x + N_y + N_z$ is the number of mols in the mixture, and N_x , N_y , and N_z (etc.) represent the number of mols of each individual gas. We remember that $N_x/N_m = B_x$. Notice that equations (f) and (i) can also be written in terms of the total number of mols, after equation (l).

Equations analogous to (j), (k), and (l) can be written for enthalpy. Gibbs' principle is that *the total entropy of a mixture is the sum of the entropies of the individual constituents of the mixture*. Hence, we may write equations for entropy s and \bar{s} analogous to equations (j), (k), and (l). Note, however,

that if two gases which are separated by a partition are allowed to mix by removing the partition, an increase of entropy results because the diffusing process is irreversible. Treating each component separately is often the convenient way, but if many calculations are to be made for a particular mixture it would be easier to get the specific heats (λ , R , etc.) of the mixture and treat the mixture as we have already learned to treat air.

251 Gas-Vapor Mixtures There are many instances where the engineer must deal with mixtures of gases and vapors—that is, mixtures in which one component may condense and settle from the mixture or an additional amount may evaporate into the gaseous mixture. There is the mixture of air and gasoline vapor from the carburetor of an automobile engine. In the products of combustion of hydrocarbon fuels there is a relatively large percentage of water vapor much of which will condense if the products are cooled to atmospheric temperatures. There are many industrial processes of gaseous mixtures some of whose components may be easily condensable. Of more general interest is normal atmospheric air which invariably contains some water vapor usually superheated steam. You have inferred by now that the expression a mixture of gases and vapors means that one or more components of the mixture may condense in more or less amount during a process which the mixture is undergoing or at least one component is acting so unlike an ideal gas as to make some of the ideal gas equations inapplicable. Mostly we shall be concerned below with mixtures where some condensation or evaporation occurs during the process.

All the gases in the mixture the noncondensing components we shall consider as the dry gas abbreviated dg. The condensable component shall be spoken of as the vapor abbreviated v. Sometimes in discussing air mixtures we shall use dry air da to mean the noncondensing components of the atmosphere. There are certain laws and principles which we shall assume to be true. Try to fix the following in mind.

1 Dalton's law (§ 247) the sum of the partial pressures of the constituents is equal to the total pressure of the mixture (liquid is not included of course)
 $p_m = p_v + p_g + p_s + \dots$ where the constituents are V, 1, and Z

2 The volumetric percentage of vapor is $\lambda_v/(\lambda_v + \lambda_g) = \lambda_v/\lambda_m$ the volumetric percentage of dry gas is $\lambda_g/(\lambda_v + \lambda_g) = \lambda_g/\lambda_m$ where λ_v = mols of vapor, λ_g = mols of dg, λ_m = total mols of mixture [Equation (d) § 246]

3 The partial pressure of a constituent in a mixture of ideal gases is equal to its fraction by volume (or mol fraction) multiplied by the pressure p_m of the mixture (total pressure) § 247. We have

$$(m) \quad p_v = B_v p_m = \frac{\lambda_v p_m}{\lambda_v + \lambda_g} \quad \text{and} \quad p_g = B_g p_m = \frac{\lambda_g p_m}{\lambda_v + \lambda_g}$$

where λ_v = mols of vapor, λ_g = mols of dg, p_v and p_g are partial pressures of the vapor and dry gas respectively, p_m = total pressure (See § 246)

4. Each gas or vapor in a mixture occupies the same volume; $V_m = V_x = V_a$, etc. The volume of any one of the constituents at its partial pressure is the volume of the mixture. Often this principle affords the easiest way to find the volume of a mixture.

5. Each gas and vapor in a mixture is at the same temperature—internal thermal equilibrium obtains; $T_m = T_x = T_y$, etc.

6. The density of a mixture is the sum of the densities of the individual constituents; $\rho_m = \rho_x + \rho_y + \rho_z + \dots$.

7. The per cent or fraction by weight (gravimetric percentage) of a particular constituent in a mixture is the density of the constituent *divided by* the density of the mixture; ρ_x/ρ_m , ρ_y/ρ_m , etc.

252. Dew Point. Imagine a container *A*, Fig. 166(b), in which there is superheated vapor (say, H_2O), state 1, Fig. 166(a). If this container is placed in colder surroundings so that heat flows outward, the superheated

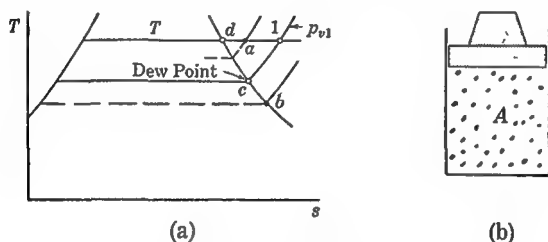


Fig. 166. Dew Point and Relative Humidity.

vapor is cooled at, say, constant pressure, path 1-c. If heat is further rejected after arrival at state *c*, where the vapor is saturated, some of the vapor will condense. The state *c* is the condensation point or *dew point* at constant pressure with respect to the initial state 1. Thus, any superheated vapor has a dew point which depends on the initial state and the process by which the saturated vapor line is reached.

In atmospheric air, the H_2O is generally superheated steam. If this air is cooled at a total constant pressure, the steam cools at constant pressure (all components of the mixture being taken as ideal gases). Some temperature is eventually reached below which condensation of the H_2O in the air occurs; this temperature is said to be the *dew point of the air*, but from the preceding paragraph we see that it is the dew point of the H_2O in the air. As explained later, we have means of determining the partial pressure p_{v1} of the H_2O vapor in the air. Knowing p_{v1} , we find the dew point from the vapor tables by looking up the saturation temperature corresponding to p_{v1} ; and that is it. (You have often noticed drops of water on the cold windowpane in the winter because the adjacent air has been cooled below its dew point.)

If atmospheric air is cooled at total constant pressure below its dew point *c* (say, to state *b*, where the temperature is t_b), some of the vapor

necessarily condenses but the vapor which remains at t_1 is saturated and at a pressure lower than p_{s1} the vapor pressure p_{s2} is the saturation pressure corresponding to the temperature t_2 . During cooling below the dew point at constant total pressure the partial pressures do *not* remain constant.

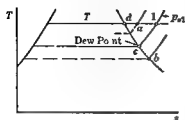


Fig 166(a) Repeated

If p_m is the total pressure of the mixture and p_v is the vapor pressure the pressure of the dry gas p_g is taken as $p_g = p_m - p_v$.

We shall generally assume that the liquid formed by cooling a gas vapor mixture below its dew point settles out which it will do given time and that the vapor left is saturated vapor (Atmospheric mists contain small bits of liquid H_2O which means that the vapor is wet vapor.)

253 Relative Humidity Consider again the container *A* Fig 166 with superheated vapor in it. Suppose a small amount of liquid is injected and the system is manipulated in such a way as to maintain constant temperature (This can be done by letting the system come into thermal equilibrium with its environment and then giving it time to do so again after additional liquid has been injected.) Perhaps the first injection is such an amount that it all vaporizes and the final equilibrium state is *a*. If just the right amount of liquid is introduced and vaporized at constant temperature the equilibrium state becomes saturated vapor at *d*. What the foregoing discussion suggests is that a saturated vapor state can be reached at constant temperature by evaporating more liquid into the space *A* Fig 166. The ratio of the vapor pressures at *1* and *d* p_1/p_{s1} is called the *relative humidity* ϕ . Notice that the relative humidity of any superheated vapor can therefore be expressed as the actual pressure of the vapor divided by the saturation pressure corresponding to the temperature of the vapor.

Returning to a gas vapor mixture we conclude that if additional liquid is vaporized into the mixture (at constant temperature t_m and total pressure p_m) the vapor becomes less superheated its partial pressure increases and if the process is continued the vapor in the gas becomes saturated vapor.

In applying these notions to atmospheric air we say that air which contains saturated steam (state *d* *c* *b* or any other on the saturated vapor curve) is *saturated air* but we mean that the *steam* is saturated. However such air (or other gas) is saturated with steam (or other vapor) in the sense that *there cannot be an increase in the amount of steam (or vapor) in the air (or gas) as long as the total pressure and temperature of the mixture remain the same*.

The partial pressure of the steam in the air is ordinarily a fraction of a pound per square inch (0.5069 psia for saturated air at 80°F less if the

"air" is not saturated; see your steam tables). At such low pressures, the molecules are so far apart that they exert a negligible force on one another and occupy negligible space; thus, the steam acts very much like an ideal gas,

$$pv \approx RT, \quad pV \approx N\bar{R}T = 1545NT, \quad \text{and} \quad p \approx \rho RT,$$

where N is the number of mols. We shall find the density $\rho = 1/v$ of convenience in this chapter. Thus, for the vapor at states 1 and d , we have $p_{v1}/p_{vd} \approx (\rho_{v1}R_vT)/(\rho_{vd}R_vT)$, and the relative humidity is

$$(86) \quad \phi = \frac{p_{v1}}{p_{vd}} \approx \frac{\rho_{v1}}{\rho_{vd}} = \frac{v_{vd}}{v_{v1}},$$

where the subscripts refer to Fig. 166, and where the approximation is often suitably accurate at low pressures. For steam, the ideal gas laws yield reasonably good results when its pressure is below 1 psia. At temperatures below 32°F, the partial pressures of saturated steam are given in Keenan and Keyes (71). Accurate properties of air-steam mixtures are found in the ASHVE *Guide* (127).

254. Humidity Ratio. We shall find it convenient to base calculations on a unit mass of dry gas (dg.), because the mass of the vapor, and therefore of the mixture, often varies; but the mass of dry gas remains constant. Thus, a convenient term is the *mass of vapor per pound of dry gas*, called the *humidity ratio* ω in connection with air-steam mixtures; $\omega = w_v/w_g$. (The name may be extended to include other mixtures.) This value can be found from

$$(87) \quad \omega = \frac{\rho_v}{\rho_g} \rightarrow \frac{\text{lb. vapor/cu. ft.}}{\text{lb. dg./cu. ft.}} = \frac{\text{lb. v.}}{\text{lb. dg.}},$$

in which $\rho_g = p_g/(R_gT_g)$, where p_g is the *partial* pressure of the dry gas; $p_g = p_m - p_v$. The density of saturated vapor is $\rho_v = 1/v_v$, in which v_v is taken from vapor tables. The density of superheated vapor can be obtained from equation (86) when the relative humidity is known; $\rho_{v1} = \phi\rho_{vd}$, where ρ_{vd} is the density of *saturated* vapor at the actual temperature of the superheated vapor, t_1 and T , Fig. 166. Since $\rho = 1/v$, we also have $\omega = v_g/v_v$.

When the vapor is in a state approximating an ideal gas, we may use $\rho = p/(RT)$ in equation (87) to get

$$(n) \quad \omega = \frac{\rho_v}{\rho_g} = \frac{(p_v)(R_gT_g)}{(R_vT_v)(p_g)} = \frac{p_vR_g}{p_gR_v} = \frac{p_vR_g}{R_v(p_m - p_v)} \frac{\text{lb. v.}}{\text{lb. dg.}},$$

where for internal equilibrium, $T_g = T_v$, and where the partial pressure of the gas is equal to the pressure of the mixture minus the pressure of the vapor, $p_g = p_m - p_v$. Equation (n) may be applied to gas-vapor mixtures

in any state, provided that the partial pressure of the vapor is low. Another useful form of equation (n) is obtained by solving for p_v ,

$$(o) \quad p_v = \frac{p_m \omega R_v}{R_a + \omega R_v},$$

where the units of p_v and p_m are the same. Equation (o) is applied to a particular state, as *l* or *d* or *c*, Fig. 166. If equations (n) and (o) are applied to an air-steam mixture, we use $R_a = R_g = 53.3$ and $R_v = \frac{1545}{18} = 85.7$ for H_2O , whose molecular weight is approximately 18, and find

$$(p) \quad \omega = \frac{53.3 p_v}{85.7 p_a} = \frac{0.622 p_v}{p_m - p_v} \quad \text{or} \quad p_v = \frac{p_m \omega}{0.622 + \omega};$$

[AIR-STEAM MIXTURE ONLY]

where ω is pounds of vapor per pound of dry gas. The percentage by volume, or the mol fraction of vapor in the mixture, is $[\omega \text{ lb v} / (M_v \text{ lb/mol})] = \text{mol of vapor, 1 lb da} / (M_v \text{ lb/mol}) = \text{mol of air}]$

$$B_v = \frac{\omega/18}{\omega/18 + 1/28.97}, \quad \text{also} \quad B_v = \frac{p_v}{p_m}$$

255 Wet-Bulb Temperature The relative humidity of atmospheric air, that is the relative amount of steam in the air, is easily found via an experimental determination of a *wet-bulb temperature* t_w . The actual temperature of the air is called the *dry-bulb temperature* t_d , because it is of course read from a thermometer whose bulb is dry. The instrument used for the purpose of determining t_w is called a *psychrometer*, of which there are several types. The commonest one is a sling psychrometer, which consists of two thermometers attached to a handle so that they may be easily whirled about the axis of the handle. One thermometer has on its bulb a wet gauze and is called a *wet-bulb thermometer*. The temperature t_w indicated by the wet bulb thermometer depends upon the rate of evaporation of moisture from the wet gauze. The heat necessary to evaporate this moisture is supplied by the air passing over the gauze (see § 256). Consequently, this air (and water) are cooled below the atmospheric temperature, the amount by which it is cooled, $t_d - t_w$, being called the *wet-bulb depression*.

The rate of evaporation of the water depends in part on the amount of steam already in the air. If the air is saturated, none of the water on the gauze evaporates because the air is already a saturated mixture with respect to the steam, the wet bulb and dry bulb temperatures are the same. The less "moisture" (steam) carried by the air, the more that must be evaporated in order to result in saturation, hence, the rate of evaporation being

greater, the wet-bulb temperature will be lower. The higher the relative humidity, the slower the rate of evaporation and the higher the wet-bulb temperature. With the wet-bulb and dry-bulb temperatures, we may enter a *psychrometric chart*, Fig. 167 (see *Problems on Thermodynamics* for a working chart), and find the amount of steam in the air, the relative

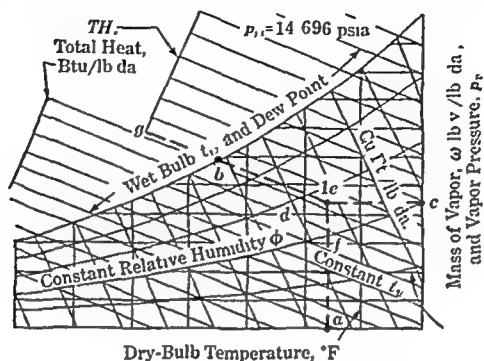


Fig. 167. *Form of Psychrometric Chart.* This chart may be entered with various data. Suppose the wet- and dry-bulb temperatures are known; find the wet-bulb temperature at *b*, the dry-bulb at *a*, and follow constant temperature lines until they meet at state 1. Move to right or left to *c* and read values of ω and $p_{v,i}$; estimate the volume between the lines *d* and *e*; estimate ϕ between lines *e* and *f*; follow constant wet bulb to *g* and read value of total heat.

humidity, and other useful information. In the absence of a psychrometric chart, or if the atmospheric pressure is significantly different from standard (14.696 psia), the wet- and dry-bulb temperatures may be used in the equations for an *adiabatic saturation process* (§ 256), an application of usual principles of thermodynamics. This process is an irreversible one, path

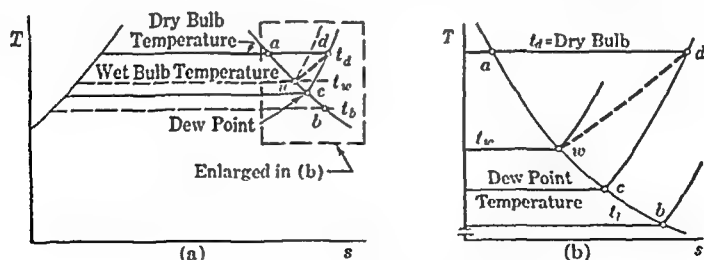


Fig. 168. *Adiabatic Saturation on Ts Plane.*

dw, Fig. 168. Note that the wet-bulb temperature is above the dew point *c* and that the amount of moisture in the air is continuously increasing from *d* to *w*.

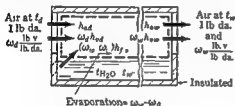
256. Adiabatic Saturation Process. Processes which are approximately adiabatic saturation and similar steady flow processes are often encountered

in practice The manner in which the adiabatic saturation process occurs is suggested by Fig 169(a) For convenience the basis of computations is one pound of dry air If the water is not initially at temperature t_w it becomes t_w when steady state has been attained If in the beginning the water is hotter than t_w it cools off by supplying some energy to evaporate the water If the water is colder than t_w it is warmed by energy transferred from the air At the steady state condition the energy used to evaporate the water comes from the entering air (let the make up water be added at temperature t_w)

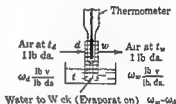
$$(q) \left. \begin{array}{l} \text{Energy given up by entering} \\ \text{air in cooling from } t_d \text{ to } t_w \end{array} \right\} = \left\{ \begin{array}{l} \text{latent heat of the evaporated} \\ \text{water at } t_w \end{array} \right.$$

[ADIABATIC SYSTEM = AIR AND WATER $\Delta K = 0$]

It is clear from this word equation why the process is called adiabatic However, for purposes of an energy diagram, we may take the boundaries of the system as outlined with heavy dashes in Fig 169(a), equating energy



(a) Adiabatic Saturation



(b) Wet Bulb Thermometer

Fig 169 The vessel in (a) is long enough that ample time is allowed for the air and water to reach equilibrium with respect to one another (which is at the adiabatic saturation temperature t_w) The process could be helped along by spraying the water into the air stream

entering to energy leaving the system (no change of stored energy) Each point of view results in the same equation From the energy diagram of Fig 169, we have

$$(r) \quad h_{ad} + \omega_d h_{sd} + (\omega_w - \omega_d) h_{fw} = h_{aw} + \omega_w h_{sw} \text{ Btu/lb da,}$$

where h_{ad} = enthalpy of 1 lb da at dry bulb t_d

h_{aw} = enthalpy of 1 lb da at wet bulb t_w

h_{sd} = enthalpy of 1 lb steam as it exists in the atmosphere at t_d (see § 257)

h_{sw} = enthalpy of 1 lb saturated steam at wet bulb t_w

h_{fw} = enthalpy of 1 lb saturated water at wet bulb t_w

ω_w = humidity ratio at the wet bulb t_w (saturated air) $\omega_w = p_s/p_a$ where $p_s = 1/v_s$, v_s being taken at t_w

ω_d = humidity ratio of the original air at t_d $\omega_w - \omega_d$ = mass of H_2O evaporated lb v/lb da

We note in equation (r) that $h_{sw} - h_{fw} = h_{gw} - h_{fd} = h_{fgw}$ the change of enthalpy (latent heat) during evaporation at t_w and we use $\omega_w h_{fgw}$ for

$\omega_2(h_{f2} - h_{f2})$. Then, solving for the *humidity ratio* ω_2 of the original atmospheric air, we get (see Fig. 16S for the state points of the steam)

$$(SS) \quad \omega_2 = \frac{\omega_1 h_{f1} + h_{12} - h_{f2}}{h_{f2} - h_{f1}} \text{ lb. v./lb. da.,}$$

where $h_{12} - h_{f2} = -0.24(t_2 - t_w)$, the change in specific enthalpy for the dry air. Knowing the total pressure p_1 and the humidity ratio ω_2 at any state, the corresponding partial pressure of the steam may be computed from equation (o) or (p). Observe that the conditions under which equation (SS) applies are those which *define the adiabatic saturation process*: to wit, the process is steady flow, the total pressure remains (virtually) constant, the heat transferred is zero, and the temperature of the make-up water is the wet-bulb temperature t_w .

The process in connection with a wet-bulb thermometer is seen to be much the same as the adiabatic saturation process [Fig. 169(b)]. The wet-bulb reading is affected by radiation to the thermometer, by the velocity of air relative to the thermometer, by the design of the instrument, and by the rate of diffusion of the evaporated water into the air stream (131). Fortunately, under ordinary circumstances, the heat absorbed by radiation from the surroundings is nearly compensated by providing a rapid movement of air over the bulb (convection), so that the wet-bulb temperature is a good approximation of the adiabatic saturation temperature. Thus the adiabatic saturation process is nearly one of *constant wet-bulb temperature*.

The psychrometric chart provides a quick answer to the same problem to which equation (SS) would be applied. But since the engineer is interested in learning principles, he will master the use of equation (SS), which is the basic energy equation for the construction of psychrometric charts. W. H. Carrier developed from equation (SS) an expression for computing the partial pressure of the steam in atmospheric air, after making some simplifying assumptions. Such a derivation yields the following equation,

$$(s) \quad p_1 = p_{f2} - \frac{(p_1 - p_{f2})(t_2 - t_w)}{2830 - 1.44t_w}, \quad [\text{CARRIER'S EQUATION}]$$

which is different from Carrier's original equation only in the constants.*

257. Enthalpy of Superheated Steam at Low Pressure. Since steam tables do not give enthalpies of superheated steam at pressures below 1 psia, another means of obtaining h_{12} in equation (SS) is needed. Having observed that the steam in the air may be treated as an ideal gas with little error, we recall that the enthalpy of an ideal gas is a function of temperature only (§ 59). Thus, the easiest way to determine the enthalpy of superheated steam at pressures below 1 psia is to use the enthalpy of saturated steam at

* Equation (s) is derived in some detail in the earlier editions reference (159).

the same temperature In Fig 170, for instance,

$$(t) \quad h_{s1} = h_{s6} \text{ Btu/lb } v$$

This same principle may be applied to other vapors when the vapor pressure is low An equation which gives satisfactory answers for the enthalpy of steam in atmospheric air is

$$(u) \quad h_s = 1061 + 0.444t_d \text{ Btu/lb } v.,$$

where t_d is the dry bulb temperature, °F

258 Example The dry and wet bulb temperatures of air are found to be $t_d = 83^\circ\text{F}$ and $t_w = 68^\circ\text{F}$ The barometer is $p_m = 29.4$ in Hg Determine (a) the humidity ratio, (b) the partial pressure of the vapor (from theory and from Carrier's equation) (c) the relative humidity, (d) the dew point, (e) the density of the air, (f) the density of dry air at the same p_m and t_d (Refer to Fig 170 steam table values are from Keenan and Keyes)

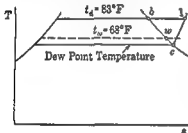


Fig 170

SOLUTION (a) *Humidity Ratio* Let the state be represented by 1, Fig 170 First obtain values needed for the solution of equation (88) From the steam tables at $t_w = 68^\circ\text{F}$, $h_{fsw} = 1055.5$ Btu/lb. and $h_{gsw} = 36.04$ Btu/lb At 83°F ,

$$h_s = 1097.9 \text{ Btu/lb},$$

which is closely the value of h_{s1} use $h_{s1} = 1097.9$ To get ω_w find ρ_{sw} and ρ_{aw} At $t_w = 68^\circ\text{F}$, the partial pressure of the steam is $p_{sw} = 0.339$ psia (from steam tables) and $v_g = 925.9$ cu ft/lb, or $\rho_{sw} = 1/925.9$ lb/cu ft The pressure of the dry air at saturation, state w, Fig 170 is $(p_m - (0.491)(29.4)) = 14.45$ psia

$$p_{aw} = p_m - p_{sw} = 14.45 - 0.339 = 14.111 \text{ psia}$$

$$\rho_{aw} = \frac{p_{aw}}{R_a T_s} = \frac{(14.11)(144)}{(53.3)(528)} = 0.0721 \text{ lb/cu ft}$$

$$\omega_w = \frac{\rho_{sw}}{\rho_{aw}} = \frac{1}{(925.9)(0.0721)} = 0.015 \text{ lb } v/\text{lb } da,$$

which is the humidity ratio after adiabatic saturation state w Fig 170 Using these various values in equation (88) we get the humidity ratio of the original air as

$$\omega_1 = \frac{(0.015)(1055.5) - (0.24)(83 - 68)}{1097.9 - 36.04} = 0.0115 \text{ lb } v/\text{lb } da,$$

or $(0.0115)(7000 \text{ grains/lb}) = 80.5 \text{ grains } v/\text{lb } da$

(b) *Partial Pressure of Vapor* According to equation (p), the vapor pressure p_{v1} Fig 170 is

$$p_{v1} = \frac{\omega_1 p_m}{0.622 + \omega_1} = \frac{(0.0115)(14.45)}{0.622 + 0.0115} = 0.262 \text{ psia}$$

From Carrier's equation (s), we have

$$\begin{aligned} p_{r1} &= p_{rv} - \frac{(p_m - p_{rv})(t_d - t_w)}{2830 - 1.44t_w} \\ &= 0.339 - \frac{(14.11)(15)}{2830 - (1.44)(68)} = 0.2615 \text{ psia.} \end{aligned}$$

(c) *Relative Humidity.* As defined by equation (86), the relative humidity is

$$\phi = \frac{p_{r1}}{p_{rb}} = \frac{0.262}{0.5588} = 46.9\%,$$

where $p_{rb} = 0.5588$ is the saturation pressure corresponding to the dry bulb temperature $t_b = 83^\circ\text{F}$.

(d) *Dew Point.* At c , the vapor pressure is $p_{rc} = p_{r1} = 0.262$ psia. The saturation temperature corresponding to this pressure is found in the steam tables as 60.6°F , the dew point.

(e) *Density of Mixture.* The density of the mixture is the sum of the densities of the constituents; $\rho_m = \rho_a + \rho_r$. The pressure and density of the dry air in state 1. Fig. 170, are

$$\begin{aligned} p_{a1} &= p_m - p_{r1} = 14.45 - 0.262 = 14.188 \text{ psia.} \\ \rho_{a1} &= \frac{p_{a1}}{R_a T_a} = \frac{(14.188)(144)}{(53.3)(543)} = 0.0706 \text{ cu. ft./lb.} \end{aligned}$$

From equation (86), we get the density of the vapor at 1 as

$$p_{r1} = \phi p_{rb} = \frac{\phi}{v_{rb}} = \frac{0.469}{577.4} = 0.00081 \text{ lb./cu. ft.,}$$

where $v_{rb} = 577.4$ cu. ft./lb. is the volume of saturated vapor at state b , Fig. 170.

$$\rho_m = \rho_{a1} + \rho_{r1} = 0.0706 + 0.00081 = 0.07141 \text{ lb./cu. ft.}$$

(f) *Density of Dry Air.* For dry air at 29.4 in. Hg = 14.45 psia and 83°F , the density is

$$\rho_a = \frac{P_a}{R_a T_a} = \frac{(14.45)(144)}{(53.3)(543)} = 0.0719 \text{ lb./cu. ft.}$$

It is interesting to note that dry air at a particular temperature and pressure is heavier than atmospheric air at the same temperature and pressure, $0.0719 > 0.07141$. For practice, the student should check the foregoing answers by a psychrometric chart, noting that the chart is constructed for a $p_m = 14.696$ psia. It is possible for chart values to be corrected for other than standard atmospheric pressure (13, 127), but the innate inaccuracies in reading a chart are often greater than those due to an atmospheric pressure which differs little from standard.

259. Enthalpy and Total Heat of an Air-Steam Mixture. It is the practice in air-conditioning computations to reckon the enthalpy of dry air from 0°F and the enthalpy of steam from the usual 32°F datum. Thus, the enthalpy of a gas-vapor mixture in general may be taken as

$$(89) \quad h_m = h_a + \omega h_r = c_p t + \omega h_r \text{ Btu/lb. dg.,}$$

where the vapor component h_v is measured from its usual datum, $c_p = 0.24$ Btu per lb for air, t is in degrees Fahrenheit, ω is the humidity ratio

Tables are available (127) which afford the easiest means of determining the properties of mixtures of air-steam at $p_m = 14.696$ psia. If the air-steam mixture is not saturated, a new term is needed, *degree of saturation* μ , which is defined as the humidity ratio of the actual air divided by the

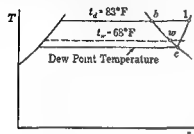


Fig 170 Repeated

humidity ratio of saturated air at the same dry bulb temperature and total pressure, or in terms of the state points on Fig 170,

$$(v) \quad \mu = \omega_1 / \omega_b$$

Together with other properties, the tables give the enthalpy of dry air and a property designated by h_{sa} , Btu per lb da, which is the difference between the enthalpy of saturated air at a particular temperature

minus the specific enthalpy of dry air at the same temperature and total pressure,

$$h_{sa} = h_{sat} - h_{dry} = \omega_b h_{gb} \text{ Btu per lb da}$$

We have $\omega_1 h_{v1} = \omega_b h_{v1} (\omega_1 / \omega_b) = \omega_b h_{gb} \mu = \mu h_{sa}$ when $h_{v1} = h_{gb}$, the subscripts being as defined in Fig 170. This approximation is good at air temperatures $t_d < 150^\circ\text{F}$ (127), so that (h_a = specific enthalpy of dry air)

$$(w) \quad h_m = h_a + \mu h_{sa} \text{ Btu/lb da}$$

There is another property which is commonly used in air conditioning called *total heat* TH , defined by*

$$(x) \quad TH = h_{aw} + \omega_w h_{gaw} = 0.24t_w + \omega_w h_{gaw} \text{ Btu/lb da},$$

where h_{gaw} is the enthalpy of saturated steam and ω_w is the humidity ratio, both at the wet-bulb temperature t_w . Actually, equation (1) is the enthalpy of a saturated mixture at the wet-bulb temperature, $h_{mw} = h_{aw} + \omega_w h_{gaw}$. Notice that since *total heat* is a function of the wet-bulb temperature only, it can be included on a psychrometric chart without complication (Fig 167). The total heat is nearly equal to the enthalpy of a mixture, so that where fine accuracy is not important, as in some air-conditioning problems, it is accurate enough to use $Q \approx \Delta TH$ instead of $Q = \Delta h$ in steady flow processes where $W = 0$ and $\Delta K = 0$. Observe that the right-hand side of equation (x), p 304, is the so-called *total heat*, that the enthalpy of the given mixture on the left-hand side is $h_m = h_a + \omega_d h_{sa}$, where the subscripts refer to Fig 169. Thus, total heat may be corrected to enthalpy by subtracting

* The language is somewhat confused here. This property is sometimes incorrectly called *enthalpy*.

the term $(\omega_w - \omega_d)h_{fw}$;

$$(y) \quad h_m = TH - (\omega_w - \omega_d)h_{fw} = h_{aw} + \omega_w h_{vw} - (\omega_w - \omega_d)h_{fw} \text{ Btu/lb. da.}$$

Compare equations (r), (89), and (x).

260. Example. For the mixture in the example of § 258 ($t_d = 83^\circ\text{F}$, $t_w = 68^\circ\text{F}$), determine the enthalpy by equation (89) and by the tables in the *ASHAE Guide* (127), and also compute the total heat for comparison.

SOLUTION. Using equation (89), we get

$$h_m = (0.24)(83) + (0.0115)(1097.9) = 32.55 \text{ Btu/lb. da.}$$

$$h_m = 32.55/1.0115 = 32.2 \text{ Btu/lb. mixture,}$$

where $1097.9 = h_g \approx h_{ts}$ (Fig. 170) is taken from the steam tables and is the enthalpy of saturated steam at the dry-bulb temperature; $h_{v1} = h_{ts}$.

From the tables in the *ASHVE Guide* (127) for 83°F , we get $\omega_b = 0.02471$; corresponding to which $\mu = \omega_1/\omega_b = 0.0115/0.02471 = 0.466$; and

$$h_m = h_a + \mu h_{as} = 19.942 + (0.466)(27.1) = 32.57 \text{ Btu/lb. da.,}$$

where the values of h_a and h_{as} have been taken from the tables. This answer is a close check of the preceding answer.

The total heat of the mixture is

$$TH = (0.24)(68) + (0.015)(1091.5) = 32.69 \text{ Btu/lb. da.,}$$

where $1091.5 = h_g$ at $t_w = 68^\circ\text{F}$ and $\omega_w = 0.015$ from § 258. Check this answer against a psychrometric chart and note that it is somewhat different from the computed values of h .

261. Internal Energy and Entropy. The internal energy of a mixture is the sum of the internal energies of the constituents, and as in the case of enthalpy, it is not necessary to measure the individual internal energies from the same datum; thus,

$$(z) \quad u_m = u_g + \omega u_v = c_v t + \omega \left(h_v - \frac{p_v v_v}{J} \right) \text{ Btu/lb. dg.,}$$

where u_g for the dry gas is measured above 0°F and the vapor properties are found as usual from vapor tables.

The *change of entropy* for a mixture is the sum of the changes for dry gas and vapor considered separately. Compute these changes in accordance with principles already established.

262. Mixtures Other than Air-Steam. The foregoing principles apply to any gas-vapor mixture, but vapor properties are not always at hand. Chemistry handbooks will contain enough data for certain purposes, data such as latent heats, specific heats, saturation pressures, and temperatures.

The curves of Fig. 171 are typical of available data. The use of this information is illustrated by the following example.

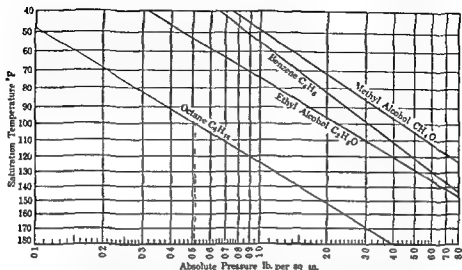


Fig. 171 Pressures and Temperatures of Saturated Vapors. To use: Enter chart for, say, a known temperature of 100°F. Move toward the right along a constant-temperature line until the desired curve is reached. Move down to read the corresponding saturated-vapor pressure. Following the dotted line for octane, find a pressure of 0.51 psia.

263 Example: Fuel-Air Mixture A fuel-air mixture in an intake manifold consisting of octane (C_8H_{18}) and air, is at 122°F and a total pressure of $p_m = 12$ psia. Determine the pounds of air per pound of fuel (a) if the mixture is saturated with respect to the octane, (b) if the relative humidity of the octane is 21%, but first determine the dew point at this humidity.

SOLUTION. (a) From Fig. 171 the partial pressure of the saturated vapor of octane at 122°F is found to be $p_{s2} = 0.95$ psia. See Fig. 172. The molecular weight of octane is 114.14 from which its gas constant may be taken as

$$R_{\text{octane}} = \frac{1545}{114.14} = 13.5$$

From the characteristic equation of an ideal gas we find the approximate density of octane vapor to be

$$\rho_{s2} = \frac{p_1}{RT} = \frac{(0.95)(144)(1)}{(13.5)(582)} = 0.0174 \text{ lb./cu. ft.}$$

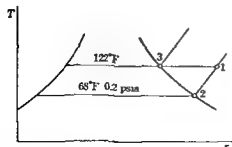


Fig. 172

The partial pressure of the air at state 3 is $12 - 0.95 = 11.05$ psia, whence the density of the air is

$$\rho_{a2} = \frac{pV}{RT} = \frac{(11.05)(144)(1)}{(53.3)(582)} = 0.0513 \text{ lb./cu. ft.}$$

Then, the mass of octane vapor per pound of air (or the humidity ratio) is

$$\omega_3 = \frac{\rho_{v3}}{\rho_{a3}} = \frac{0.0174}{0.0513} = 0.339 \text{ lb. v./lb. da.}$$

This ratio is equivalent to $1/0.339 = 2.95$ lb. of air/lb. of fuel. Such a mixture is not likely in practice because there is not enough air to complete the combustion of the fuel.

(b) Since the dew point is that temperature at which the vapor becomes saturated during a constant pressure cooling, we find first the partial pressure of the octane from

$$\phi = \frac{p_{v1}}{p_{v3}} = 0.21 = \frac{p_{v1}}{0.95};$$

or $p_{v1} = 0.2$ psia (Fig. 172). From Fig. 171, the corresponding saturation temperature is $t_2 = 68^\circ\text{F}$, the dew point.

The density of saturated vapor at 122°F has been found as $\rho_{v3} = 0.0174$ lb./cu. ft. Using equation (86), $\phi = \rho_{v1}/\rho_{v3}$, we have

$$\rho_{v1} = \phi \rho_{v3} = (0.21)(0.0174) = 0.00363 \text{ lb./cu. ft.}$$

The partial pressure of the air with the vapor in condition 1 is $12 - 0.2 = 11.8$ psia whence

$$\rho_{a1} = \frac{(11.8)(144)}{(53.3)(582)} = 0.0548 \text{ lb./cu. ft.}$$

$$\frac{1}{\omega_1} = \frac{\rho_{a1}}{\rho_{v1}} = \frac{0.0548}{0.00363} = 15.1 \text{ lb. air/lb. v. (fuel).}$$

This ratio is typical of mixtures found in internal combustion engines.

264. Example: Constant Total Pressure (Steady Flow). An atmospheric air mixture in state 1 has the following properties: $p_m = 29.92$ in. Hg, $t_d = 100^\circ\text{F}$, and $\omega_1 = 0.0295$ lb. v./lb. da. This air is cooled to 60°F , state 2, Fig. 173, and then heated to 85°F , state 3. (a) Determine the relative humidity of the original mixture. (b) How much moisture is deposited during the cooling to 60°F ? (c) How much heat is removed during cooling? (d) For a flow of 100,000 cfm, how many tons of refrigeration are ideally required? (e) What is the volume of the original 100,000 cfm after cooling? (f) What is the relative humidity of the air in state 3 (Fig. 173)?

NOTE. The events of this problem are fairly typical of the events in summer air conditioning of habitations. Since the various answers may be obtained with the aid of a psychrometric chart, the reader should use the chart and check all results.

SOLUTION. (a) To find the relative humidity, first find the partial pressure p_{v1} of the vapor from equation (p);

$$p_{v1} = \frac{\omega_1 p_m}{0.622 + \omega_1} = \frac{(0.0295)(14.7)}{0.622 + 0.0295} = 0.666 \text{ psia.}$$

From the steam tables, we find $p_{a1} = 0.9492$ psia for 100°F and then

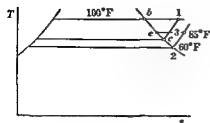


Fig. 173

$$\phi_1 = \frac{p_{a1}}{p_{a1}} = \frac{0.666}{0.9492} = 70.2\%$$

(b) The condensed H_2O is obtained by subtracting ω_2 from ω_1 . To get an idea of where state 2 for the H_2O is, we enter the steam tables and find the dew point t_c (Fig. 173) corresponding to $p_{a1} = 0.666$ as $t_c = 88.4^\circ\text{F}$. Thus, the air has been cooled below the dew point and H_2O has been condensed. At 60°F , $p_{a2} = 0.2563$ psia is

taken from the tables, then from equation (p), we get

$$\omega_2 = \frac{0.622 p_{a2}}{p_m - p_{a2}} = \frac{(0.622)(0.2563)}{14.7 - 0.2563} = 0.01105 \text{ lb } \nu / \text{lb } \text{da}$$

The condensation is therefore

$$\omega_1 - \omega_2 = 0.0295 - 0.01105 = 0.01845 \text{ lb } \nu / \text{lb } \text{da}$$

(For the 100,000 cfm, this condensation amounts to

$$\frac{(6770)(0.01845)}{8.4 \text{ lb/gal } \text{H}_2\text{O}} = 14.85 \text{ gpm,}$$

which is equivalent to 892 gal/hr. See part (d) for computation of 6770 lb/min.)

(c) The heat transferred at constant pressure or in a steady flow process where $W = 0$ and $\Delta h = 0$ is $Q = \Delta h$. The enthalpy of the vapor in the original air is $h_{a1} = h_{a0} = 1105.2$ Btu/lb ν at 100°F , or

$$\omega_1 h_{a1} = (0.0295)(1105.2) = 32.6 \text{ Btu/lb } \text{da}$$

At state 2, the enthalpy of the H_2O should account for the condensed steam, that is,

$$\begin{aligned} h_{a2} &= \omega_2 h_{a2} + (\omega_1 - \omega_2) h_{f2} \\ &= (0.01105)(1088) + (0.01845)(23.06) \\ &= 12 + 0.518 = 12.518 \text{ Btu/lb } \text{da} \end{aligned}$$

where h_{a1} and h_{f2} correspond to a temperature of $t_2 = 60^\circ\text{F}$. Now using equation (89), we find

$$\begin{aligned} Q &= \Delta h_m = c_p(t_2 - t_1) + h_{a2} - h_{a1} \\ &= 0.24(60 - 100) + 12.518 - 32.6 = -29.7 \text{ Btu/lb } \text{da} \end{aligned}$$

The heat removed per pound of original mixture is $29.7/1.0295 = 28.85$ Btu/lb mixture. This value of Q should be closely approximated by ΔTH .

(d) If there are 100,000 cfm of atmospheric air, there are also 100,000 cfm of dry air and 100,000 cfm of vapor. The mass of 100,000 cfm of dry air flowing is

$$\dot{m}_a = \frac{p_{a1} V_{a1}}{R_a T_1} = \frac{(14.7 - 0.666)(144)(100,000)}{(53.3)(560)} = 6770 \text{ lb/min}$$

For example, a cubic inch of silica gel in its crushed and prepared condition exposes some 50,000 sq ft (127) While silica gel and activated alumina have a particular affinity for H_2O , activated carbon prefers organic vapors and is used, for example, in refrigerators to reduce objectionable vapors with odors After they have become *saturated*, both absorbents and adsorbents are *reactivated* by heating them

THE TRANSFER OF HEAT

266. Introduction. We have repeatedly used transferred heat Q and described some heat exchangers without discussing the details of the phenomena by which heat is transferred. The purpose of this chapter is to give the reader a general idea of the nature of the problems and to provide an introductory background for further study. The developments in this field are too extensive to be covered fully in a book on thermodynamics.

Heat exchanger is a name of general connotation applied to any device which effects a transfer of heat from one substance to another. One way in which this transfer of heat is brought about is by the irreversible mixing of cold and hot fluids in some exchangers, such as open feedwater heaters, jet condensers, desuperheaters, wherein an equilibrium condition is estimated by equating the energy lost by one fluid to the gain in energy of the other fluid. In other cases (Figs. 174, 175), heat is transferred from one fluid on one side of a wall or partition to and through the wall to another fluid on the other side. Numerous heat exchangers already mentioned come to mind; as examples, we have: steam boilers, surface condensers, evaporators, closed feedwater heaters, and the automobile radiator. Those factors which determine the coefficient of heat transfer between a fluid and the adjacent surface will not be discussed in detail; and we shall have little to say about radiation. The idea is to obtain a general understanding of the mechanisms of heat transfer.

267. Conduction. Methods of heat transfer are defined in § 19, which should be read again now. We recall that heat is *conducted* from one part of a system to another because faster moving atoms or electrons or molecules in the hotter part induce by impacts an increased activity of adjacent atoms or electrons or molecules (increase their kinetic energy); thus *heat flows* (there is a heat flux) from the hotter to the colder parts.

Heat may be conducted at uneven rates the rate increasing or decreasing, that is, the temperature difference in the path of heat flow may be increasing or decreasing. This condition is an unsteady state, as during the warm up period of an engine, and it must be analyzed via infinitesimal heat flows $dQ/d\tau$, where $d\tau$ is the duration of time during which heat dQ passes a certain section. However, this study will be confined to systems in a steady

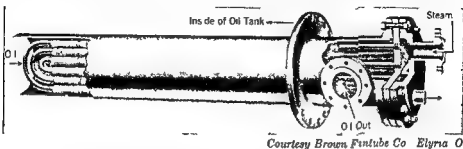


Fig 174 Heat Exchanger, Extended Surface Used for heating viscous oils—reducing the pumping work. Oil enters from the oil tank at the open left end. Steam circulates through the tubes, two-pass flow through tubes, single pass flow through the shell. See Fig 175(b).

state, wherein each point of the system remains at a constant temperature and heat transfer is constant at a rate of Q energy units per unit of time. Also it is assumed that the flow of heat is unidirectional.

268 Fourier's Law For steady state, unidirectional flow, Fourier's equation (1822) gives the heat conduction as

$$(90) \quad Q = -kA \left(\frac{dt}{dL} \right) \text{ Btu/hr,}$$

where Q Btu per hr is the heat conducted across a surface of A sq ft, through a wall thickness of dL in, and with a temperature drop of $dt^\circ\text{F}$ through the distance dL . The ratio dt/dL is called the *temperature gradient*.



Courtesy Foster Wheeler Corp, New York
(a) Circumferential extended surface



Courtesy Brown Fintube Co Elyria O
(b) Longitudinal extended surface

Fig 175 Extended Surface Elements

along the path. The negative sign in equation (90) is used because the temperature decreases in the direction of heat flow (that is $\int dt = t_2 - t_1$ is a negative number and the negative sign makes Q positive for convenience).

The symbol k represents the *thermal conductivity*,* which is the amount of heat (Btu per hr) transmitted in unit time across unit area (sq ft)

* Do not confuse the symbol k with k c_p/c_v . Familiarity with the subject will make it easy to attach the correct meaning to the symbol by its context.

through unit thickness (both in. and ft. are used) for unit temperature change (1°F). Any set of units for k , as defined by equation (90), may be used, but the foregoing units are the usual ones. Values of k given and used in this chapter are for a 1-in. thickness; or

$$(a) \quad k \rightarrow \frac{\text{Btu-in.}}{\text{ft}^2\text{-}^{\circ}\text{F-hr.}}$$

If the thickness L is in feet, then

$$(b) \quad k \left(\frac{\text{Btu-in.}}{\text{ft}^2\text{-}^{\circ}\text{F-hr.}} \right) \left(\frac{\text{ft.}}{12 \text{ in.}} \right) = k \frac{\text{Btu-ft.}}{\text{ft}^2\text{-}^{\circ}\text{F-hr.}}$$

It will soon be evident that extraordinary care must be exercised in the matter of units in heat transfer, and because units are diverse in the literature, the reader is cautioned to check thoroughly the units of all factors entering into any equation.

269. Variation of Thermal Conductivity. Thermal conductivity varies widely, just as electrical conductivity does. Study the values in Table XII. Not only is there great variation between materials, but particular materials may also have widely different conductivities in different states. For example, k for aluminum at $212^{\circ}\text{F} = 672^{\circ}\text{R}$ is about 1440 Btu-in. per $\text{ft}^2\text{-hr-}^{\circ}\text{F}$, but at about 18°R it goes up to about 35,000 Btu-in. per $\text{ft}^2\text{-hr-}^{\circ}\text{F}$ (148). However, the conductivity of solids varies so little with the more usual temperatures that a single average or typical value may be used with small error. In most applications, the *thermal resistivity*, which is the reciprocal of the conductivity, is so small for metals as compared with other resistances to heat flow that a small variation of k from its true value has little effect on the over-all conductance or resistance. If the relevant temperatures are extreme, high or low, it is advisable to look into more detailed sources for test values. As we shall see, the largest resistance is in the fluid film adjacent to the solid. King (149) makes the following generalizations concerning conductivities of solids:

1. The conductivities of all homogeneous solid materials are relatively high; practically all good (heat) insulators are porous, cellular, fibrous, or laminated materials.
2. In general, the conductivity increases with the density and elasticity.
3. With rare exceptions, the conductivity of insulating materials increases very materially with the temperature.
4. The absorption of moisture greatly impairs the insulating value of porous materials.

The conductivities of liquids and gases are more sensitive to temperature changes. Moreover, the difficulty of eliminating convection currents in tests on liquids and gases has somewhat complicated the determination of conductivities. For relatively small temperature variations, the con-

ductivity of solids, liquids, and gases may be assumed to vary linearly with the temperature. For this assumption, the k in equation (90) may be taken as the arithmetic average for the temperature range involved (or the value of k for the mean temperature), and the integration made with k constant. If the variation is not linear, and a plot of k values is available, the mean value between two temperatures may be estimated from the curve (as suggested for specific heats in connection with Fig 19). Lacking other data, interpolate between the values given in Table XII for k at the average temperature of the conducting body.

270. Conduction through a Plane Wall. After a steady state of unidirectional heat flow has been reached in a single homogeneous material whose thermal conductivity k is constant, the temperature gradient dt/dL for a plane wall is constant (a straight line—not true when k varies with temperature). Accordingly, integration of equation (90) gives

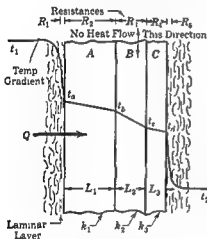


Fig 176 Temperature Gradients in a Composite Wall

upon the value of the thermal conductivity k . Applying equation (c) to a composite wall made up of three homogeneous materials A, B, C, Fig 176, we have

$$(d) \quad Q_A = \frac{k_1 A (t_a - t_b)}{L_1}, \quad Q_B = \frac{k_2 A (t_b - t_c)}{L_2}, \quad Q_C = \frac{k_3 A (t_c - t_d)}{L_3} \text{ Btu/hr}$$

Solving for the temperature difference from each of these equations, and noting that $Q_A = Q_B = Q_C = Q$ Btu per hr for steady-state flow, we find

$$(e) \quad t_a - t_b = \frac{QL_1}{k_1 A}, \quad t_b - t_c = \frac{QL_2}{k_2 A}, \quad \text{and} \quad t_c - t_d = \frac{QL_3}{k_3 A}$$

Next, equating the sum of the left-hand terms to the sum of the right hand

$$Q \int_a^L dL = -kA \int_{t_a}^{t_b} dt,$$

$$(c) \quad Q = \frac{kA(t_a - t_b)}{L} = \text{Btu/hr},$$

where t_a and t_b are the surface temperatures of a partition such as A, Fig 176 and k is the average value for the given conditions. We see that the rate of flow of heat depends (1) directly upon the temperature difference (potential) between the two surfaces of the wall, (2) directly upon the area of surface A through which transmission occurs, (3) indirectly upon the thickness L of the wall, and (4) directly

Table XII. CONDUCTIVITIES

The units of k are Btu-in./ft²-hr-°F; at atmospheric pressure; k for a solid changes little with pressures below 200 psi, but significant changes of k for liquids and vapors may accompany pressure changes. The values are from various sources, here selected largely from McAdams (137). Straight-line interpolations are permissible between the temperatures given. Nearby extrapolations may give satisfactory results.

Material	Temp. °F	k	Material	Temp. °F	k
<i>Solids</i>			<i>Liquids (continued)</i>		
Aluminum	32	1400	Kerosene	68	1.03
	392	1490		167	0.97
Aluminum piston alloy	0-400	1290*	Petroleum oil, average	68	1.0
Asbestos, 29 lb./ft. ³	-200	0.865	Sodium	212	590
Asbestos, corrugated, 4 plies/in.	300	0.828		410	550
Bearing metal, white	68	164	Sulfur dioxide	5	1.53
Brickwork, low density	68	5		68	1.33
Cast iron, grey	0-400	360*	Water	32	4.1
Copper, pure	32	2690*		200	4.7
	212	2616		300	4.75
				620	3.3
Concrete, 1-4 dry		5.4	<i>Gases</i>		
Cork board	86	0.3	Air	-148	0.109
Glass window	3	6-7.4		32	0.168
Gold	64	2028		572	0.312
Graphite	32	1165	Ammonia	-58	0.107
	392	910		32	0.151
Gypsum	68	3		212	0.23
Gypsum plaster		3.3		212	0.23
Magnesia (85%)	100-300	0.43*	Carbon dioxide	-58	0.077
Mineral wool (glass and rock wool)	86	0.27		32	0.101
Monel	68	242		212	0.154
Plaster on wood lath, ¾-in. total thickness		2.5	Freon F12	32	0.057
				212	0.096
Steel	0-400	312*	Hydrogen	32	1.06
Wallboard, insulating	70	0.34		572	2.04
Wood, balsa	86	0.32	Nitrogen	32	0.167
Oak, maple	59	1.44		572	0.306
White pine	59	1.05	Oxygen	32	0.17
				212	0.226
<i>Liquids</i>			Steam	212	0.163
Ammonia	68	3.13		932	0.394
	140	3.48	Sulfur dioxide	32	0.06
				212	0.0827

* Average for temperature range given.

Table XIII. CONDUCTANCES AND TRANSMITTANCES

The units of k/L and U are Btu per hr-ft²-°F. These values are intended as representative, suggesting the order of magnitude in the various situations. They are not to be used in actual design unless it is known that they apply. Since the values of U are not particularized, the reference area is not meaningful. However, if these values are used for problem work, let the reference area be the internal tube or pipe area in such cases. (a) From McAdams (197). (b) From ASHAE (197)

Construction and Materials	$\frac{k}{L}$	U
Air space, $\frac{3}{4}$ in. or more in width	(b) 1 10	
Air space, $\frac{3}{4}$ in. or more in width, bounded by aluminum foil	(b) 0 46	
Ammonia condenser, 2x3-in. double pipe, water inside at $v = 6$ fps, NH ₃ in annular space, $\Delta t_m = 3.5^\circ\text{F}$, clean	(a)	1100
Asphalt shingles	(b) 0 50	
Brick wall, 8 in. thick, plaster inside	(b)	0 46
Brick veneer, frame wall with wood sheathing, $\frac{1}{2}$ in. plaster on gypsum lath	(b)	0 27
Brick veneer as above, plus 2 in. mineral wool insulation	(b)	0 097
Concrete blocks, 8 in., hollow gravel aggregate	(b) 1 00	
Feedwater heaters, closed, steam condensers, free convection	(a)	50-200
" , forced convection	(a)	150-800
Insulating board, $\frac{1}{2}$ in. thick	(b) 0 66	
Heat exchanger air in tube, condensing steam outside tubes (on outside surface area)	(a)	8
Heat exchanger, cooling oil with water in tubes		50
Steam condensing, to air, free convection	(a)	1-2
" , forced convection	(a)	2 10
Steam condensing, to boiling water, free convection	(a)	300-800
Steam condensing, to liquid oil, free convection	(a)	10-30
" , forced convection	(a)	20-60
Superheaters, steam, free convection	(a)	1 6-2
" , forced convection	(a)	2-6
Water to gas and liquid to gas (hot water radiators, air coolers, economizers steam boilers), free convection	(a)	1-3
" , forced convection	(a)	2-10
Water to water, free convection	(a)	25-60
" , forced convection	(a)	150-300

terms of these equations, we get

$$(f) \quad t_a - t_d = \frac{Q}{A} \left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} \right],$$

or

$$(g) \quad Q = \frac{A(t_a - t_d)}{L_1/k_1 + L_2/k_2 + L_3/k_3} = \frac{A \Delta t}{\Sigma(L/k)} \text{ Btu/hr.}$$

for the composite wall in which the temperature drop from surface to surface is Δt ($\Delta t = t_a - t_d$ for the wall of Fig. 176; observe that in this usage, the Δt is the *first* temperature minus the second, counting in the direction in which heat flows). It is readily seen that if another section were added to the wall, the only change necessary in equation (g) would be the addition of another term L_4/k_4 to the sum of the other L/k 's and the interpretation of t_d as being the temperature of the final surface. This sum of L/k values is simply represented by $\Sigma L/k$, as shown.

For any one section of the wall (Fig. 176), $C = kA/L$ in equation (d) is called the *conductance*, which is seen to include the effect of size and shape of the conducting body. The conductance is the rate of heat transfer per unit temperature difference, $Q/\Delta t$. The reciprocal of the conductance is the *thermal resistance*, $R = L/(kA) = \Delta t/Q$. The *conductance per unit area*, or the *unit conductance* k/L , is often given for particular bodies. See Table XIII.) The *unit resistance* (per unit area) is L/k . For the composite plane wall (Fig. 176), the conductance from surface to surface is $A/\Sigma(L/k)$ [equation (g)], the unit conductance is $1/\Sigma(L/k)$, and the resistance is $\Sigma(L/k)/A$.

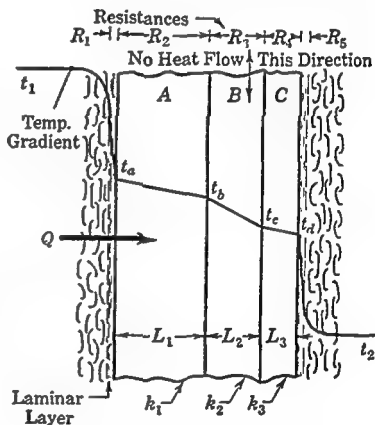


Fig. 176. Repeated.

271. Film Coefficient. On each side of the composite wall of Fig. 176 is a fluid, the nature of which is of no concern at the moment. On the hot side of the wall, the fluid is hotter than the surface at some temperature $t_1 > t_a$. On the cold side, the fluid is colder than the surface, its temperature being $t_2 < t_d$. Thus, through thin films of the fluids adjacent to the surfaces, there are temperature drops $t_1 - t_a$ and $t_d - t_2$. The unit rate of heat flow through these films is called the *film coefficient* (and other names such as *film conductance*, *surface conductance*) and is represented by h , whose units in this book are Btu per hr-ft²-°F; that is, h is the rate of heat flow (Btu in 1 hr.) through an area of 1 sq. ft. when the temperature potential across the film is 1°F. Its magnitude depends on variables which are too

many to be considered at this time. At this moment, we are interested in the film coefficient in order to obtain a picture of the over-all, fluid to-fluid flow of heat. From the definition of h , we see that the heat passing across the films of Fig. 176 is

$$(b) \quad Q = h_1 A (t_1 - t_a) \quad \text{and} \quad Q = h_2 A (t_a - t_2) \text{ Btu/hr}$$

As examples of the surface coefficients, we have from ASHAE (127),

Inside building walls, still air, design value,	$h = 1.65 \text{ Btu/hr-sq ft-}^\circ\text{F}$,
Outside walls, 15-mph wind, design value,	$h = 6.00 \text{ Btu/hr-sq ft-}^\circ\text{F}$,
Evaporating refrigerants in tube, typical value,	$h = 200 \text{ Btu/hr-sq ft-}^\circ\text{F}$,
Condensing steam in tube, typical value,	$h = 2000 \text{ Btu/hr-sq ft-}^\circ\text{F}$

Actual values of h may be quite different from the foregoing.*

272. Heat Transfer from Fluid to Fluid. If it were simple and convenient to measure surface temperatures, equation (g) would be the one to use. However, in practice, the fluid temperatures are easy to find, unless the speed of the fluid is high (152), and it is therefore desirable to express the heat flow in terms of these temperatures. To find the corresponding equation, solve for the temperature differences in equation (h), adding them to those in equation (e) as previously explained, and obtain

$$Q = \frac{A(t_1 - t_2)}{1/h_1 + L_1/k_1 + L_2/k_2 + 1/h_2} \text{ Btu/hr}$$

Generalizing from this equation, we write for the steady-state, unidirectional flow of heat,

$$(91A) \quad Q = \frac{A \Delta t}{\Sigma(1/h) + \Sigma(L/k)} = \frac{\Delta t}{\Sigma R} \text{ Btu/hr},$$

where Q is the rate of heat transfer from fluid to fluid through a wall of area A sq ft. When the temperature difference between the fluids is $\Delta t = t_1 - t_2$, ΣR is the total thermal resistance, and the total unit resistance to heat flow is

$$(i) \quad \frac{1}{U} = \Sigma \frac{1}{h} + \Sigma \frac{L}{k}$$

in which $\Sigma(1/h)$ is the sum of all the surface-layer fluid resistances, and $\Sigma(L/k)$ is the sum of all the unit resistances of the materials of the wall. The over-all unit resistance is represented by $1/U$ and the reciprocal of this over-all resistance is called the *transmittance* U , also the *over-all coefficient of heat transfer* and the *over-all (unit) conductance*.† Since from equation

* Do not confuse the symbol h with that for enthalpy h .

† Terms ending in *-ity* designate characteristics of materials, normally independent of size or shape, sometimes called *specific properties*. Examples are *conductivity* and *resistivity*. Terms ending in *-ance* designate properties of a particular object, depending not only on the material, but also upon size and shape, sometimes called *total quantities*. Examples are *conductance* and *transmittance*. Terms ending in *-ion* designate time rate of

(i), $U = 1/[\Sigma(1/h) + \Sigma(L/k)]$, equation (91A) may be written

$$(91B) \quad Q = UA \Delta t \text{ Btu/hr.}$$

See § 275. Table XIII shows some representative values of U .

It will be helpful to draw an analogy between the flow of heat and the flow of electricity. You recall Ohm's law as $I = E/R$, where I is the current flowing (analogous to the rate of heat flow), E is the electromotive force or electrical potential (analogous to the temperature difference or thermal potential which is the "driving force" for heat), and R is the resistance to the flow of electricity (analogous to the thermal resistances of $1/hA$ and L/kA). Note that the total thermal resistance is the equivalent of a series connection of electrical resistances, in which case the total resistance is the sum of the series connected resistances; $R = R_1 + R_2 + R_3 + \dots$, where R_1, R_2, R_3 , etc. are the individual resistances. Compare with Fig. 176, p. 321. For R constant, the greater the voltage drop, the greater the current flow (the greater the temperature drop, the greater the heat flow). If the total resistance is increased by adding more resistances in the denominator of equation (91A), or by increasing the size of one or more resistance, heat flows at a slower rate. This effect is the desired one in providing insulation for steam lines, cold-storage rooms, etc. On the other hand, if a greater rate of heat flow is desired, as in apparatus designed for the purpose of transferring heat, we endeavor to reduce the resistance. If one resistance is very much larger than any of the others, there will be little benefit derived from reducing any of the resistances except the largest; thus, if the largest resistance cannot be reduced, practically the best conditions have been attained.

The engineer has to keep in mind that the transmittance may *decrease* significantly during the operation of a heat exchanger because of the accumulation of deposits on the transmitting surfaces. On the other hand, the transmittance of insulated walls may *increase* due to the deterioration of the insulating material. These materials often lose a large part of their insulating value because they become wet from seepage or from moisture deposited from air cooled below the dew point.

273. Conduction through Curved Wall. Since the area through which heat flows in a curved wall is not constant, we must reconsider Fourier's equation (90) in this connection. Consider a thick cylinder, Fig. 177, for which the temperature on the inside surface is t_a , the temperature on the outside surface is t_b , and the thermal conductivity is k . The heat flows radially, say, from the inside to the outside, and in doing so, a given quantity of heat passes across larger and larger areas, since the cylindrical area

the process of transfer . . . Examples are *conduction* and *transmission*. *Transmission*, *transmissivity*, *transmittance* usually refer to transfer by one or more of the processes of conduction, convection, and radiation." From A.S.A. Standard, Z 10.4-1943.

increases with the radius of the cylinder. Consider a length of cylinder z and take a very thin element of the cylinder of thickness dr with a radius of r inches. The area of this thin cylindrical surface is $2\pi rz$. The change in temperature across dr is a differential amount dt . Thus Fourier's equation gives

$$Q = -k2\pi rz \frac{dt}{dr}$$

Separating the variables and integrating we get

$$Q \int_r^{r_0} \frac{dr}{r} = -2\pi zk \int_{t_0}^{t_1} dt,$$

$$Q \ln \frac{r_0}{r_1} = 2\pi zk(t_1 - t_0),$$

or

$$(j) \quad Q = \frac{2\pi rk(t_1 - t_0)}{\ln(r_0/r_1)} = \frac{2\pi zk \Delta t}{\ln(D_0/D_1)},$$

Fig 177 Curved Wall. The ends are insulated so that there is no longitudinal flow of heat

D_0/D_1 . In equation (j) note that the resistance for the curved wall is

$$(k) \quad R = \frac{\Delta t}{Q} = \frac{\ln(r_0/r_1)}{2\pi zk}$$

Setting up an equation for the heat flow through a composite curved wall such as an insulated pipe is simple when it is done by summing resistances ΣR in accordance with equation (91A) $Q = \Delta t / \Sigma R$. Consider Fig 178 which represents a pipe λ with insulation Y wherein the inside film coeff

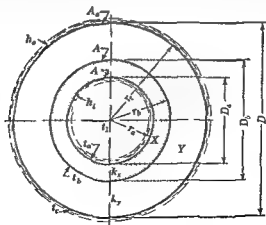


Fig 178 Composite Curved Wall

cient is h , and the outside value is h_o ; the temperature of the fluid on the inside is t_1 , which, in the following discussion, is greater than t_2 , the temperature of the fluid on the outside. The film resistances are (assuming no film resistance at the intermediate area A)

$$R_i = \frac{1}{A_i h_i} \quad \text{and} \quad R_o = \frac{1}{A_o h_o}.$$

Applying equation (k) to X and Y (see the dimensions in Fig. 178— $D_b/D_a = r_b/r_a$), we get their resistances as

$$R_x = \frac{\ln (D_b/D_a)}{2\pi z k_x} \quad \text{and} \quad R_y = \frac{\ln (D_c/D_b)}{2\pi z k_y}.$$

Thus, the heat flow is $Q = \Delta t / \Sigma R$, or

$$Q = \frac{\Delta t}{\frac{1}{A_i h_i} + \frac{\ln (D_b/D_a)}{2\pi z k_x} + \frac{\ln (D_c/D_b)}{2\pi z k_y} + \frac{1}{A_o h_o}};$$

or generalizing for any number of laminated cylinders, each with certain inside and outside diameters of D_i and D_o , with a conductivity of k , and z ft. long, we have for a steady state

$$(92A) \quad Q = \frac{\Delta t}{\sum \frac{1}{Ah} + \sum \frac{\ln (D_o/D_i)}{2\pi z k}},$$

where $\Delta t = t_1 - t_2$ is the temperature drop from fluid to fluid, $\Sigma(1/Ah)$ is the sum of all film resistances in the path of heat flow, and the other term in the denominator sums all other resistances. The heat is flowing radially only.

IMPORTANT NOTE: The value of k in equation (92) must have the units Btu-ft per hr-ft²-°F in order to get heat in Btu per hr. Compare with the units given in Table XII.

In applying $Q = UA \Delta t$ to curved walls, the area A becomes a convenient reference area; for a single pipe, either the outside or the inside pipe area. Inasmuch as a certain amount of heat Q is passing through the wall under steady-state operation no matter which area is the reference area, we have

$$(92B) \quad Q = U_o A_o \Delta t \quad \text{or} \quad Q = U_i A_i \Delta t,$$

wherein, by comparison with equation (92A),

$$U_o A_o = U_i A_i = \frac{1}{\sum \frac{1}{Ah} + \sum \frac{\ln (D_o/D_i)}{2\pi z k}}.$$

Thus, if an over-all transmittance U is given for a curved wall, the corresponding reference area should be stated.

274 Example. An insulated steam pipe, located where the ambient temperature is 90°F , has an internal diameter of 2 in and an external diameter of $2\frac{1}{2}$ in. The outside diameter of the corrugated asbestos insulation is 5 in and the surface coefficient on the outside is $h_o = 2 \text{ Btu/hr-sq ft-}^{\circ}\text{F}$, a value intended to include the effect of radiated heat where the movement of the air is that due to natural circulation (free convection). On the inside, the steam has a temperature of 300°F and $h_i = 1000 \text{ Btu/hr-sq ft-}^{\circ}\text{F}$. (This value of h_i is commonly used for saturated or wet steam flowing in a pipe. Note that this value corresponds to a low resistance to heat flow, so that some inaccuracy here has little effect on the transmittance.) Compute (a) the heat loss per foot of pipe length, and (b) the surface temperature on the outside of the insulation.

SOLUTION (a) Refer to Fig 178. From Table XII, we find

$$k_s = 312 \text{ Btu-in/hr-ft}^2\text{-}^{\circ}\text{F} \text{ for steel}$$

$$k_v = 0.828 \text{ Btu-in/hr-ft}^2\text{-}^{\circ}\text{F} \text{ for the corrugated asbestos insulation}$$

Converting the inch unit to feet, we have $k_s = 312/12$ and $k_v = 0.828/12$. The true value of k for steel will vary with the composition of the steel, but since the

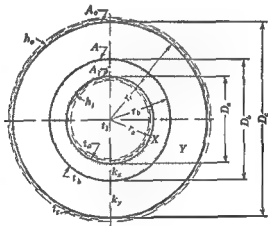


Fig 178 Repeated

resistance of any steel to the flow of heat is relatively small in a case like this, whatever error is involved will be seen to have negligible effect after numerical results have been obtained. First, compute the various resistances per foot of pipe length. The area of a cylinder is πDz , where $z = 1 \text{ ft}$ is its length and the area is to be in square feet ($2/12 \text{ ft}$ in R_1 , $5/12 \text{ ft}$ in R_4).

$$\text{Inside film } R_1 = \frac{1}{A_i h_i} = \frac{12}{(\pi 2)(1000)} = 0.00191 \text{ hr-}^{\circ}\text{F/Btu}$$

$$\text{Pipe } R_2 = \frac{\ln(D_o/D_i)}{2\pi z k} = \frac{\ln(2.5/2)}{(2\pi)(312/12)} = 0.001365 \text{ hr-}^{\circ}\text{F/Btu}$$

$$\text{Insulation } R_3 = \frac{\ln(D_o/D_i)}{2\pi z k} = \frac{\ln(5/2.5)}{(2\pi)(0.828/12)} = 1.6 \text{ hr-}^{\circ}\text{F/Btu}$$

$$\text{Outside film } R_4 = \frac{1}{A_o h_o} = \frac{12}{(\pi 5)(2)} = 0.382 \text{ hr-}^{\circ}\text{F/Btu}$$

Summing these resistances, we have $R = 1.985275 \text{ hr-}^\circ\text{F/Btu}$, where it is evident that the last several digits are not significant. Observe the small effect of the first two resistances. For $\Delta t = 300 - 90 = 210^\circ\text{F}$, we get

$$Q = \frac{\Delta t}{\Sigma R} = \frac{210}{1.985} = 106 \text{ Btu/hr. for each foot of length.}$$

(b) The heat $Q = 106 \text{ Btu/hr.}$ flows through each cylindrical lamination and will be equal to

$$Q = \frac{\text{temperature difference between two sections}}{\text{sum of the resistances between the same sections}}.$$

For a temperature difference to the outside surface of $t_1 - t_c = 300 - t_c$, Fig. 178, and a resistance to the outside surface of $R_1 + R_2 + R_3 = 1.603$, we get

$$106 = \frac{300 - t_c}{1.603},$$

from which $t_c = 130^\circ\text{F}$, a safe value for the outside surface.

275. Logarithmic Mean Temperature Difference. Our discussion of the transfer of heat through walls from one fluid to another has been on the tacit assumption that the hot fluid remains at a constant temperature t_1 and the cold fluid remains at a constant temperature t_2 . However, in many instances, either the cold fluid or the hot, or both, undergo a change of temperature in passage through the heat exchanger.

For purposes of explanation, consider a double-pipe type of exchanger (Fig. 179). Let the hot fluid flow through the annular space and the cool fluid through the inner pipe. As the temperatures of the fluids change, the difference between the temperatures of the fluids changes. Thus we find different differences between the temperatures of the hot and cold fluids at different sections of the exchanger. In such cases, the *logarithmic mean temperature difference* (LMTD) represented by Δt_m , is used in place of $t_1 - t_2$. Since the rate of change of temperatures of the substances is not constant, the LMTD is not the same as the arithmetic mean, as shown in equation (93) below.

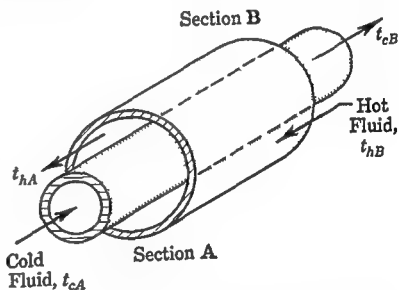


Fig. 179. Double-Pipe Arrangement. Observe the notation. One end of the exchanger is designated as section A and the other end, section B. In this illustration, a counterflow arrangement, t_{cA} is the initial temperature of the cold fluid and t_{hA} is the final temperature of the hot fluid.

At this time, we shall consider two types of flow, *parallel flow*, in which case the fluids flow in the same direction through the heat exchanger, and

counterflow, where the fluids flow in opposite directions. In a general way, the variation of temperatures is as indicated in Fig 180. In case of parallel flow, both fluids enter at section *A*, and as they pass to section *B* their temperatures approach one another. At and near section *A* [Fig 180(a)] the temperature difference is a maximum, and consequently the rate of flow of heat and the rate of decrease of temperature difference are a maximum. As the temperatures of the fluids approach one another, the rates of change of the temperatures decrease and the curves flatten out. In Fig 180(b), a counterflow arrangement, the cold fluid enters at section *B*, leaves at *A*. In contrast to parallel flow, heat transfer is taking place between the fluids at the moment when each is in its coldest state and when each is in its hottest state. The transfer of heat in a counterflow exchanger tends to conserve available energy and makes

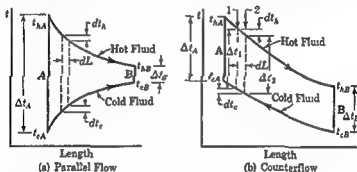


Fig 180 Temperature Variations, Parallel Flow and Counterflow. In each case, the temperature difference at section *A* is $\Delta t_A = t_{hA} - t_{cA}$, and at section *B* the temperature difference is $\Delta t_B = t_{hB} - t_{cB}$.

possible a higher final temperature of the fluid being heated than obtainable with parallel flow.

To find the equation for the mean temperature difference consider a differential area of transmitting surface dA in a counterflow exchanger. As the cold fluid flows past this area, its temperature increases a differential amount dt_c due to the transfer of a differential quantity of heat dQ the amount of which is $dQ = w_c c_c dt_c$, where w_c is the mass of cold fluid flowing in a unit time and c_c is the specific heat of the cold fluid. Assuming no heat loss from the exchanger and no change of kinetic energy we conclude that the heat loss of the hot fluid $dQ = w_h c_h dt_h$ is the same as the gain of the cold fluid. Note that the $\int dt_h$ is a negative number, so that

$$dt_c + dt_h = d(\Delta t)$$

is the change in the difference of temperatures, $\int d(\Delta t) = \Delta t_2 - \Delta t_1$ (Fig 180) across a differential area dA of infinitesimal length dL between sections 1 and 2. From the definition of the transmittance U , we have also

$$dQ = U dA \Delta t$$

Using the conclusions of the foregoing discussion, we get

$$dt_c + dt_h = d(\Delta t) = dQ \left(\frac{1}{w_c c_c} - \frac{1}{w_h c_h} \right) = U dA \Delta t \left(\frac{1}{w_c c_c} - \frac{1}{w_h c_h} \right),$$

where dQ is a positive number and the negative sign on the right-hand side is necessary to produce an arithmetic difference. Rearranging the foregoing equation, we have

$$\int_{\Delta t_A}^{\Delta t_B} \frac{d(\Delta t)}{\Delta t} = U \left(\frac{1}{w_c c_c} - \frac{1}{w_h c_h} \right) \int_0^A dA,$$

wherein the transmittance U and specific heats c are taken as constant. Integration of this equation gives

$$\ln \frac{\Delta t_B}{\Delta t_A} = -\ln \frac{\Delta t_A}{\Delta t_B} = \frac{UA}{w_c c_c} - \frac{UA}{w_h c_h} = - \left(\frac{UA}{w_h c_h} - \frac{UA}{w_c c_c} \right).$$

Now using $Q = UA \Delta t_m = w_c c_c (t_{cA} - t_{cB}) = w_h c_h (t_{hA} - t_{hB})$ in this equation, we find

$$\ln \frac{\Delta t_A}{\Delta t_B} = \frac{t_{hA} - t_{hB}}{\Delta t_m} - \frac{t_{cA} - t_{cB}}{\Delta t_m} = \frac{(t_{hA} - t_{cA}) - (t_{hB} - t_{cB})}{\Delta t_m},$$

from which

$$(93) \quad \Delta t_m = \frac{\Delta t_A - \Delta t_B}{\ln (\Delta t_A / \Delta t_B)},$$

the value of the log mean temperature difference between sections A and B when the difference in temperature at A is Δt_A , and the difference at B is Δt_B . It does not matter which end of the heat exchanger is taken as section A and which B ; the same result will be obtained from equation (93). A similar analysis of the case of parallel flow will yield the same equation (93). Hence this equation applies to either case and it serves also whenever the temperature of either fluid is constant, as during evaporation or condensation. When Δt_A is equal to or nearly equal to Δt_B , use the arithmetic average for Δt_m . Where there is cross flow, as around baffles in heat exchangers, and where there are fins, the mean temperature difference is not given by equation (93); nor in case w , c , or U varies. For such other situations, see McAdams (187). Equation (91) may now be written,

$$(91C) \quad Q = UA \Delta t_m.$$

In taking values of U for curved walls from the literature, notice whether they are based on the inside A_i or outside A_o surfaces.

276. Thermal Radiation. All bodies *radiate* heat. If two bodies were completely isolated in a vacuum from all other bodies, but not from each other, the colder body would radiate heat to the hotter body, and the hotter

body would radiate heat to the colder body. For purposes of explanation let all the radiant heat from the hotter body go to the colder body and all the radiant heat from the colder body to the hotter body. Then the colder body would be heated because due to its lower temperature it radiates less heat than it receives from the body with the higher temperature. Radiant heat or thermal radiation is an emanation of the same nature as light and radio waves. Like other waves in striking a body some of them will be reflected (the fraction being indicated by the *reflectivity*) some will be absorbed (the fraction being indicated by the *absorptivity*) and if the body is transparent to the waves some will be transmitted through the body (the fraction being given by the *transmissivity*). The sum of the reflectivity plus absorptivity plus transmissivity is equal to 1 or 100%. Bodies which are opaque to light are for the most part opaque to radiant heat (true of most solids). For opaque substances the energy of the radiant heat is either absorbed or reflected in which case the *reflectivity plus absorptivity is equal to one*. At the other extreme some gases and some glass transmit nearly all the radiant heat. Brightly polished metals are such good reflectors that most of the radiant heat may be reflected.

A body which absorbs all the impinging radiant heat is called a *black body*, a hypothetical conception in which the absorptivity is unity. A black body is also the best radiator. At a particular body temperature actual bodies radiate less heat than a black body. The ratio of the radiation from an actual body to the radiation from a black body is called the *emissivity* ϵ . The emissivity is not a constant property but usually it increases with the temperature of the radiating body.

Kirchoff's law may be stated as follows: the absorptivity a and the emissivity ϵ of a body are the same when the body is in thermal equilibrium with its surroundings. From this statement we can see that the emissivity of highly polished metals may be quite low.

The nearest approach to a black body is obtained by a hollow vessel penetrated only by a small pin hole through which radiant heat may pass to the inside. Once inside little of this radiant heat has a chance to be reflected or reradiated back through the pin hole. Thus for practical purposes this vessel absorbs all the energy entering the hole.

277 Stefan-Boltzmann Law This law states that the amount of radiation from a black body is proportional to the fourth power of the absolute temperature $Q_R = \sigma A T^4$ where $\sigma = (0.1713)(10^{-8})$ is the Stefan Boltzmann constant when Q is in Btu per hour and A sq ft is the radiating area of the black body whose surface temperature is $T^\circ R$. As usual this equation for an ideal situation must be modified to care for actual situations. As previously stated net heat transferred by radiation is the result of an interchange of radiation: the radiation from the hot body to the cold body minus the radiation from the cold body. Also the effective emissivities

and the fact that all the radiation from a certain source will not strike the surface must be considered. Introducing these corrections into the Stefan-Boltzmann law, we have

$$(94) \quad Q_r = 0.171 F_E F_A A \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right] \text{ Btu/hr.,}$$

where F_A is a view factor, sometimes called an angle factor, which allows for the average solid angle through which one surface "sees" the other; that is, it allows for the exchange of radiation which is direct (not reflected from intermediate surfaces);

F_E is a factor which depends on the individual emissivities ϵ (epsilon) and, in some instances, on the ratio of the areas.

A is the area in square feet of a surface.

Further detail on radiation is beyond the intended scope of this book.

278. Closure. At this stage of development of the science of heat transfer, the computation of the heat flux is dependent largely on experimental results, as indicated in the previous discussions. On this account, it is often desirable to search the literature for experiments related to the particular situation in which one is interested; or to verify one's design by experiment. If the effects of contaminants in the fluids and on the surfaces are included, the over-all coefficients vary widely for seemingly similar circumstances, which suggests a thoughtful approach to actual designs.

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APPENDIX A. PROBLEMS

NOTE 1 *If desired, additional problems, together with convenient tables of properties and working charts, are available in Problems on Thermodynamics by Faires, Brewer, Simmang, published by The Macmillan Co*

NOTE 2 *Unless otherwise stated in these problems, the atmospheric pressure should be taken as 14.7 psia (29.92 in Hg)*

NOTE 3 *The answers given are usually slide-rule answers and are subject to inherent errors*

1. THERMODYNAMIC PROPERTIES

1. Work, energy, and moment (or torque) are all measured in foot-pounds or similar units of space and force. May we add energy and moment? Explain fully.

2. Weight and mass may both be measured in pounds. If the weights given are as obtained on Mt. Ramer, is it correct to add the weights in pounds and masses in pounds? Is there any circumstance where such a sum would give either the total mass or the total weight? Discuss fully.

3. A body weighs 1000 lb at standard gravity. What would it weigh on top of Pikes Peak ($g \approx 32.12 \text{ fps}^2$)? at sea level at the equator ($g \approx 32.26 \text{ fps}^2$)? at sea level at the North Pole ($g \approx 32.09 \text{ fps}^2$)?

4. Five masses are as follows: m_1 is 500 gm of mass, m_2 weighs 800 gm, m_3 weighs 4 poundals, m_4 weighs 32.2 lb, and m_5 is 2 slugs. The weights are at standard gravity. What is the total mass expressed (a) in pounds, (b) in slugs, (c) in grams?

Ans: (a) 99.59 lb, (b) 3.1 slugs, (c) 45,200 gm

5. A 10-cu ft tank contains 3.22 lb of a gas. What is the density of the gas in a slugs/cu ft?

Ans: 0.01

6. (a) If the pressure gage on a tank reads 20 in Hg gage, what is the absolute pressure in inches Hg, in psi, and in psf? (b) If the gage reads 20 in Hg vacuum, what is the absolute pressure in inches Hg,

in psi, and in psf? (c) Is pressure an intensive or extensive property?

7. Suppose pressure is being measured by a column of water whose density is 62 lb/cu ft. Explain how to convert a column height z in feet to psi and determine the conversion constant.

8. Steam at a pressure of 150 psia and a temperature of 400°F occupies a volume of 3.223 cu ft/lb. (a) What is its density in lb/cu ft and in slugs/cu ft? (b) Convert the pressure to inches Hg and feet of water (60°F). (c) Convert the temperature to °R, °K, and °C. Is temperature an intensive or an extensive property?

Ans: (a) 0.31, 0.00965, (b) 306, 5085, (c) 860, 478, 204.4

9. The same as 8 except that the steam is at 200 psia, 500°F, 2.726 cu ft/lb.

10. What are the units of the constant 0.96 in $v = 0.96(as)^{1/2}$, where v is in mph, a in ft/sec², and s in ft?

Ans: mph/ft/s

11. A certain equation in heat transfer is $h = C \Delta T^{0.266} / (D^{0.2} T^{0.181})$, where h is the heat transfer coefficient for free convection in Btu/ft²-hr-°R, ΔT is a temperature difference in °R, D is a characteristic dimension in inches, and T is temperature in °R. What are the units, if any, of C ?

Ans: Btu-in^{0.2}/ft²-hr-°R^{1.085}

12-20. These numbers may be used for other problems.

2. CONSERVATION OF ENERGY

NOTE *Consider all energy problems from two viewpoints, the energy equation and the energy diagram.*

21. An inventor claims that he will be able to get 3,890,000 ft-lb of work from his engine in a specified time. An engineer investigating the claim finds that

during the same time, fuel is supplied to the engine which releases 4100 Btu of heat. What conclusion does the engineer draw concerning the inventor's claims? What is the basis of this conclusion?

22 A closed system which is undergoing cyclic changes of state receives heat at the rate 13 550 Btu/min, and discharges 17,110 Btu/min of heat. Over a complete cycle there is no change of stored energy which is entirely internal energy. The only other energy crossing the boundary is work. How much work is done? Is it done on or by the system?

Ans -59 33 Btu/sec

23 A closed system undergoes a process during which it receives 2250 Btu of heat and does 3220 Btu of work. What is the change of stored energy?

Ans -755,000 ft-lb

24 A closed system consisting of helium is compressed by 373 Btu of work while the internal energy increases 299 Btu. How much heat, if any, flows? in or out?

25. Assuming that there are no heat effects and no frictional effects find the kinetic energy and velocity of a 3220-lb body (mass of water) after it falls 778 ft from rest. Start with the steady flow equation, deleting energy terms which are irrelevant.

Ans $v_2 = 223.7$ fps

26 Water leaves a nozzle at 72 mph. If the jet is directed vertically upward, how far above the nozzle will the water go, ideally? This is an open system without frictional effects. Start with the steady flow equation, deleting energy terms which are not applicable.

Ans 173.5 ft

27 (a) A 64,400 lb airplane is traveling at 1000 fps (682 mph). How much is its kinetic energy in hp-hr? (b) If it noses vertically upward at this speed with power off and in a vacuum (no friction) at standard gravity, through what vertical distance would it move?

Ans (a) 1 288,000 Btu, (b) 2 94 mi

28 During a steady flow process the pressure of the working substance drops from 200 to 20 psia, the velocity increases from 200 to 1000 fps, the internal energy of the open system decreases 25 Btu/lb, and the volume increases from 1 to 10 cu ft/lb. No heat is transferred. Sketch an energy diagram and apply the steady flow equation for 10 lb/sec to determine

the work per lb. Convert to hp. Is it done on or by the substance?

Ans +5.85 Btu/lb

29 An air compressor (an open system) receives 600 lb/min of air at 14.4 psia and a specific volume of 2 cu ft/lb. The air flows steadily through the compressor and is discharged at 100 psia and 0.4 cu ft/lb. The initial internal energy of the air is 12 Btu/lb, at discharge the internal energy is 47 Btu/lb. The cooling water circulated around the cylinder carries away 33 Btu/lb of air. The change in kinetic energy is a 2 Btu/lb increase. $\Delta P = 0$. Sketch an energy diagram. Compute the work (by or on?) in Btu/lb and in hp.

Ans -72 Btu/lb

30 Steam enters the blades of a single-stage impulse turbine with a velocity of 1400 fps and leaves with a velocity of 460 fps. Assuming that there is no change in $h = u + pv/J$ and no transferred heat, apply the energy equation and calculate the work delivered to the blades per pound of steam flowing. Compute the hp for a flow of 42.4 lb/min. Sketch an energy diagram indicating the flow of energy.

Ans 34.9 Btu/lb

31 An air compressor picks up air for which $h = 20$ Btu/lb and delivers it when $h = 70$ Btu/lb. While passing through the compressor the air rejects 10 Btu/lb. Neglect the change of kinetic energy and compute the work (in Btu and hp) for compressing 50 lb/min. Apply the energy equation and then show the energy diagram.

Ans -3000 Btu/min

32 (a) A boiler is a steady flow device. Let the system be the H_2O between entry and exit. Entering energy consists of the internal energy of the water, the flow work, and the heat transferred to generate the steam. Departing energy consists of internal energy and flow work. The change of kinetic energy is generally negligible. Show an energy diagram and set up an expression for the heat to generate 1 lb of steam. (b) Water enters a boiler with $h = 128$ Btu/lb. The steam leaving has internal energy $u_2 = 1111$ Btu/lb and does flow work passing the boundary of $W_{f2} = 83.9$ Btu/lb. What is the transferred heat?

33 (a) Apply the energy equation to a turbine where the transferred heat is negligible, and set up the expression for the work of 1 lb of steam. Show the energy

diagram. (b) Steam enters a turbine with $h = 1197.5$ Btu/lb. It leaves with $h_2 = 985$ Btu/lb.; $\Delta K = 0$. What is the work done in one hour when the steam flows through at the rate of 1 lb./sec.? What is the corresponding hp?

Ans. 763,920 Btu/hr.

34. Steam enters a turbine with a velocity of 50 fps and a value of $h_1 = 1292$ Btu/lb. The exit velocity is 117 fps and $h_2 = 1098$ Btu/lb. Loss by radiation is 13 Btu/lb. (a) What is the work in Btu/min. and hp if the amount of steam flowing is 2 lb./sec.? (b) The change of kinetic energy is what percentage of Δh ? Sketch an energy diagram.

Ans. (a) 21,690 Btu/min.

35. A combustor on a gas turbine, Fig. 80, p. 152, receives air with an enthalpy of 208 Btu/lb. and, at the same time, fuel which carries with it 323 Btu/lb. of air, mostly chemical energy which is released by combustion. The heat loss is 1.4 Btu/lb. of air passing through, and the amount of energy leaving in the exit stream in the form of unburned fuel is 16 Btu/lb. of air. What is the enthalpy of the departing gases? Sketch an energy diagram.

Ans. 501 Btu.

36. A gas in steady flow at the rate of 1000 lb./min. performs a process in which $Q = 0$ and the enthalpy decreases 19.2 Btu/lb. (a) If this process occurs in a nozzle where $W = 0$, what is the change of kinetic energy? If the initial velocity

is negligible, what is the final velocity? (b) Considering the process in the turbine as a whole where the change of kinetic energy is zero, determine the work in Btu/min. and in hp.

37. Jet engines provide a thrust of 10,000 lb. on a plane moving at 500 mph. What is the rate at which work is being expended on the plane in Btu/sec., in hp, in kw, and in hp-hr./sec.?

38. (a) Determine the $\int p dV$ for a process during which the pressure remains constant and evaluate it in Btu for a pressure of 778 psi during a volume change from 2 to 10 cu. ft. What does this integral evaluate for a reversible nonflow process? (b) Using the $\int p dV$, determine the nonflow work while the volume remains constant.

39. A nonflow process occurs for which $p = V^2 + 8/V$ psia, where V is in cu. ft. If the volume expands from 5 to 10 cu. ft., what is the work? *Ans.* 55.1 Btu.

40. A process occurs during which $p = 6V + 20$ psia while the volume V changes from 2 to 3 cu. ft. What is the $\int p dV$ in Btu? What is this in a nonflow process?

41. The same as 40 except that $pV^n = 778$, where p is in psia, V in cu. ft., and $n = 1.28$.

42-50. These numbers may be used for other problems.

3. ENERGY RELATIONS

51. The heat rejected by 10 lb. of each of the following gases is 200 Btu: nitrogen, helium, argon. For each gas, what is the change of temperature (a) if pressure remains constant, (b) if volume remains constant?

52. While the pressure remains constant, 3 lb. of a gas are reversibly cooled from 340°F to 60°F. Assuming constant c_p , determine the change of entropy if the gas is (a) air, (b) argon, (c) helium.

Ans. (a) -0.31 Btu/°F, (b) -0.161 Btu/°F, (c) -1.61 Btu/°F.

53. The same as 52 except that $V = C$.

54. From the curves of Fig. 19, estimate the mean specific heat of O_2 between 1000°F and 4000°F and find the heat

necessary to change the temperature of 1 lb. through this temperature range while the pressure is constant. What percentage error would there be in using the value of c_p from Table I in this computation?

55. The constant pressure specific heat of a certain gas is $c_p = 0.3 + 0.0037T$ Btu/lb., where T is °R, applicable from 0°F to 600°F. The corresponding $c_v = -0.3 + 0.0037T$. Determine the heat to raise the temperature of 3 lb. from 40°F to 540°F (a) at constant pressure, (b) at constant volume. (c) Compute the mean specific heat for the specified temperature range. *Ans.* (a) 3825 Btu, (b) 2925 Btu.

56. How much heat is necessary to raise the temperature of 10 lb. of CO_2 from

400°F to 4000°F during a constant pressure process if

$$c_p = 0.276 + 1.4687/10^5 - 0.994/T^{1/2}$$

$$\text{Ans } 10,590 \text{ Btu}$$

57. What are the instantaneous specific heats at constant pressure and at constant volume, Btu/mol, Btu/lb and ft lb/lb, at a temperature of 3510°F for each of the following gases CO_2 , N_2 , air? By what percentages do these values vary from those at normal atmospheric temperatures?

58. If the temperature of a substance remains constant at 280°F while it receives 1480 Btu of heat in an internally reversible process, what is ΔS (a) for air, (b) for hydrogen? $\text{Ans } (a) 2 \text{ Btu}/^\circ\text{F}$

59. If a reversible process occurs such that $T = 100 + 300S$, what heat, reversibly transferred, will change the entropy from 0.5 to 0.6 Btu/°F? Between what two temperatures does this process take place? Roughly sketch this process on the TS plane. $\text{Ans } 19.1 \text{ Btu}$

60. The $\dot{p} dV = 20,000 \text{ ft lb/min}$ in a steady flow process of helium during which the pressure changes from 75 to 25 psia while the volume changes from 5 to 9.68 cfm. (a) If this process occurs in a nozzle determine the change of kinetic energy and the exit velocity when $A_1 = 0$. Consider equation (k) § 4.1. (b) If this process is applied to a turbine with $\Delta K = 0$, what is

the work? (c) Determine ΔH . (d) Integrate $-\dot{V} dp$ for $pV^k = C$ and determine its magnitude. (e) What is Q ? Consider equation (m), § 4.2.

$\text{Ans } (a) \Delta K = 62 \text{ Btu}, (e) \text{ Approx zero}$

61. For the conditions of 40, namely, $p = 6V + 20 \text{ psia}$, $V_1 = 2$, and $V_2 = 3 \text{ cu ft}$, make the integral $-\dot{V} dp$ and evaluate it in Btu. What does this integral represent when the change of kinetic energy is negligible?

62. (a) If $p = -28.8V + 900 \text{ psia}$ while the volume changes from 10 to 20 cu ft, determine $\int p dV$ and $-\dot{V} dp$ by integrations. Sketch the curve on the pV plane and designate the areas which represent these integrals. Name the energies which these integrals represent. (b) If the system should be a diatomic gas what heat is transferred?

$\text{Ans } (a) 866 \text{ Btu}, 800 \text{ Btu} (b) 1031 \text{ Btu}$

63. (a) During a certain reversible process of 5.49 lb of argon $Q = -164.5 \text{ Btu}$ and $\Delta H = 216.5 \text{ Btu}$. What are the values of $-\dot{V} dp$, ΔU , and $\int p dV$? (b) If this is a nonflow process what is the work (on or by)? (c) If the temperature range is 40°F to 350°F what is the mean specific heat of the process?

$\text{Ans } (a) -381 \text{ Btu}, 130 \text{ Btu}, -204.5 \text{ Btu} (c) -0.095 \text{ Btu/lb } ^\circ\text{R}$

64-70 These numbers may be used for other problems.

4 THE IDEAL GAS

71. The temperature of an ideal gas remains constant while the pressure changes from 14.7 psia to 100 psia. (a) If the initial volume was 3.2 cu ft, what is the final volume? (b) If the mass of gas is 0.2 lb, determine the change of density.

$\text{Ans } (a) 0.410 \text{ cu ft}, (b) 0.425 \text{ lb/cu ft increase}$

72. Hydrogen at 400°R occupies 30 cu ft. (a) If the volume is increased to 90 cu ft at constant pressure, what is the final temperature in °K and °C? (b) If the pressure is next tripled while the volume remains constant what is the temperature in °F?

$\text{Ans } (a) 667^\circ\text{K} (b) 3140^\circ\text{F}$

73. Compute the density of the following gases at 100 psia and 60°F. (a) air,

(b) hydrogen, (c) carbon dioxide. (d) oxygen. $\text{Ans } (c) 0.574 \text{ lb/cu ft}$

74. An automobile tire contains 3730 cu in of air at 32 psia and 80°F. (a) What mass of air is in the tire? (b) In operation this air temperature increases to 145°F. If the tire is inflexible what is the resulting percentage increase in gage pressure? (c) What mass of the 145°F air must be bled off to reduce the pressure back to its original value?

$\text{Ans } (a) 0.504 \text{ lb}, (b) 17.5\%, (c) 0.04 \text{ lb}$

75. A tank 6 in in diameter and 48 in long contained acetylene at 250 psia and 80°F. After some acetylene was used the pressure was 200 psia and the temperature was 70°F. (a) What percentage of the acetylene was used? (b) What volume

would the used acetylene occupy at 14.7 psia and 60°F?

Ans. (a) 18.5%, (b) 2.38 cu. ft.

76. A spherical balloon is 40 ft. in diameter and surrounded by air at 60°F and 29.92 in. Hg abs. (a) If the balloon is filled with hydrogen at a temperature of 70°F and atmospheric pressure, what total load can it lift? (b) If it contains helium instead of hydrogen, other conditions remaining the same, what load can it lift? (c) Helium is nearly twice as heavy as hydrogen. Does it have half the lifting force?

Ans. (a) 2380 lb.

77. An engineer estimates that for proper ventilation of an industrial plant 40,000 cfm of air are needed when the outside temperature is 100°F and the atmospheric pressure is 14.3 psia. What volume of air is being handled as measured under standard conditions of 60°F and 14.7 psia?

78. A tank contains air at 800 psia and 80°F. After the removal of an amount of air which measures 700 cu. ft. at standard atmospheric pressure and 60°F, the pressure is found to be 300 psig and the temperature 80°F. What is the volume of the tank?

Ans. 22 cu. ft.

79. A closed vessel *A* contains $V_A = 2$ cu. ft. of air at 600 psia and 100°F. This vessel connects with *B* which contains an unknown volume of air V_B at 20 psia and 40°F. When the valve separating these vessels is opened, internal equilibrium of the mixture is established at 250 psia and 60°F. What is V_B ?

Ans. 2.67 cu. ft.

80. The volume of a 6×12-ft. tank is 339.2 cu. ft. It contains air at 200 psig and 75°F. How many 1-cu. ft. drums can be filled to 50 psig and 65°F if it is assumed that the air temperature in the tank remains at 75°F? The drums have been sitting around in the atmosphere which is at 14.7 psia and 65°F.

Ans. 992.

81. Using the universal gas constant \bar{R} and molecular weights, determine the gas constant in (a) ft-lb./lb.-°R, (b) Btu/lb.-°R, (c) Btu/mol.-°R, and (d) atm-ft.³/mol.-°R: for air and hydrogen.

Ans. (c) 1.986, (d) 0.73.

82. During a process of 4 lb. of air, the temperature increases from 40°F to 340°F. Find (a) ΔU , (b) ΔH . (c) If $Q = 0$, what are the numerical values of $\int p dV$ and $-\int V dp$? See § 51.

Ans. (a) 205.5 Btu, (b) 288 Btu.

83. During a certain process of helium, the state changes from $p_1 = 75$ psia and $V_1 = 5$ cu. ft. to $p_2 = 25$ psia and $V_2 = 9.68$ cu. ft. What are ΔU and ΔH ?

Ans. -37.4 Btu, -62 Btu.

84. For a certain ideal gas, $R = 35$ ft-lb./lb.-°R and $k = 1.3$. It is at 60 psig and 90°F. (a) What are c_p and c_v ? (b) What volume will 5 lb. of this gas occupy? (c) If 20 Btu are added at constant volume, what are the resulting temperature and pressure?

Ans. (a) $c_p = 0.195$, (b) 8.94 cu. ft., (c) 117°F and 63.7 psig.

85. The same as 84 except that $R = 77.8$ and $k = 1.4$.

86. The gas constant of a certain ideal gas is 400 ft-lb./lb.-°R; $c_v = 0.8$ Btu/lb.; $w = 5$ lb. From the state $p_1 = 100$ psia and $V_1 = 40$ cu. ft., the gas undergoes a constant pressure process until the temperature becomes 1000°R. Find (a) c_p and k , (b) ΔU , (c) ΔH , (d) Q , (e) ΔS , and (f) the $\int p dV$. What does this integral represent in a nonflow process? Does the nonflow energy equation balance?

Ans. (b) 2848 Btu, (c) 4680 Btu, (e) 1832 Btu.

87. The same as 86 except that $R = 77.8$ and $c_v = 0.2$.

88-100. These numbers may be used for other problems.

5. PROCESSES OF IDEAL GASES

NOTE. In each of the following problems, indicate clearly whether the work is done on or by the substance, whether the heat is transferred to or from the substance, and whether there is an increase or a decrease of internal energy, enthalpy, and entropy. Moreover, a sketch of the process on the pV and TS planes and energy diagrams should be included in all solutions.

101. Ten cu. ft. of air at 300 psia and 400°F is cooled to 140°F at constant vol-

ume. What is (a) the final pressure, (b) the work, (c) the change of internal energy,

(d) the transferred heat (e) the change of enthalpy (f) the change of entropy?

Ans (a) 219.5 psia (c) -420 Btu (e) -587 Btu (f) -0.581 Btu/°R

102 If 500 Btu of heat are added to 5 lb of helium at constant volume when the initial temperature is 100°F, find (a) the final temperature (b) ΔH (c) ΔS (d) ΔU (e) W (f) If this state change of the system is brought about by paddle work instead of heat, what work is required and what are the changes in the properties? Draw an energy diagram. Qualitatively how do the entropy changes of the surroundings compare for the reversible and irreversible processes? Explain.

Ans (a) 232.5°F, (b) 828 Btu (c) 0.706 Btu/°R

103 Same as 102 except that the gas is one for which $R = 77.8$ and $k = 1.4$.

104 The pressure on 3 lb of air is decreased at constant volume from 200 to 50 psia. What is ΔS ?

Ans -0.713 Btu/°R

105 The pressure on 5 cu ft of hydrogen being heated at constant volume changes from 40 psia to 200 psig. Find (a) the work (b) ΔU (c) Q (d) ΔH (e) ΔS Btu/lb °R (f) If this constant volume process is irreversible and $Q = 0$, $W = 399$ Btu of paddle work, what are ΔU and ΔS of the hydrogen system? Draw an energy diagram. Qualitatively how do the entropy changes of the surroundings compare for the reversible and irreversible processes? Explain.

Ans (a) 399 Btu (d) 560 Btu (e) 4.09 Btu/lb °R

106 The temperature of 5 lb of air decreases from 300°F to 40°F while $p = C$. Compute (a) ΔU (b) ΔH (c) Q (d) ΔS (e) $\int p dV$ (f) $-\int V dp$ (g) For an internally reversible nonflow process, what is the work? (h) If the nonflow process is irreversible with $W = 75$ Btu, what is Q for the same state change (energy diagram)? (i) If the process is steady flow with $W = 0$ and $\Delta K = +12$ Btu, what is the heat? (j) Does the energy equation $Q = \Delta H - \int V dp/J$ balance?

Ans (b) -312 Btu (d) -0.5025 Btu/°R, (e) 89.2 Btu

107 While the pressure remains constant at 100 psia, the volume of a system of air changes from 20 cu ft to 10 cu ft. What are (a) ΔU (b) ΔH (c) Q (d) ΔS ?

(e) If the process is nonflow and internally reversible, what is the work? (f) If the process is steady flow with $W = 0$ and $\Delta K = -5$ Btu, what is Q ?

Ans (a) -463 Btu (b) -648 Btu (e) -185 Btu

108 A certain gas with $c_p = 0.529$ and $R = 96.2$ ft lb/lb °R expands from 5 cu ft and 80°F to 15 cu ft while $p = 15.5$ psia = a constant. Compute (a) T_2 (b) ΔH (c) ΔU (d) ΔS (e) For an internally reversible nonflow process, what is the $\int p dV$ and the work? (f) For an irreversible nonflow process during which $W = 20$ Btu, what is Q ? (g) For a steady flow process in which $W = 0$ and $\Delta K = -12$ Btu, what is Q ?

Ans (a) 1160°F (b) 122.8 Btu (c) 94 Btu (d) 0.125 Btu/°R, (e) 28.7 Btu

109 The heat transferred to a certain quantity of air during a constant pressure process is 48 Btu. What are ΔU , ΔH and W ?

Ans $W = 13.8$ Btu

110 During an isothermal process at 88°F, the pressure on 3 lb of air drops from 80 psia to 5 psig. For an internally reversible process, determine (a) the $\int p dV$ and the work of a nonflow process (b) the $-\int V dp$ and the work of a steady flow process during which $\Delta K = 0$ (c) Q (d) ΔU and ΔH (e) ΔS .

Ans (a) 421 Btu (e) 0.768 Btu/°R

111 While its temperature remains constant at 80°F, 2.5 lb of a certain gas whose $c_p = 0.28$ and $c_v = 0.2$ Btu/lb °R rejects 300 Btu. $p_1 = 85$ psia. Determine (a) V_1 , V_2 and p_2 (b) W for a reversible nonflow process (c) ΔS (d) ΔU and ΔH (e) the change of kinetic energy if the process is a reversible steady flow during which $W = -350$ Btu.

112 During the isothermal expansion of 5 lb of air, the gain in entropy is 0.552 units. What is (a) the expansion ratio V_2/V_1 (b) the ratio p_2/p_1 ? Ans (a) 5

113 From a state defined by 300 psia, 100 cu ft and 240°F, helium undergoes an isentropic process to 0.3 psig. Find (a) V_2 and t_2 (b) ΔU and ΔH (c) $\int p dV$ (d) $-\int V dp$ (e) Q and ΔS . What is the work (f) if the process is nonflow (g) if the process is steady flow and $\Delta K = +10$ Btu?

Ans (a) 608 cu ft 247°F (b) -5870 -9740 Btu (g) 9730 Btu

114 The same as 113 except that the gas is methane.

115. During an isentropic process of 4 lb. of air, the temperature increases from 40°F to 340°F. Compute (a) ΔU and ΔH , (b) $\int p dV$, (c) $-\int V dp$, (d) Q and ΔS . What is the work (e) if the process is nonflow? (f) if the process is steady with $\Delta K = -10$ Btu? (g) For an irreversible adiabatic process from the same initial state to the same final pressure, the final temperature is 400°F. What are the works of the nonflow and steady flow ($\Delta K = -10$) processes and the change of entropy? Is $\int p dV/J = -\Delta U$? Find m , p , T , and evaluate the integral for $pV^m = C$.

Ans. (a) 205.5, 288 Btu, (g) -247, -335.5 Btu.

116. The same as 115 except that the gas is argon.

117. From 85.3 psig and 0.5 cu. ft., 4 lb. of an ideal gas, for which $c_p = 0.124$ and $c_v = 0.093$ Btu/lb.-°R, undergo an isentropic process to a volume of 3 cu. ft. What are (a) R and t_2 , (b) ΔU and ΔH , (c) $\int p dV$, (d) $-\int V dp$, (e) Q and ΔS ? (f) What is the work if the process is nonflow? (g) If the process occurs in a nozzle, $W = 0$, what is ΔK ? Also compute v_2 if K_1 is negligible. (h) The foregoing expansion between the stated pressures is actually an irreversible adiabatic with a final temperature of 240°F. Compute W' (nonflow) and $\Delta K'$ as asked for above. Is $\int p dV/J = -\Delta U$? Find m , p , T , and evaluate the integral for $pV^m = C$.

Ans. (a) 24.1, 194°F, (b) -50.2, 67 Btu, (h) $m = 1.3$.

118. The same as 117 except that $c_p = 0.7$ and $c_v = 0.5$ Btu/lb.-°R.

119. During a polytropic process, 10 lb. of an ideal gas, whose $R = 40$ and $c_p = 0.25$, changes state from 20 psia and 40°F to 120 psia and 340°F. Determine (a) n , (b) ΔU and ΔH , (c) ΔS , (d) Q , (e) $\int p dV$, (f) $-\int V dp$. (g) If the process is steady flow during which $\Delta K = 0$, what is W ? What is ΔK if $W = 0$? (h) What is W for a nonflow process?

120. The same as 119 except that $R = 155.6$ and $c_p = 0.7$.

121. A polytropic process of argon occurs from 14.7 psia and 50 cu. ft. to 125 psia and 9.6 cu. ft. Find (a) n , (b) ΔU and ΔH , (c) $\int p dV$, (d) $-\int V dp$. (e) Compute the value of Q from the specific heat equation and then check it from the energy equations $Q = \Delta U + \int p dV/J$ and

$Q = \Delta H - \int V dp/J$. (f) What are the works of a reversible nonflow process and of a reversible steady flow during which $\Delta K = -20$ Btu? Does the area "behind" the curve represent this work? Explain.

Ans. (a) 1.295, (b) 130, 216.5 Btu, (e) -164.5 Btu, (f) -294.5, -361 Btu.

122. A polytropic process of air from 150 psia, 300°F, and 1 cu. ft. occurs to $p_2 = 20$ psia in accordance with $pV^{1.3} = C$. Determine (a) t_2 and V_2 , (b) ΔU , ΔH , and ΔS , (c) $\int p dV$ and $-\int V dp$. (d) Compute the heat transferred from the polytropic specific heat and check by energy equations $Q = \Delta U + \int p dV/J$ and $Q = \Delta H - \int V dp/J$. (e) Find the nonflow work, and (f) the steady flow work for $\Delta K = 0$.

Ans. (a) 17°F, 4.71 cu. ft., (b) -25.85, -36.2 Btu, +0.0142 Btu/°R, (d) 8.62 Btu, (e) 34.45, 44.8 Btu.

123. The nonflow work of reversibly compressing a certain amount of air in accordance with $pV^{1.32} = C$ is 60,000 ft.-lb. Compute (a) ΔU , (b) ΔH , (c) Q from the specific heat equation. (d) If a reversible steady flow with $\Delta K = 0$ (instead of nonflow) takes place between the same states, what is the work?

Ans. (a) 61.8 Btu, (b) 86.5 Btu, (c) -15.45 Btu, (d) -102 Btu.

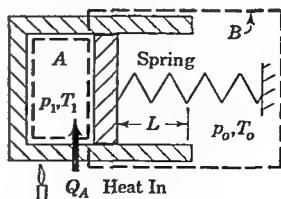
124. The same as 123 except that the gas is methane.

Ans. (a) 81.8 Btu, (b) 106.6 Btu, (c) 5.12 Btu.

125. If 1 lb. of air is throttled from 60 psig to 12 psia, what is the change of entropy? Ans. 0.1253 Btu/lb.-°R.

126. The same as 125 except that the gas is helium.

127. Given a system A (see figure) which is 0.1 lb. of air at $p_1 = 14.7$ psia and



Problem 127.

$t_1 = 80^\circ\text{F}$. The movable piston, whose area is 30 sq. in., does not permit the passage of matter or heat. The only heat is

Q_A added to system A until the spring has been compressed from its free length (zero force) an amount $L = 6$ in. after which the state of A is defined by p_1 and t_1 . System B includes part of the atmosphere and the spring whose scale is 500 lb/in. $p_0 = 14.7$ psia and $t_0 = 80^\circ\text{F}$ which values do not change. Make energy diagrams and write the energy equations (a) for system A between the states 1 and 2 and (b) for system B while A changes from 1 to 2 in the form of statement (5B). § 25 (c) How much energy is stored in the spring (Btu)? (d) What work (Btu) is

done in displacing part of the mass in system B across its boundaries? (e) Sketch the p - v curve for system A . What work is done by system A ? (f) What is the final temperature of system A and its change of internal energy? Let the average specific heat at constant volume be 0.2 Btu/lb $^\circ\text{F}$. (g) What is the heat added? (h) What mass of air is displaced from the system B ?

Ans (c) 0.964 Btu (e) 1.249 Btu (f) 79.8 Btu (g) 81.0 Btu

128-140 These numbers may be used for other problems

6. CYCLE ANALYSIS AND THE SECOND LAW

141 During a Carnot cycle the working substance receives 500 Btu at 540°F and rejects heat at 140°F . Find (a) the thermal efficiency (b) the work (c) the rejected heat (d) ΔS during the isothermal processes

Ans (a) 40% (b) 200 Btu (d) 0.5 Btu/ $^\circ\text{R}$

142 A Carnot power cycle operates on 2 lb of air between the limits of 70°F and 500°F . The pressure at the beginning of isothermal expansion is 400 psia and at the end of isothermal expansion is 185 psig. Determine (a) the volume at the end of the isothermal compression (b) ΔS during an isothermal process (c) Q_A (d) Q_R (e) W (f) e (g) the ratio of expansion during isothermal heating and the overall ratio of expansion (h) the mep

Ans (a) 7.82 cu ft (b) 0.0948 (c) 91 Btu (d) 50.3 Btu (h) 15.88 psi

143 During a single Carnot cycle the change of entropy of the working substance during the addition of 400 Btu of heat is 0.01 Btu/ $^\circ\text{R}$. The efficiency of the cycle is 30%. Determine (a) the temperatures ($^\circ\text{F}$) at which heat is transferred (b) the rejected heat (c) How many cycles must be completed per minute for an output of 100 hp?

144 Air is used as a working substance in an Ericsson cycle. Properties at the beginning of isothermal expansion are 100 psia 5 cu ft and 540°F . For a ratio of isothermal expansion of 2 and a minimum temperature of 40°F find (a) ΔS during the isothermal process (b) Q_A (c) Q_R (d) net W (e) e (f) the volume at the end

of isothermal expansion and the overall ratio of expansion (g) p_m

Ans (b) 64 Btu (g) 3.35 ps

145 Air is made to pass through a Stirling cycle which consists of two isothermal processes and two regenerative constant volume processes. At the beginning of isothermal expansion $p_0 = 100$ psia $V_0 = 2$ cu ft and $t_0 = 600^\circ\text{F}$. The ratio of isothermal expansion is $V_1/V_0 = 1.5$ and the minimum temperature is $t_1 = 80^\circ\text{F}$. Calculate (a) ΔS during the isothermal processes (b) Q_A (c) Q_R (d) net W (e) e and (f) mep

Ans (b) 15.75 Btu (e) 49.1% (f) 41.7 psi

146 From 20 psia and 80°F 10 cu ft of oxygen are isothermally compressed 1.2 to 80 psia. Next heat is supplied at constant pressure 2.3 until the entropy returns to its original value after which a reversible expansion 3.1 closes the cycle. (a) Sketch this cycle on the p - v and T - S planes and give equations for Q_A , Q_R , W , e and p_m . (b) Find numerical values of V_1 , T_2 , p_2 , ΔS_{1-2} , net W (from p - v and T - S planes), e and p_m . (c) If the rate at which the gas is circulated through the cycle is 10 cfm at 1 what horsepower is being developed?

Ans (b) $\Delta S_{1-2} = -0.095$ Btu/ $^\circ\text{R}$ $W = 11.4$ Btu $e = 18.2\%$ $p_m = 8.18$ psi (c) 0.269 hp

147 Ten cu ft of helium at 20 psia and 80°F are compressed isentropically 1.2 to 80 psia. The helium is then expanded polytropically 2.3 with $n = 1.35$ to the initial temperature. An isothermal rejection of heat 3.1 returns the helium to the

initial state. (a) Sketch this cycle on the pV and TS planes and give equations for Q_A , Q_R , W (from pV and TS planes), e , and p_m . (b) Find numerical values for T_2 , V_2 , p_2 , Q_A , Q_R , ΔU_{1-2} , ΔU_{2-1} , ΔU_{1-2-1} , net W , and ΔS_{3-1} .

Ans. (b) $p_2 = 9.52$ psi, $Q_A = 36.55$ Btu, $\Delta U_{1-2} = 41.4$ Btu, $W = 9.1$ Btu.

148. An engine operates on 0.3 lb. of air as a working substance, which is initially at 25 psia and 100°F. There occurs a constant volume heating 1-2 to 125 psia, after which there is an isentropic expansion 2-3 to 25 psia. Finally, a constant pressure process 3-1 returns the air to the initial condition. (a) Sketch this cycle on the pV and TS planes and give equations for Q_A , Q_R , W (from pV and TS planes), e , and p_m . (b) Find numerical values for these quantities.

Ans. (b) $Q_A = 115.3$ Btu, $W = 28.3$ Btu, $p_m = 28.4$ psi.

149. One lb. of argon at 20 psia and 200°F (state 1) is compressed polytropically with $pV^{1.2} = C$ to 100 psia (state 2). It is next cooled at constant pressure to the initial temperature (state 3), then expanded isothermally to the initial state. (a) Sketch this cycle on the pV and TS planes and give equations for Q_A , Q_R , W (from pV and TS planes), and p_m . (b) Find the numerical values of V_1 , V_2 , T_2 , Q_A , Q_R , W , and p_m .

Ans. (b) $V_2 = 2.32$ cu. ft., $Q_A = 52.9$ Btu, $W = -8.2$ Btu, $p_m = 6.24$ psi.

150. In a reversed cycle, there is a work input of 100 Btu/min., a work output of 50 Btu/min., while 150 Btu/min. are rejected. What is the heat added to the system and the COP if the cycle is used for (a) refrigeration, (b) heating? (c) What horsepower is required to drive the cycle?

Ans. (a) $COP = 2$, (b) $COP = 3$.

151. A Carnot engine has a thermal efficiency at 25% as a power engine. It is reversed and does 900 Btu of refrigeration. (a) Determine the work and COP . (b) If this system is used for heating, what is the COP ? Is more work required than in (a)?

Ans. (a) $COP = 3$, (b) $COP = 4$.

152. The COP of a reversed Carnot cycle is 5.35 when the refrigeration is done at 0°F. The change of entropy during the isothermal processes is 0.2 Btu/°R. Find (a) the amount of refrigeration done, (b) the temperature at which heat is rejected,

(c) the net work. (d) If this cycle is used for heating, what is the COP ?

Ans. (a) 92 Btu, (b) 86°F, (c) -17.2 Btu, (d) 6.35.

153. A reversed Carnot cycle acts as a heat pump to deliver 4600 Btu/min. to a heated space at 75°F, having received heat from a 5°F cold body. (a) What are W (in hp) and the COP ? (b) If this same system is used as a refrigerating machine, what is its COP ? (c) Prove that the COP of a reversed cycle used for heating, COP_h , is the COP of the same cycle used for cooling, COP_c , plus one; i.e., $COP_h = COP_c + 1$.

Ans. (a) 14.15 hp, 7.67, (b) 6.67.

154. The layman sometimes points out that there are about 19,000 Btu of heating value in a pound of gasoline, that his automobile engine produces, say, 4000 Btu of useful work per pound of fuel, and that when the engine is perfected, it will produce almost 19,000 Btu of work per pound of fuel. Refute this idea.

155. From a source at a constant temperature of 3000°F, there are transferred 2000 Btu to a system which accepts this heat at a constant temperature of 300°F. The available sink temperature is 80°F. Determine the change of entropy (a) of the source, (b) of the system. What is the net change? (c) Compute the available part of the 2000 Btu at the source temperature and at the system temperature. What is the change of available energy? Does this check $T_a \Delta S$? (d) Suppose that the system received the 2000 Btu of energy as paddle work (instead of from the source). What are the net increases in unavailable energy and entropy?

Ans. (b) 2.057 net change, (c) 1108 Btu.

156. While the pressure is constant, 5 lb. of air at 140°F receive 1200 Btu of heat. (a) If the sink is at 80°F, how much of the heat supplied is unavailable? (b) What cycle would convert all of the available energy into work (i.e., sketch on TS plane)? What is its thermal efficiency? (c) Under what circumstance may all of this transferred heat be converted into ideal work? (d) If the energy received by the system, 1200 Btu, had its origin as paddle work (instead of heat), what would be the increase of unavailable energy?

Ans. (a) 636 Btu, (b) 47%.

157. Steam at 456°F with a specific heat of $c_p = 0.6$ Btu/lb.-°R is to be heated

to 708°F in a heat exchanger by heat from a gas whose $c_p = 0.24$ and whose initial temperature is 1500°F. The rate of flow of the gas is 52.5 lb/sec, and of the steam 25 lb/sec. Let the flow be reversible and compute (a) the final temperature of the gas, (b) the change in entropy of the steam, (c) the change in entropy of the gas, (d) the net change of entropy. (e) What is the net loss of available energy of the system for $T_0 = 530^\circ\text{R}$? (f) How does the entropy change in any actual process?

Ans (a) 1200°F, (d) 1.54 Btu/°R-sec, (e) 816 Btu/sec

168 An economizer which may be taken as a thermally isolated system and heat exchanger, has a through flow of 405,800 lb/hr of water and 536,000 lb/hr of flue gas whose $c_p = 0.24$ Btu/lb-°R. The heat from the cooling of the gas from 767°F to 386°F goes to raise the water

temperature from 263°F to 384°F. (This "preheats" the water on its way to the boiler and thus saves heat in the boiler.) If the system processes are internally reversible (no friction of flow), what is (a) the net change of entropy due to the irreversible heat flow and (b) the net increase of unavailable energy for a sink temperature of 40°F? (c) How much available energy is lost to the sink in the gas leaving the economizer (at 386°F)? (d) If the hot gas (767°F) is discharged to the sink (without going through the economizer) what is the loss of available energy? Consider that the heat is rejected at constant pressure. Has available energy been conserved by use of the economizer?

Ans (b) 7.6×10^6 Btu/hr (d) 106×10^6 Btu/hr

169-170 These numbers may be used for other problems

7. COMPRESSION AND EXPANSION OF GASES

NOTE In general, it would be best to work these problems by using constant specific heats and then to check the results by using the air table, p 146. While the differences in the answers so obtained for the low temperatures involved should be small, some difference is to be expected.

171 An indicator with a 60-lb spring is used for cards from a double-acting 12x16 in compressor running at 200 rpm. The length of all cards is 3.5 in. the average area of the head-end cards is 2.55 sq in. and of the crank end cards 2.5 sq in. The diameter of piston rod is 2 in. Compute the indicated mep's on each side of the piston, the corresponding ihp's, and the total ihp.

172 The capacity of a compressor is 14.1 cfm measured at intake where $p_1 = 14$ psia and $t_1 = 80^\circ\text{F}$. Discharge is at 84 psia with negligible change of kinetic energy. (a) For isentropic compression determine the mass of air delivered (lb/min), the temperature at the end of compression, and the conventional work and horsepower. (b) The same as (a) except that the compression is polytropic with $n = 1.35$. (c) The same as (a) except that the compression is isothermal. (d) The same as (a) except that the process is an irreversible adiabatic with $m = 1.45$. (e) In the various fore-

going situations, would it take more or less or the same work to compress the same volume of hydrogen?

Ans (a) $t_2 = 442^\circ\text{F}$, 143.5 hp, (b) 139.5 hp, (c) 109.5 hp (d) 159 hp

173 Hydrogen enters a compressor at 80°F after a compression according to $pV^{1.4} = C$. (a) If 20 lb/min are handled, what is the conventional horsepower of the compressor? (b) Would it take more or less or the same work to compress the same mass of air between the same temperatures? Ans (a) 5.44 hp

174 A centrifugal air compressor handles 5000 cfm measured at intake where $p_1 = 14$ psia and $t_1 = 75^\circ\text{F}$. Discharge is at 40 psia. $\Delta K = 0$. What is the work if the process is (a) an irreversible adiabatic for which $m = 1.48$, (b) isentropic, (c) an internally reversible polytropic with $n = 1.32$? (d) What is the increase of availability of the air in (a) and in (b)?

Ans (a) -18,400 Btu/min, (b) -15,850 Btu/min, (c) -15,500 Btu/min

175. Measured at intake of 14.6 psia and 80°F, 28,000 cfm of air are pumped to 28.5 psia. At discharge the velocity is 100 fps and the temperature is 200°F. If the initial velocity is negligible and the process is an irreversible adiabatic one, find the horsepower required to drive the compressor, mechanical friction neglected.

Ans. 1400 hp.

176. Same as 175 except that at discharge $p_2 = 38$ psia and $t_2 = 270^\circ\text{F}$.

177. Consider that 3500 cfm of air at 14.5 psia and 75°F (the state at intake and also the dead state) enter a compressor and are discharged at 25 psia and 190°F. The initial velocity is zero and the final velocity is 120 fps. The process is an irreversible adiabatic following $pV^n = C$. Find (a) m , (b) ΔH , (c) W , (d) ΔS . (e) What is the increase of availability of the air during compression? (f) What would be the increase of availability if the compression were isentropic between the stated pressures? In which case is the increase of availability of the system greater? Is this what you expected? Is the greater work of the irreversible compression compensated fully by its greater increase of availability? Discuss.

Ans. (a) 1.555, (b) 7070 Btu/min., (c) -7144 Btu/min., (d) 2.39 Btu/°R, (e) 5790 Btu/min.

178. A reciprocating air compressor with a clearance of 4.5% is to handle 1000 cfm of free air when atmospheric conditions are $p_a = 14.6$ psia and $t_a = 76^\circ\text{F}$. At the end of the suction stroke, the air is at $p_1 = 14$ psia and $t_1 = 95^\circ\text{F}$. For a discharge pressure of 65 psia, for $n = 1.33$ and $\Delta K = 0$, what should be the displacement of the compressor?

Ans. 1200 cfm.

179. A reciprocating air compressor with a 6% clearance pumps 50 lb./min. of air from 14.7 psia and 80°F to 50 psig. The compression process is polytropic with $n = 1.33$. With respect to the system of 50 lb./min., compute (a) T_2 , (b) ΔH , (c) Q , (d) W in hp. (e) Determine the required displacement as based on the conventional volumetric efficiency.

Ans. (a) 320°F, (b) 2880 Btu/min., (d) 78.2 hp, (e) 775 cfm.

180. A reciprocating compressor is expected to have an adiabatic compression efficiency of 77% when pumping air from 14.7 psia and 65°F to 300 psia. The clear-

ance is 8%, $\Delta K = 0$, and 42.4 lb./min are to be delivered. Find (a) the required shaft horsepower, (b) the displacement as obtained from the conventional volumetric efficiency. (c) Comment on the magnitude of the discharge temperature. Would there be danger of igniting the cylinder lubricant? Is the volumetric efficiency good or bad? What alternative would you suggest as a better design for actual use? Ans. (a) 225 bhp, (b) 905 cfm.

181. The same as 180 except that the gas is helium.

182. An electric motor is delivering 170 hp to the shaft of a double-acting reciprocating compressor which is turning 270 rpm and compressing propane from 14.7 psia and 90°F. Bore by stroke is 20x12 in.; volumetric efficiency is 80%; adiabatic compression efficiency is 78%; $\Delta K = 0$. Using the value of the frictional horsepower as given by equation (1), § 121, estimate (a) the ihp of the compressor, (b) the mechanical efficiency, (c) the isentropic work of the corresponding conventional card, Btu/min., (d) the mass of gas delivered. (e) For the foregoing conditions, compute the discharge temperature after isentropic compression.

Ans. (a) 149 ihp, (c) 8090 Btu/min., (e) 408°F.

183. A two-stage compressor without clearance receives 80 lb./min. of air at 14 psia and 60°F and delivers it at 120 psia. The compression is polytropic, $pV^{1.3} = C$, and the intercooler cools the air back to 60°F. For each piece of equipment, $\Delta K = 0$. Find (a) t_2 , t_4 , and the optimum intermediate pressure, (b) the conventional work, (c) the heat transferred in the various processes (sketch on TS plane), (d) the isentropic horsepower for a single-stage machine, (e) the corresponding percentage saving for the two-stage machine, (f) the mass of water to be circulated through the intercooler if its temperature rise is 15°F.

Ans. (a) 41 psia, (b) 161 hp, (c) 4169 Btu/min., (d) 199.5 hp, (e) 17.8%, (f) 188 lb./min.

184. (a) A compressor delivers 50 lb./min. of air, receiving it at 65°F and compressing it isentropically to 786°F. Draw the energy diagram and determine the work. (b) For an adiabatic compression efficiency of 75%, what is the actual horsepower and the temperature at the end of

the actual compression (same state 1)?
 (c) During the actual compression, the system (50 lb/min) undergoes an increase of available energy and the surroundings experience a loss of available energy. What are the gain of E_a by the system, the

loss of E_a by the surroundings and the net loss of E_a ?

Ans (a) 8660 Btu/min, (b) 272 hp, 1030°F, (c) 10,430, 11,520, 1090 Btu/min

185-190. These numbers may be used for other problems

8. THE GAS TURBINE AND TURBO-JET

NOTE Because the air table allows for the variation of specific heat and because of the high temperatures in gas turbines, the results are much more accurate if the air table is used. However, there is something to be said for the practice in using specific heats. Both methods of solution are desirable. Answers are for table solutions unless otherwise indicated in the statement.

191. (a) If the highest permissible temperature in a gas turbine is 1540°F and $t_1 = 70^\circ\text{F}$, what compression ratio and pressure ratio result in the maximum ideal work (constant specific heats)? (b) For the ratios found in (a), what are the net work and thermal efficiency? (Note that while $p_2/p_1 = p_3/p_1$, it is not true that $v_2/v_1 = v_3/v_1$ when specific heats vary.) (c) If the compression ratio found above is doubled, what would be the work and thermal efficiency of the ideal air cycle (minimum and maximum temperatures the same)? What percentage is this work of the maximum work?

Ans (a) 5.44, 10.52, (b) 120.9 Btu/lb, 47.2%, (c) $\epsilon = 58.4\%$

192. A Brayton cycle has an adiabatic compression ratio of 4, the process beginning at 80°F and 15 psia. At the end of the heating, $t_3 = 1080^\circ\text{F}$. Considering the working substance as 1 lb of cold air (k is constant at 1.4) and using specific heats compute (a) t_2 and p_2 , (b) the works of the compressor and turbine, and the net work, (c) Q_A , Q_R , ΣQ . (d) Find the mep from the work computed above. Also write an equation for the mep as obtained from the pV plane and solve it for p_m . (e) Is this cycle the one for which the work is a maximum for the specified temperature limits? Explain. (f) If the flow of air is 100,000 cfm at the initial state, what ideal horsepower is being developed?

Ans (a) 480°F, 104.2 psia, (b) $W = 61.2$ Btu/lb, (d) 17.9 psi, (f) 10,850 hp

193. The same as 192 except that the working substance is 1 lb of hot air with an average $k = 1.32$.

194. The design of a gas turbine is to be based on an intake at 14.7 psia and 80°F,

and a maximum temperature of 1100°F. For 1 lb of air in an ideal cycle, determine (a) the approximate (that is for constant c_p) temperature and pressure at the end of compression for maximum work, (b) the heats added and rejected, (c) the works of the compressor and turbine, (d) the net work (from $W_1 - W_2$ and ΣQ), (e) the thermal efficiency. (f) If 1 lb/min of fuel oil ($q_f = 17,400$ Btu/lb) is consumed, what is the corresponding ideal amount of air and of air and fuel lb/min? What is the consequent net work in hp?

Ans (a) 458°F, 95.7 psia, (d) 86.35 Btu/lb, (f) 107 lb/min, 189.5 hp

195. The same as 194 except that the maximum temperature is 1540°F.

196. Let the basic data be as given in 194. The compression efficiency is 82%, the engine efficiency is 84%. Determine (a) the actual enthalpy h_2 after compression, (b) the actual enthalpy h_4 after expansion, (c) the net work W , (d) the actual thermal efficiency. What is the ratio of the actual ϵ' to ideal ϵ ?

197. The following data correspond approximately to those for a General Electric gas turbine 4690 bhp $q_f = 17,400$ Btu/lb, intake at 14.5 psia and 80°F, pressure ratio = 6, turbine inlet temperature = 1400°F, compression efficiency = 83%, engine efficiency = 85%. For the air cycle with friction, find (a) actual net work of the fluid W' , h_2 , and ϵ' (ignore the effect of the mass of fuel), (b) h_4 , t_4 , (c) the percentage of ideal net work lost by virtue of the imperfect processes in compressor and turbine, (d) Assume that the shaft (brake) work is 95% of the fluid work computed above and determine the

air flow, lb./sec., (c) fuel used, lb./hr. and lb./bhp-hr., (f) the heat rate, Btu/bhp-hr.

198. In a gas turbine unit, air from the compressor enters the combustor at 400°F and leaves at 1510°F . Of a fuel with $q_1 = 18,600$ Btu/lb., 5% passes through unburned. The heat loss from the combustor is 800 Btu/lb. of fuel. Sketch an energy diagram showing a complete energy balance. Now ignoring the enthalpy of the liquid fuel and the effect of the mass of fuel, and treating the products as though they have the properties of air, determine the amount of fuel per pound of air, and the efficiency of the combustor.

Ans. 0.0177 lb., 90.5%.

199. A jet-engine driven plane is moving at a constant speed of 700 mph at an altitude of 30,000 ft. where $p_a = 4.36$ psia

and $t_a = 412^{\circ}\text{R}$. The engine thrust is 3200 lb.; air flow is 79 lb./sec.; the ram coefficient is 80%; $r_p = 1.2$ in compressor; turbine inlet temperature is 1500°F ; engine efficiency of turbine is 75%. Find (a) the propulsive power, (b) the exhaust jet velocity relative to the plane. For the ideal cycle, compute (c) the pressure at entry to the compressor (stagnation effect), (d) ideal compressor work, (e) ideal exhaust temperature of the turbine, (f) the ideal enthalpy drop in the jet nozzle and the ideal relative velocity at exit from jet nozzle (expansion to atmospheric pressure from zero initial velocity).

Ans. (a) 5960 hp, (b) 2330 fps, (c) 7.71 psia, (d) 60.95 Btu/lb., (e) 1277°F , (f) 2710 fps.

200-210. These numbers may be used for other problems.

9. INTERNAL COMBUSTION ENGINES

NOTE. Unless otherwise stated, the answers to the following problems are for some average constant specific heats. However, where appropriate, it would be interesting and instructive to solve them also by use of the air table (the variable specific heat air standard), p. 146. If the Gas Tables are at hand, see the examples in the back for the method of making a real mixture analysis (exclusive of dissociation effects), if it is desired to make such an analysis. In an Otto air cycle, the computed temperature at the end of the heat addition process is likely to go beyond the top limit of the air table for normal air/fuel ratios.

211. An Otto cycle operates on 0.1 lb./cc. of air from 13 psia and 130°F at the beginning of compression, state 1. The temperature at the end of combustion is 5000°R ; compression ratio is 5.5; hot air standard, $k = 1.3$. (a) Find e_c , V_1 , p_2 , t_2 , p_1 , V_2 , t_1 , and p_1 . (b) Compute Q_A , Q_R , W (from TS and pV planes), e , and the corresponding horsepower.

Ans. (a) $p_1 = 605$ psia, $T_1 = 3000^{\circ}\text{R}$, (b) $Q_A = 91.6$ Btu/sec., $e = 10\%$, 51.8 hp.

212. What would be the efficiency of the cycle in 211 if the working substance had the properties of cold helium at all times? Is there any advantage in an Otto cycle of such a substance?

213. From state 1 of 100°F , 13.8 psia, and 1 cu. ft., the compression in an Otto cycle is to 250 psia. The heat supplied per cycle is 80 Btu. Using an average $k = 1.31$ for the hot air standard, find (a) the compression ratio, (b) the percentage clearance, (c) T_2 and p_2 at the end of combustion, (d) Q_R , W , and e , (e) the displace-

ment per cycle and the mep. (f) If this is a 6-cylinder, 4-stroke-cycle engine, turning at 300 rpm, determine the number of cycles completed per minute and the ideal horsepower.

Ans. (a) 9.09, (c) 6510°R , 1470 psia, (d) $e = 49.5\%$, (e) $p_m = 240$ psi, (f) 811 hp.

214. The same as 213 except that the air table is to be used in the solution (do not use $k = 1.31$). Compare answers.

Ans. (a) 8.08, (c) 6529°R (obtained by extrapolation of unabridged table), 1300 psia, (d) $e = 48.3\%$, (e) $p_m = 212$ psi, (f) 835 hp.

215. A Diesel cycle operates with a compression ratio of 13.5 and with the cutoff occurring at 6% of the stroke. State 1 is defined by 14 psia, 140°F . For the hot-air standard with $k = 1.34$ and for an initial 1 cu. ft., compute (a) t_2 , p_2 , V_2 , t_3 , p_3 , V_3 , p_4 , and t_4 , (b) Q_A , Q_R , (c) W computed from both TS and pV planes, (d)

e and mep (e) For a rate of "circulation" of 1000 cfm, compute the horsepower

Ans (a) $p_2 = 427$ psia, $t_4 = 805^\circ\text{F}$, (c) 10 Btu, (d) $p_m = 58.6$ psi, (2) 237 hp

216 In an ideal, single-cylinder, 2-stroke-cycle Diesel engine, operating at 300 rpm, compression is from 14.7 psia, 80°F , 1.43 cu ft to 500 psia. Then 16 Btu/cycle is added as heat. Make computations for the cold air standard and find (a) p , V , T at each corner of the cycle, (b) W , (c) e and the mep , (d) hp

Ans (a) $T_1 = 2120^\circ\text{R}$, (b) 9.7 Btu, (c) 60.7%, 39.9 psi, (d) 68.6 hp

217. The same as 216 except that computations are to be made for hot air with $k = 1.33$

218. The charge in a Diesel engine consists of 0.015 lb of fuel, whose lower heating value is 17,400 Btu/lb, and 1.2 lb of air and products of combustion. From $t_1 = 100^\circ\text{F}$, the compression ratio is 12 (a) Compute the cutoff ratio of the corresponding hot air standard when $c_p = 0.27$ Btu/lb (b) What is the thermal efficiency and the work done?

Ans (a) 2.78, (b) 47%, 368 Btu/charge of fuel

219 A 6-cylinder, $3\frac{5}{8} \times 3.6$ -in automotive engine with a compression ratio of 8, shows a fuel consumption of 0.45 lb per bhp-hr at 3000 rpm, $bhp = 113$, $fhp = 20$ (let $ihp = bhp + fhp$) Let the standard for comparison be the hot air standard with $k = 1.3$. The heating value of the fuel is $q_1 = 18,500$ Btu/lb. Compute (a) the mechanical efficiency, e_m , and e_t , (b) the brake and indicated engine efficiencies,

(c) the brake and indicated mep 's (d) What is the heat rate, in Btu/bhp-hr and in Btu/min?

Ans (a) $e_t = 36\%$, (b) $e_m = 77.4\%$, (c) $p_{mb} = 134$ psi, (d) 15,700 Btu/min

220 A 12.5×16 -in, single-cylinder, 2-stroke-cycle gas engine develops 40 bhp at 200 rpm while using a natural gas with a lower heating value of 1000 Btu/cu ft. Its compression ratio is 6 and its mechanical efficiency is 81%, and at the stated load, the heat rate is 10,000 Btu/bhp-hr. Let the hot air Otto cycle, $k = 1.33$, be the standard of comparison. Determine the brake and indicated thermal efficiencies, engine efficiencies, and mep 's

Ans $e_i = 31.5\%$, $e_t = 70.6\%$, $p_{mi} = 64.5$ psi

221. A 12.5×13 -in, single-acting 2-stroke-cycle oil engine is running at 326 rpm. An indicator with a 350 lb spring produces a card with an average area of 0.396 sq in and length of 2.97 in. Crank case compression is used for scavenging. An indicator card from the crank case has an area of 0.5 sq in and a length of 2.97 in when the spring scale is 12 lb. Mechanical efficiency has been determined as 80%. Compute (a) the mep of the working cylinder and of the crank case, (b) the corresponding ihp 's and net ihp , (c) the bhp

Ans (b) net $ihp = 58.4$, (c) 49.6 hp

222 The mep of an ideal hot-air ($k = 1.34$) Diesel cycle is 100 psi $p_1 = 14$ psia, $r_k = 12$. What is r_c ? Ans 2.47

223-230 These numbers may be used for other problems

10 LIQUIDS AND VAPORS

NOTE The answers given to the problems on steam are based on Steam Tables published by Combustion Engineering Inc. If other tables are used some differences in answers are to be expected. Combustion Engineering may be willing to furnish their tables for loan to students. A copy of their tables is found in Problems on Thermodynamics. In some of the problems, also in later chapters, the data are chosen for a convenient fit with these tables.

231 (a) What are the temperature, volume, enthalpy, entropy, and internal energy of 3 lb of saturated steam at 350 psia? (b) The same as (a) except that the steam is at 350 psia and 520°F (c) The same as (a) except that the steam is wet with 10% moisture

232 (a) A 10-cu ft drum contains saturated steam at 80.3 psig. What are the temperature and mass of steam in the drum? Determine the enthalpy, entropy, and internal energy (b) The same as (a) except that the drum contains 3 lb of steam (c) The same as (a) except that the mass is 1.241 lb of steam

233. For practice in interpolation, find the enthalpy, volume, entropy, and internal energy of 1 lb. of steam at 462 psia and 625°F.

234. (a) Steam at 500°F has a specific entropy of 1.815 Btu/lb.-°R. What are its pressure, specific volume, and enthalpy? (b) The same as (a) except that $s = 1.4$ Btu/lb.-°R. (c) The same as (a) except that $s = 1.433$ Btu/lb.-°R.

235. (a) Steam at 120 psia has a specific enthalpy of 1410 Btu/lb. What are its temperature, specific volume, entropy, and internal energy? (b) The same as (a) except that $h = 1190.6$ Btu/lb. (or 1190.4 for Table X). (c) The same as (a) except that $h = 1000$ Btu/lb.

236. (a) Ammonia at 100°F has a specific volume of 2.8 cu. ft./lb. Determine its pressure, enthalpy, entropy, and internal energy. (b) The same as (a) except that $v = 1.419$ cu. ft./lb. (c) The same as (a) except that $v = 1.2$ cu. ft./lb.

237. (a) Ammonia at 140 psia has an enthalpy of 650 Btu/lb. What are its temperature, volume, entropy, internal energy, and degrees of superheat? (b) The same as (a) except that $h = 600$ Btu/

lb. (c) The same as (a) except that $h = 629.9$ Btu/lb.

238. (a) Compressed liquid water is at 3000 psia and 200°F. For 1 lb., what are its enthalpy, entropy, and volume? (b) Saturated water at 200°F is pumped to 3000 psia. Find the approximate increase of enthalpy (pump work), using $\int v dp$. Show these points on a large-scale Ts plane.

239. The same as 238 except that the temperature is 600°F.

240. (a) Water at 3000 psia and 300°F enters a steam generator and leaves as superheated steam at 3000 psia and 800°F. How much heat is added per pound? (b) Saturated water at 300°F is pumped to 3000 psia and then heated to steam at 3000 psia and 800°F. How much heat is added? See equation (j), § 168. (c) The same as (b) except that $p = 85.3$ psig. (d) Is it advisable in either (b) or (c) to account for the enthalpy increase during compression of the liquid?

Ans. [from K & K] (a) 992.1, (b) 993, (c) 1159.2 Btu/lb.

241–250. These numbers may be used for other problems.

11. PROCESSES OF VAPORS

REMINDER. The student should sketch all processes on the Ts and/or pv planes and sketch energy diagrams, whether or not the problem says so.

251. An internally reversible isobaric process occurs from water at 200°F and 160 psia to 600°F. (a) For 1 lb., determine Δh , Δs , Δv , and Δu . (b) If the process is nonflow, what are the $\int p dv/J$, W , and Q ? Check by the nonflow energy equation. (c) For a steady flow process through a steam generator with $\Delta K = 0$, what are the $\int v dp$ and Q ? What is W ?

Ans. (a) $\Delta u = 1042.7$, (b) $W = 113.4$, (c) $Q = 1156.1$ Btu/lb.

252. Steam with an entropy of 1.302 Btu/lb.-°R undergoes a constant pressure process at 400 psia until the final entropy is 1.6393 Btu/lb.-°R. (a) Determine y_1 , Δh , Δv , Δu . (b) If the process is steady flow through a heat exchanger wherein $\Delta K = -5$ Btu/lb., how much heat is transferred? (c) If the process is nonflow, what are $\int p dv/J$, W , and Q ? What statement of the law of conservation of energy best describes the energy balance in this one?

253. Ammonia at 20 psia with a specific enthalpy of 136.8 Btu/lb. changes state at constant pressure until the temperature becomes 0°F. For 10 lb. of NH_3 , find (a) x_1 , ΔH , ΔV , ΔS , and ΔU . (b) What are the work and heat of a nonflow system? (c) What are the work and heat of a steady flow system of the NH_3 flowing through an evaporator in which $\Delta K = 0$?

Ans. (a) $\Delta V = 114.8$ cu. ft., (b) 425 Btu, 4787 Btu.

254. Five pounds of steam occupy a 14.02-cu. ft. tank at 190 psia. After some cooling of the tank, its pressure drops to 80 psia. Compute (a) y_2 , ΔS , and ΔU . (b) What is W ? What amount of heat is transferred? (c) At what pressure was the steam in the tank dry and saturated?

Ans. (a) $y_2 = 49\%$, $\Delta U = -2309$, (c) 161.9 psia.

255. Ammonia at 20 psia and $x_1 = 20\%$ is trapped in a vessel and receives heat

until its pressure rises to 160 psia. For 1 lb, find (a) t_2 , Δs , Δh , and Δu (b) What are W and Q ? (c) If the volume of the vessel is 15 cu ft, what is the mass in it before and after this process?

256 During an isentropic process, 5 lb of steam are compressed from 10 to 150 psia. The initial total entropy is $S_1 = 7.8490$ Btu/°R. (a) Compute the property changes ΔH , ΔV , and ΔU . (b) If the process is steady flow with $\Delta K = -100$ Btu, what work is required? (c) What are the work and heat for a nonflow process? Sketch the process on the Ts plane after the solution is completed.

Ans (a) $y_1 = 14.46\%$, $\Delta U = 847.5$ Btu, (b) -865.5 Btu

257. If 6 lb/sec of steam pass through an isentropic process from 200 psia and 450°F to 5 psia, find (a) y_2 , ΔH , ΔU , and ΔS . (b) If the process is nonflow, what are Q and W and $\int p dV/J$? (c) If the process is steady flow with $W = 0$, what is $\Delta \dot{A}$? Check by Mollier chart showing sketch of solution. (d) If the process is steady flow with $\Delta \dot{A} = 200$ Btu/sec, what is W ? The horsepower? (e) If the process is an irreversible adiabatic with the entropy increasing to $s_2 = 1.8437$ Btu/lb °R, what are the nonflow work and the steady flow work ($\Delta K = 200$ Btu/sec)?

258 An isentropic process of 10 lb of steam occurs from 175 psia and 4231 cu ft/lb to 20 psia. Find (a) y_2 or t_2 , (b) ΔH , ΔU , Q , and ΔS . (c) What is the work of a nonflow process? (d) For $\Delta K = 0$, what is the work of a steady flow process? Check by the Mollier chart, showing sketch of solution. (e) If the process is an irreversible steady flow adiabatic from the same initial state with the entropy increasing 0.0817 Btu/lb °R, what is the work ($\Delta K = 0$)? (f) How much work is lost because of the irreversibility? (g) In (e) what is the increase of unavailable energy for a sink temperature of 70°F? (h) Show areas on the ts plane which represent respectively the lost work and the increase of unavailable energy.

Ans (b) $\Delta U = -1794$ Btu, (d) 2336 Btu, (e) 1631 Btu, (f) 705 Btu, (g) 449 Btu

259 Ammonia in a refrigeration machine is compressed isentropically from 20 psia to 200 psia and 200°F. For 1 lb, find (a) y_1 , Δh , Δu , Q , and Δs . (b) For a

nonflow compression, what is the work? (c) What is the work during steady flow if $\Delta K = 0$? (d) For a compression efficiency [in (c)] of 78.2% and 4 lb/min flow, what is the actual work (in hp) and the actual discharge temperature of the NH_3 ?

Ans (a) $y_1 = 5.83\%$, $\Delta u = 107.4$ Btu/lb, (c) -131.9 Btu/lb, (d) 15.9 hp, 260°F

260 During an irreversible adiabatic process from 250 psia and 420°F to $p_2 = 5.3$ in Hg abs, the entropy increases 0.3 Btu/lb °R, $\Delta K = 0$. Calculate (a) y_2 and Δh , (b) W and Q , (c) the work lost due to irreversibility, and (d) the increase of unavailable energy for a sink at 80°F.

Ans (b) $W = 125.7$ Btu/lb, (c) 179 Btu/lb (d) 162 Btu/lb

261 During an isothermal compression, steam at 80 psia and 600°F rejects 781 Btu/lb. (a) Compute x_2 , Δh , and Δu . (b) If the process is nonflow, what is the work? (c) What is the work of a steady flow process with $\Delta A = 0$?

Ans (a) $x_2 = 45\%$, (b) -392.7 Btu/lb (c) -315.5 Btu/lb

262 Saturated steam at 400°F expands isothermally to 60 psia. For 10 lb, (a) what are ΔS , ΔH and ΔU ? Determine Q and W (b) for a nonflow process, (c) for a steady flow with $\Delta K = 0$.

Ans (a) $\Delta U = 253.1$ Btu, (b) $Q = 1603$ Btu

263 Saturated steam at 130 psia expands polytropically to 230°F in accordance with $pv^{1.18} = C$. (a) Compute t_2 , Δh , Δs , and Δu . Now sketch the process on the Ts plane to match these computations. (b) If this expansion occurs nonflow in a steam engine cylinder, what are W and Q during the process? (c) Consider the process as steady flow and apply the energy equation to determine an expression for W . Solve for W from this equation ($\Delta K = 0$) and from $\int v dp$. Do they check? (d) What is the change of availability of the steady flow system for $t_s = 70^\circ\text{F}$? Does the unavailable energy of the surroundings increase or decrease?

Ans (a) $s_2 = 17$ cu ft/lb, $\Delta u = 134.2$ Btu/lb, (b) 117.2, -17 Btu/lb, (c) 135 Btu

264 Steam at 480 psia and 1000°F expands polytropically to 80 psia and 360°F. The mass is 10 lb. (a) Determine n , ΔH , ΔU , $\int p dv$, and $-\int v dp$. (b)

What are Q and W for a nonflow process? (c) Set up the energy equation and energy diagram for a steady flow process. What is Q for this process? If $\Delta K = 10$ Btu/lb., what is the work? What does $-\int v dp$ represent in this process? Show a numerical check. If the flow is 10 lb./min., what is the horsepower? (d) What is the unavailable portion of the heat transferred for a sink at 500°R? Is it an increase or decrease? Show area on sketch. What is the change of availability of the system in steady flow?

Ans. (a) $n = 1.49$, $\Delta U = -2381$ Btu, $\int p dv = 111,500$ ft-lb./lb., (b) -946 , 1435 Btu, (c) 48.1 hp, (d) -421 , -2664 Btu.

265. Starting from 30 psia and 10°F, ammonia is compressed to 160 psia during an internally reversible process for which $pv^{1.255} = C$. (a) Calculate v_2 , Δh , Δu , Δs . Use the nearest whole degree for t_2 . (b) Find W and Q for a nonflow process. (c) Find W (from $\int v dp$) and Q for a steady flow ($\Delta K = 0$). How do these values of Q compare? If the rate of flow is 20 lb./min., what power is required? (d) What is the change of availability of the steady flow system when $T_0 = 500^\circ\text{R}$?

Ans. (a) $t_2 = 210^\circ\text{F}$, $\Delta u = 74.7$ Btu/lb., (b) -83.6 , -8.9 Btu/lb., (c) 49.5 hp.

266. Ammonia at 20 psig and 5% liquid ($y = 5\%$) is compressed, internally re-

versibly, until it is all saturated vapor at 150 psia. The mass is 3 lb. (a) Determine n and the change of properties ΔH , ΔU . (b) What is the nonflow work? (c) What is the steady flow work if $\Delta K = -15$ Btu? (d) How much heat is transferred? into or out of the system?

Ans. (a) $n = 1.09$, $\Delta U = 116$ Btu, (b) -216 Btu, (c) -221 Btu, (d) -100 Btu.

267. A throttling calorimeter receives a sample of steam from a steam main, in which the pressure is 100 psia. After throttling, the steam is at 14.7 psia and 250°F. What is the percentage moisture in the original steam? Solve by using (a) the superheat tables, (b) the saturated steam tables and the specific heat of steam, (c) the Mollier chart, making a sketch to show your solution.

268. The same as 267 except that throttling occurs from 160 psia to 15 psia and 250°F.

Ans. 3.06%.

269. In a refrigerating system, liquid ammonia enters the expansion (throttling) valve at 150 psia and 70°F. If it emerges at 25 psia, compute (a) x_2 and Δs ; (b) also the loss of available energy during throttling.

270-280. These numbers may be used for other problems.

12. VAPOR CYCLES AND ENGINES

REMINDER. Show sketches for each problem depicting the processes on some convenient plane and sketch energy diagrams for steady flow problems. Use the Mollier chart as a check on the steam table solution wherever possible.

281. (a) A Rankine engine receives saturated steam at 110 psia and exhausts it at 15 psia. For 1 lb., calculate the work. (b) Suppose the exhaust from the foregoing engine is passed without loss through a steam separator, which removes all the liquid. Suppose now that the remaining saturated steam at 15 psia enters another Rankine engine and expands to 1 psia. How much work is done by 1 lb.? How much for 1 lb. of original steam? (c) Does it appear that the work to be obtained by carrying the expansion of steam below atmospheric pressure is significant? Is a low pressure of 1 psia feasible?

Ans. (a) 145.6 Btu/lb., (b) 170 , 151.1 .

282. Steam is generated at 540 psia and 750°F and condensed at 90°F. Consider

1 lb. (a) For a Rankine engine operating between these limits, compute its work, thermal efficiency, steam rate, and mep. (b) Considering that a Rankine cycle occurs between the same limits, determine Q_A , Q_R , net W , and e . How much is the ideal pump work? Indicate areas on the T - s plane which represent Q_A , Q_R , and W . (c) What mass of steam is required for a net output of 50,000 kw?

Ans. (a) 492.1 , 37.16% , 5.17 lb./hp-hr., 7.11 psi, (b) $e = 37.09\%$, $W_p = 1.605$ Btu/lb., (c) $347,000$ lb./hr.

283. The same as 282 except that the expansion is from 2000 psia and 1100°F to 110°F.

284. A turbo-generator unit consumes 250,000 lb./hr. of steam while delivering

30 000 kw The steam expands from 800 psia and 900°F to 1 psia (a) Considering an ideal engine, determine the thermal efficiency, steam rate, and mep. (b) Considering 1 lb of steam in an ideal cycle between the same limits, compute the pump work, heats added and rejected, and the thermal efficiency (c) Considering the actual engine, determine the combined steam rate, combined work, combined thermal efficiency, and the combined engine efficiency Also, estimate the enthalpy of the actual exhaust if the generator efficiency is 93%

Ans (a) 38.9%, 6.34 lb/kw-hr, 10.7 psi, (b) $Q_A = 1382.8$ Btu/lb, 38.8%, (c) $W_K = 410$ Btu/lb, $\eta_s = 76.2\%$, $h_e = 113.9$ Btu/lb

285 In an incomplete expansion engine, steam expands from 160 psia and 136.45 degrees of superheat to 30 psia which is the point release The exhaust pressure is 15 psia At full load, the actual engine has a steam rate of 22.3 lb/bhp-hr and $\eta_m = 80\%$ (a) For the ideal engine, compute the efficiency, steam rate, and mep (b) For the actual engine, determine the brake thermal efficiency, the indicated engine efficiency, and the heat rate in Btu/bhp-hr

Ans (a) 16.4%, 14.22 lb/hp-hr, 73 psi, (b) 10.45%, 79.7%, 24,400 Btu/bhp-hr

286 A compound steam engine with incomplete expansion receives saturated steam at 150 psia expands it to 15 psia where release occurs Exhaust is to a condenser at 104°F The actual engine uses 2760 lb/hr of steam, it has a brake engine efficiency of 60% and a mechanical efficiency of 85% The electric generator which it drives has an efficiency of 92% (a) For the ideal engine, find the thermal efficiency and mep For the actual engine, determine (b) the brake work in Btu/lb and hp the brake thermal efficiency, (c) the indicated work, thermal efficiency, and steam rate, (d) the combined thermal efficiency and steam rate (e) What is the approximate enthalpy of the actual exhaust steam?

Ans (a) 20.35%, 54 psi, (b) 148.5 bhp, 12.2%, (c) 14.35%, (d) 11.2%, 27.1 lb/kw-hr, (e) 1033.2 Btu/lb

287. A double acting, 12x15 in., simple steam engine, turning at 200 rpm, expands steam from 185 psia and 400°F to 15 psia where the exhaust valve opens Exhaust is at 5 psia Indicator card information

is as follows head-end area is 2.17 sq in., crank end area is 2.34 sq in., average length of all is 3 in., scale of indicator spring is 100 lb, piston rod diameter is 1½ in At this load, the mechanical efficiency was 86% and the steam rate was 23 lb/bhp-hr (a) For the ideal engine, compute W and e For the actual engine, determine (b) e_b , w_i , and η_i , (c) ihp and the total steam flow in lb/hr

Ans (a) 21.1%, (b) 10.2%, 19.75 lb/ihp-hr, 56%, (c) 127.7 ihp, 3410 lb/hr

288 A turbine whose steam rate is 6.1 lb/bhp-hr receives steam with an enthalpy of 1507.5 Btu/lb Radiation loss from the turbine is 18 Btu/lb of steam and the condenser pressure is 5.3 in. Hg abs (a) If $\Delta h = 5$ Btu/lb of steam, what are the approximate enthalpy and quality of the actual exhaust? (b) What is the stagnation enthalpy of the exhaust if the entering kinetic energy is zero? (c) If the total steam flow is 610,000 lb/hr, what horsepower is being delivered?

Ans (a) 1067, 94.7%, (b) 1072 Btu/lb

289 Saturated steam leaves a steam generator at 190 psia Because of frictional losses in the pipe and throttling at the turbine entrance, the pressure drops adiabatically to 140 psia The sink temperature is 77°F What is the increase of (a) the entropy and (b) the unavailable energy? (c) What is the change of availability? (d) What is the loss of Rankine work during this process? Indicate significant areas on your Ts diagram

Ans (b) 16.75, (c) -16.75 (d) 21 Btu/lb

290 Steam is delivered by a steam generator at 570 psia and 500°F Because of frictional losses in the pipe and throttling at the turbine at reduced load the pressure drops adiabatically to 470 psia at entry to the nozzles The condenser is at 104°F, brake engine efficiency is 72% generator efficiency is 94%, work of ideal engine is 388.2 Btu/lb, sink temperature is 80°F Solve by tables with a Mollier chart check (sketch) Determine (a) the loss of available energy due to fluid friction in arriving at the nozzles, (b) the approximate enthalpy of the actual exhaust, (c) the loss of available energy during passage through the turbine and the total loss (d) the overall loss of Rankine work

Ans (a) 9.77, (b) 940.2, (c) 102.9, 112.67, (d) 117.5 Btu/lb

291. Steam enters an ideal reheat turbine at 1400 psia and 900°F and exhausts at 1 psia and a quality of 87%. The ideal work is 651 Btu/lb. What is the ideal thermal efficiency? Draw an energy diagram of engine and reheater.

Ans. 42.1%.

292. In a reheat cycle, steam enters the turbine at 630 psia and 650°F, expands to 180 psia, whence it passes through a reheater emerging at 700°F. It now expands to the condenser temperature of 101°F. For the ideal cycle, compute W_p , Q_A , Q_R , and e .

Ans. 1.94, 1421.8, 900.6 Btu/lb., 36.6%.

293. In an ideal regenerative cycle, steam is generated at 520 psia and 900°F; it then expands to 255°F and 32.53 psia where m lb. are extracted for feedwater heating. Condensation is at 81°F. For 1 lb. of throttle steam, find the fraction extracted, the total pump work, the enthalpy of water entering boiler, and the thermal efficiency.

Ans. 0.154, 1.51, 225.1, 40.9%.

294. In an ideal reheat-regenerative cycle, steam at 1650 psia and 780°F expands to 430 psia, whence all of it passes through a reheater and re-enters the turbine at 600°F. It then expands to 135 psia, at which point some is bled for regenerative feedwater heating. The remainder of the throttle flow continues its

expansion to a condenser temperature of 90°F. Compute W_p , Q_A , Q_R , and e .

Ans. 4.92 (approx.), 1112.1, 628 Btu/lb., 43.5%.

295. Indicate the following reheat-regenerative cycle on the Ts plane and set up equations for (a) the amount of extracted steam at each point, (b) the heat added, (c) the thermal efficiency: one reheat with two stages of regenerative feedwater heating *after* the reheat.

296. In an ideal reheat-regenerative cycle, steam expands from 1700 psia and 700°F to 800 psia where some steam is extracted for feedwater heating and the remainder is reheated to 750°F. The steam expands again to 145 psia where additional steam is extracted for feedwater heating, and the remainder expands to the condenser pressure of 1.202 psia. Set up the equations needed on the basis of 1 lb. and compute the following: (a) percentages of extracted steam, (b) Q_A , Q_R , and net work (find net work by ΣQ and check by sum of Rankine engine works minus pump work), (c) thermal efficiency. (d) Draw an energy diagram of the engine only and show an energy balance as a check.

Ans. (a) 20.9%, 17.8%, (b) net $W = 393$, (c) 45.7%.

297–310. These numbers may be used for other problems.

13. NOZZLES

311. A substance flows at the rate of 2 lb./sec. through a nozzle from 140 psia and 500°F to 80 psia. The expansion is isentropic. If the initial velocity is zero (state 1 is the stagnation state), find the temperature and area at the exit section when the substance is (a) steam, (b) air. (c) The same as (b) except that the initial velocity is 200 fps. Show energy diagram.

Ans. (a) 380°F, 1.068 in.², (b) 821.5°F, 0.871 in.² (air tables), (c) 0.833 in.²

312. A nozzle expands steam in equilibrium from a stagnation state of 110 psia and 400°F to 5 psia. Throat area is 0.326 sq. in. Let $p_c = 0.545p_1$. For an ideal expansion, determine (a) the mass rate of flow, and (b) the area of the exit section.

Ans. (a) about 1800 lb./hr., (b) 1.39 in.²

313. A large user of natural gas (let $R = 96.2$, $k = 1.3$) receive, it at 100 psia and 80°F but uses it at 15 psia. It is proposed that the gas be used in a turbine to generate power. If it is decided that each nozzle is to allow a flow of 1.6 lb./sec., determine the pressure, temperature, velocity, and area (a) at the throat and (b) at the exit section for an isentropic expansion (ideal gas); initial velocity negligible. Would the state of the exhaust be such that it could be used for cooling air for air conditioning purposes? Discuss.

Ans. (a) 468.5°R, 0.965 in.², (b) 348.4°R, 1.58 in.²

314. Helium flows through a nozzle whose exit area is 2.83 sq. in. and changes state from 100 psia and 90°F to 80 psia. The nozzle efficiency is 97% and the initial velocity is negligible. What are (a) the

exit temperature, specific volume, and velocity? (b) the rate of flow, (c) the value of m in $p_1 V_1^m = p_2 V_2^m$, (d) the rate of increase of unavailable energy?

Ans (a) 508.5°R, 17.03, 1687 fps, (b) 7030 lb/hr, (c) 1.54, (d) 91.7 Btu/hr

315 A nozzle, whose throat area is 1 sq in., expands steam from 200 psia and 400°F to 50 psia. The expansion is into the supersaturated state. For a nozzle efficiency of 95% and negligible initial velocity, compute (a) the actual discharge, and (b) the exit area. Assume that the actual specific volume is equal to the ideal (closely so).

Ans (a) 10,230 lb/hr, (b) 1.293 in.²

316 Steam enters a nozzle with negli-

ble velocity at 1050 psia and 650°F and expands to 450 psia. The actual exit temperature is 460°F and the rate of flow is 2 lb/sec. Determine (a) the actual exit velocity and nozzle area, (b) the nozzle efficiency, (c) the value of m in $p v^m = C$ which defines the pressure-volume relation at the end states, (d) the loss of available energy for a sink temperature of 40°F.

Ans (a) 1960 fps, 0.153 in.², (b) 96.4% (c) 1.255 (d) 3.2 Btu/sec

317 The same as 316 except that the working substance is helium and the actual exit temperature is 510°F.

318-330 These numbers may be used for other problems.

14. THE REVERSED CYCLE

331 A refrigerating cycle operates between an evaporator temperature at 10°F and a condenser pressure of 120 psia. Entering the compressor the refrigerant, which is ammonia, is 100% vapor and its temperature on the high pressure side of the expansion valve is 60°F. For 1 lb in an ideal cycle find (a) the work, (b) the COP, (c) the rate of refrigerant circulation for a 5-ton capacity, lb/min, (d) the heat rejected to the condenser water, Btu/min, (e) the horsepower per ton, (f) the piston displacement for a volumetric efficiency of 80% (g) For an adiabatic compression efficiency of 75% but ignoring other cycle losses compute the actual horsepower per ton of refrigeration.

Ans (a) 67.6, (b) 7.48 (c) 1.98, (d) 1135, (e) 0.631, (f) 18.1 cfm, (g) 0.842

332 The same as 331 except that the refrigerant is Freon 12 and $p_2 = 80$ psia (instead of 120).

333 The temperature in a refrigerating system at the expansion valve is 80°F. The evaporator is at 40°F. The substance leaves the evaporator, entering the compressor, as saturated vapor only. The cooling capacity is to be 25 tons. For comparison purposes consider the volumetric efficiency of the compressor as 100%, let $L/D = 1$, let $n = 150$ rpm let the compressor be double acting. Compute the required displacement in cfm and the bore D and stroke L when the refrigerant is (a) NH_3 , (b) SO_2 , (c) CO_2 ,

(d) Freon 12, (e) H_2O . Are all of the answers reasonable? For CO_2 , $h_{f+g} = 98.4$, $h_{f+g} = 35.6$ Btu/lb, $v_{f+g} = 0.145$ cu ft/lb. For SO_2 , $h_{f+g} = 185.37$, $h_{f+g} = 40.05$ Btu/lb, $v_{f+g} = 2.837$ cu ft/lb.

Ans (a) 40.5 cfm, 6.67x6.67 in.

334 A single acting twin cylinder, 12x12 in compressor running at 150 rpm, takes in saturated vapor at 1°F and discharges it at 180 psia. Saturated liquid enters the expansion valve. The compressor's volumetric efficiency is 82%. The brake work is 1.25 times the ideal. The water to be frozen is at 80°F and the manufactured ice is at 16°F for ice $c = 0.5$ Btu/lb-°F. If NH_3 is the refrigerant, determine (a) the capacity of the system (b) bhp per ton, (c) the tons of ice manufactured in 24 hr if there are no radiation losses.

Ans (a) 50.8 tons, (b) 1.94, (c) 36.5

335 The same as 334 except that the refrigerant is Freon 12 and $p_2 = 115$ psia (after compression).

336 A reversed vapor-compression cycle is to be used for heating. The maximum demand is expected to be 600 cfm of 40°F outside air heated to 85°F. The temperature in the evaporator is to be 25°F and the Freon 12 refrigerant is pumped to 150 psia having entered the compressor as saturated vapor. Liquid enters the expansion valve at 100°F. (a) What rate

of refrigerant circulation is needed, lb./min.? Also determine (b) the horsepower input for an ideal/actual work ratio of 70%, (c) the actual COP, (d) the cost of heating at 2 cents/kw-hr. (i) when the heat is obtained from the reversed cycle, (ii) when the heat is obtained from an electrical heating element.

337. The inhabitants of a planet where the outside temperature is 200°F decide to maintain a room at 150°F and use H_2O as the refrigerant. The steam enters the

compressor at 130°F and is compressed to saturated vapor at 220°F. The refrigerant enters the expansion valve at 210°F. The capacity is 20 tons. For an ideal cycle, determine (a) the COP, (b) the rate of circulation of refrigerant, (c) the compressor displacement for a volumetric efficiency of 80%.

Ans. (a) 6.26, (b) 4.75 lb./min., (c) 844 cfm.

338-350. These numbers may be used for other problems.

15. GASEOUS MIXTURES

NOTE. The student should check computations where possible by a psychrometric chart, thereby becoming familiar with both the basic theory and the commercial manner of obtaining a solution. Abbreviations: d.b., dry bulb; w.b., wet bulb; da, dry air; dg, dry gas; v, vapor.

351. Atmospheric air at standard pressure and 85°F d.b. has a relative humidity of 65%. Find (a) the partial pressures of the vapor and the dry air, (b) the humidity ratio, (c) the dew point, (d) the volume of the mixture, cu. ft./lb. da.

Ans. (a) 0.388, 14.312 psia, (b) 0.01685, (c) 72°F, (d) 14.1.

352. Atmospheric air at 80°F has a humidity ratio of 0.016 lb./lb. da. Determine (a) the vapor pressure, (b) the relative humidity, (c) the dew point.

Ans. (a) 0.369 psia, (b) 72.8%, (c) 70.49°F.

353. In a condenser for a steam turbine where the temperature is 92°F, there are 0.11 lb. da./lb. v. What is the vacuum pressure, referred to a barometer of 29.92 in. Hg?

Ans. 28.3 in. Hg.

354. The state of atmospheric air is defined by 27.6 in. Hg abs., 72°F d.b., 58°F w.b. (a) Using the adiabatic saturation process, determine the humidity ratio, the vapor pressure, the relative humidity, and the dew point. (b) Check the vapor pressure by Carrier's equation. (c) What is the density of the atmospheric air? What would be the density of dry air at the same total pressure? Which is heavier? (d) Check this solution by the psychrometric chart. Show sketch.

Ans. (a) $p_v = 0.1706$ psia, (c) $\rho_m = 0.0686$ lb./cu. ft.

355. Atmospheric air (29.92 in. Hg barometer) is heated at constant pressure from 36°F and 60% relative humidity to

75°F (typical of home heating). (a) What is the relative humidity in state 2? (b) How much moisture must be added to the air in the second state to bring its relative humidity to 40% (state 3)? Convert to gallons per hour for 10,000 cfm of atmospheric air (not typical of home heating to bring in this much outside air). Name all states involved in your solution on your T_s diagram. (c) What is the dew point of the air in state 3?

Ans. (a) 14.5%, (b) 27 gal./hr., (c) 49°F.

356. The following processes are similar to those in summer air conditioning. Atmospheric air at 100°F and 70% relative humidity is cooled to 60°F (state 2) and delivered to a room where 80°F (state 3) is maintained. The total pressure remains constant at 14.7 psia. Solve this problem by chart only; include all details of your chart readings on a sketch. Find (a) ω_1 and TH_1 , (b) ω_2 and TH_2 . (c) For an atmospheric air flow of 500 cfm, compute the condensed H_2O in lb./min., and the heat transferred 1-2. How many tons of refrigeration are needed?

Ans. (c) 0.73 lb./min., 6 tons.

357. The following processes are typical of those in summer air conditioning. Atmospheric air, at 90°F d.b., $\phi = 70\%$, and the barometer at 28.5 in. Hg, is cooled in a steady flow process to 50°F (state 2), after which it is reheated (perhaps by heat flow into the building) to 73°F d.b. (state 3), all at constant total pressure. Determine (a) the relative humidity and

humidity ratio in state 2 (b) the heat rejected 1 2 (ΔH including h_f at state 2) (c) the change of total heat 1 2 (Find the wet bulb temperature from the psychrometric chart) Is ΔTH a reasonable approximation of ΔH for engineering purposes? (d) How much moisture is taken out of the air? (e) What are the relative humidity and humidity ratio in state 3?

Ans (a) 0.00801 lb H_2O /lb da (b) -25.64 Btu/lb da (c) -25.84 Btu/lb da (d) 101 gr (e) $\phi = 44.3\%$

358 The volumetric analysis of a mixture of gases is 98% of N_2 and 2% of H_2O . The mixture is cooled at a constant total pressure of 60 l psia from 120°F to 80°F. Determine (a) the original humidity ratio relative humidity and dew point (b) the

rate of condensation for a flow of 1000 cfm measured at state 1

Ans (a) 0.01313 lb v/lb dg 71% 108°F, (b) 2.03 lb/min

359 It is desired to process atmospheric air at state 1, 92°F and 60% relative humidity to state 2 75°F and 40% relative humidity (a) Sketch the necessary processes on the T -s plane using refrigeration and determine the temperature to which the atmospheric air must be cooled (b) How much moisture is removed? (c) What are the wet bulb temperatures at states 1 and 2?

Ans (from chart) (a) 49°F (b) 0.01245 lb/lb da (c) 80°F 59.5°F

360-370 These numbers may be used for other problems

16 THE TRANSFER OF HEAT

371 A 4 in thick wall has an outside surface temperature of 90°F and conducts 125 Btu/hr ft² from the inside out. What is the temperature of the inside surface if the material of the wall is (a) grey cast iron $k = 360$ (b) dry concrete $k = 5.4$ (c) oak $k = 1.44$ Btu in/ft² hr °F? (d) If all three of the walls are put together in series determine the conductance from outside surface to inside surface and the inside surface temperature.

Ans (a) 91.39°F (b) 182.6°F, (c) 437°F (d) 532°F

372 (a) A 10x30-ft wall of a building is made up of 8 in of brick with $\frac{1}{2}$ in of plaster on the inside. If the temperatures are 10°F on the outside and 75°F on the inside determine the conductance of the wall, the transmittance and the rate of heat transmission. The conductivities are brick $k = 5$ plaster $k = 3.3$ Btu in/ft² hr °F. (b) The same as (a) except that there is an air space of over $\frac{3}{4}$ in between the plaster and the brick. (c) The air space reduces the heat flux by what percentage?

Ans (a) 0.57 0.396 Btu/hr ft² °F 7740 Btu/hr (b) 0.375 0.292 Btu/hr ft² °F, 5690 Btu/hr (c) 26.5%

373 A partition is made up of 1 in of aluminum ($k = 1400$) 1 in of corrugated asbestos ($k = 0.828$) $\frac{1}{2}$ in of copper ($k = 2690$) 1 in of air space and $\frac{1}{4}$ in of steel ($k = 312$ Btu in/ft² hr °F). The

film coefficients are 20 Btu/ft² hr °F on inside (aluminum) 3 Btu/ft² hr °F on the outside. (a) Determine the overall transmittance. (b) What is the resistance in hr ft² °F/Btu? (c) What percentage change in U would occur if the aluminum, copper and steel were omitted in its calculation? Would it make any significant difference if steel were substituted for the aluminum and copper which are excellent conductors?

374 (a) A residence is built as follows: brick veneer on frame walls with $\frac{3}{8}$ in of plaster 1400 sq ft glass in walls $U = 0.9$ Btu/hr ft² °F 340 sq ft ceiling with transmittance of 0.65 Btu/hr ft² °F 2000 sq ft floor with transmittance of 0.3 Btu/hr ft² °F 2000 sq ft. Inside temperature is to be 75°F. Lowest expected outside temperature is 10°F (let this be the attic temperature too). Take the temperature difference for the floor as 40°F. What is the heat loss from the house? (b) The same as (a) except that there is 2 in of rock wool insulation in the wall and 3 in in the ceiling ($U_o = 0.08$ Btu/hr ft² °F now). (c) What is the percentage reduction of the heat loss due to the insulation? (d) Compute the temperature of the inside wall surface in (a) and (b).

375 The same as 374 except that the cooling loads for an outside temperature of 100°F are desired. Let the attic tem

perature be 120°F and the temperature differential on the floor be 15°F . How many tons of refrigeration are required?

376. Feedwater is to enter an economizer at 160°F , leave at 360°F . The flue gases ($c_p = 0.242$) enter at 800°F and there are 2 lb. of flue gases per pound of water. The water flow is to be 50,000 lb./hr. and the transmittance is expected to be about 8 Btu/hr-ft²- $^{\circ}\text{F}$ based on the inside tube surface. Determine (a) the exit temperature of the gases, (b) the LMTD for counterflow and for parallel flow, (c) the transmitting area (inside tube surface) needed for counterflow and for

parallel flow. Which exchanger would be the more expensive?

Ans. (a) 387°F , (b) 321.5° , 193.5° , (c) 3890 ft.², 6460 ft.²

377. (a) A feedwater heater, for which the transmittance is 350 Btu/hr-ft²- $^{\circ}\text{F}$, uses condensing steam at 20 psia for heating 85,000 lb./hr. of water from 60°F to 215°F . What transmitting surface is required? (b) After some fouling of the tubes, the transmittance decreases to 305 Btu/hr-ft²- $^{\circ}\text{F}$. For the area found in (a), compute the exit temperature of the feedwater after fouling.

Ans. (a) 621 ft.², (b) 208°F .

NOTE. There are not 377 problems in this book. See blank numbers at the end of each chapter.

APPENDIX B. TABLES

Table XIV USEFUL CONSTANTS

Abbreviations atm = atmospheres, cal = calorie cm = centimeter gm = gram
gmol = gram-mol, kcal = kilo calorie kg = kilogram kJ = kilo joule km =
kilometer kw = kilowatt, m = meter mol = lb mol wt = watt others as usual
 $2.3 \log_{10} N = \log_e N = \ln N$

LINEAR				
12 $\frac{\text{in}}{\text{ft}}$	0.394 $\frac{\text{in}}{\text{cm}}$	30.48 $\frac{\text{cm}}{\text{ft}}$		5280 $\frac{\text{ft}}{\text{mi}}$
3 $\frac{\text{ft}}{\text{yd}}$	2.54 $\frac{\text{cm}}{\text{in}}$	3.28 $\frac{\text{ft}}{\text{m}}$		1.609 $\frac{\text{km}}{\text{mi}}$
AREA				
43.560 $\frac{\text{ft}^2}{\text{acre}}$	144 $\frac{\text{in}^2}{\text{ft}^2}$	10.76 $\frac{\text{ft}^2}{\text{m}^2}$	640 $\frac{\text{acres}}{\text{mi}^2}$	6.45 $\frac{\text{cm}^2}{\text{in}^2}$
VOLUME				
1728 $\frac{\text{in}^3}{\text{ft}^3}$	7.481 $\frac{\text{gal}}{\text{ft}^3}$	8 $\frac{\text{bt}}{\text{gal}}$		28.320 $\frac{\text{cm}^3}{\text{ft}^3}$
231 $\frac{\text{in}^3}{\text{gal}}$	1.244 $\frac{\text{ft}^3}{\text{bushel}}$	43.560 $\frac{\text{ft}^3}{\text{acre-ft}}$		3531 $\frac{\text{ft}^3}{\text{m}^3}$
ANGULAR				
6.2832 $\frac{\text{rad}}{\text{rev}}$		57.3 $\frac{\text{deg}}{\text{rad}}$		0.549 $\frac{\text{rpm}}{\text{rad/sec}}$
TIME				
60 $\frac{\text{sec}}{\text{min}}$	3600 $\frac{\text{sec}}{\text{hr}}$	60 $\frac{\text{min}}{\text{hr}}$		24 $\frac{\text{hr}}{\text{day}}$
SPEED				
1.162 $\frac{\text{mph}}{\text{knot}}$	88 $\frac{\text{fpm}}{\text{mph}}$	0.6918 $\frac{\text{mph}}{\text{fps}}$		1.467 $\frac{\text{fps}}{\text{mph}}$
FORCE PRESSURE MASS				
2000 $\frac{\text{lb}}{\text{ton}}$	1000 $\frac{\text{lb}}{\text{kip}}$	2.205 $\frac{\text{lb}}{\text{kg}}$	33.9 $\frac{\text{ft H}_2\text{O (60°F)}}{\text{atm}}$	0.49 $\frac{\text{psi}}{\text{in Hg (60°F)}}$
16 $\frac{\text{oz}}{\text{lb}}$	7000 $\frac{\text{grains}}{\text{lb}}$	453.6 $\frac{\text{gm}}{\text{lb}}$	13.6 $\frac{\text{in H}_2\text{O}}{\text{in Hg}}$	0.0361 $\frac{\text{psi}}{\text{in H}_2\text{O (60°F)}}$
32.174 $\frac{\text{lb}}{\text{slug}}$	444.800 $\frac{\text{dynes}}{\text{lb}}$	980.7 $\frac{\text{dynes}}{\text{gm}}$	29.92 $\frac{\text{in Hg (32°F)}}{\text{atm}}$	
ENERGY AND POWER				
860 $\frac{\text{cal(IT)}}{\text{wt hr Joule}}$	33.000 $\frac{\text{ft lb}}{\text{hp min ergs}}$	2544 $\frac{\text{Btu}}{\text{hp hr Btu/lb mol}}$	3412.1 $\frac{\text{Btu}}{\text{kw hr joules}}$	1.341 $\frac{\text{hp}}{\text{kw}}$
1 $\frac{\text{wt-sec}}{\text{ft lb Btu}}$	10 ⁷ $\frac{\text{joule}}{\text{ft lb hp-sec}}$	1800 $\frac{\text{kcal/gmol}}{\text{Btu}}$	1.356 $\frac{\text{ft lb}}{\text{kw hp}}$	1.055 $\frac{\text{Btu}}{\text{cal}}$
	550 $\frac{\text{ft lb}}{\text{hp-sec}}$	42.4 $\frac{\text{Btu}}{\text{hp min}}$	0.746 $\frac{\text{kw}}{\text{hp}}$	252 $\frac{\text{cal}}{\text{Btu}}$
	737.562 $\frac{\text{ft lb}}{\text{kw sec}}$			3600 $\frac{\text{kJ}}{\text{kw hr}}$
UNIVERSAL GAS CONSTANT				
1545.32 $\frac{\text{ft lb}}{\text{mol °R}}$		0.730 $\frac{\text{atm ft}^3}{\text{mol °R}}$		0.00078 $\frac{\text{hp hr}}{\text{mol °R}}$
1.986 $\frac{\text{Btu}}{\text{mol °R}}$ and $\frac{\text{cal}}{\text{gmol °K}}$		10.73 $\frac{\text{psia ft}^3}{\text{mol °R}}$		82.06 $\frac{\text{atm cm}^3}{\text{gmol °K}}$

Table XV. SATURATION PROPERTIES OF FREON 12*

Temp °F	Abs Pres psi	Volume cu ft /lb		Enthalpy Btu/lb			Entropy Btu/lb-°R	
<i>t</i>	<i>p</i>	<i>v_f</i>	<i>v_g</i>	<i>h_f</i>	<i>h_{fg}</i>	<i>h_g</i>	<i>s_f</i>	<i>s_g</i>
-10	19 19	0 01091	1 973	6 37	69 82	76 19	0 01462	0 16989
0	23 85	0 01103	1 609	8 52	68 75	77 27	0 01932	0 16888
1	24 36	0 01104	1 577	8 74	68 64	77 38	0 01979	0 16878
10	29 34	0 01116	1 324	10 68	67 65	78 33	0 02395	0 16798
20	35 74	0 01130	1 099	12 86	66 52	79 38	0 02851	0 16719
30	43 15	0 01144	0 919	15 06	65 36	80 42	0 03301	0 16648
40	51 67	0 01159	0 774	17 27	64 16	81 43	0 03745	0 16586
50	61 39	0 01175	0 655	19 51	62 93	82 44	0 04184	0 16530
60	72 43	0 01191	0 558	21 77	61 64	83 41	0 04618	0 16479
70	84 89	0 01209	0 478	24 05	60 31	84 36	0 05048	0 16434
80	98 87	0 01228	0 411	26 36	58 92	85 28	0 05475	0 16392
90	114 49	0 01248	0 355	28 71	57 46	86 17	0 05900	0 16353
100	131 86	0 01269	0 308	31 10	55 93	87 03	0 06323	0 16315

Table XVI. PROPERTIES OF SUPERHEATED FREON 12*

Saturation properties in italics

Temp °F	80 PSIA (66 21°F)			100 PSIA (80 76°F)		
	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>
	<i>0 5068</i>	<i>84 00</i>	<i>0 16450</i>	<i>0 4067</i>	<i>85 35</i>	<i>0 16389</i>
70	0 5127	84 64	0 16571			
80	0 528	86 32	0 16885			
90	0 543	87 98	0 17190	0 419	86 96	0 16685
100	0 557	89 64	0 17489	0 431	88 69	0 16996
110	0 572	91 24	0 17782	0 444	90 41	0 17300
Temp °F	115 PSIA (90 31°F)			150 PSIA (109 45°F)		
	<i>0 3537</i>	<i>86 20</i>	<i>0 16352</i>	<i>0 2697</i>	<i>87 80</i>	<i>0 16281</i>
100	0 365	87 94	0 16665			
110	0 376	89 71	0 16978	0 271	87 91	0 16299
120	0 387	91 46	0 17283	0 280	89 80	0 16629
130	0 398	93 20	0 17581	0 289	91 66	0 16947

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Table XVII SATURATED AMMONIA*

Pressure psi abs <i>p</i>	Temp of <i>t</i>	Volume		Enthalpy			Entropy	
		Liquid <i>v_f</i>	Vapor <i>v_g</i>	Liquid <i>h_f</i>	Evap <i>h_{fg}</i>	Vapor <i>h_g</i>	Liquid <i>s_f</i>	Vapor <i>s_g</i>
5.0	-63.11	0.02271	49.31	-24.5	612.8	588.3	-0.0599	1.4857
10.0	-41.34	0.02319	25.81	-1.4	598.5	597.1	-0.0034	1.4276
15.0	-27.20	0.02350	17.67	13.6	588.8	602.4	0.0318	1.3938
15.98	-25.0	0.02357	16.66	16.0	587.2	603.2	0.0374	1.3886
18.30	-20.0	0.02369	14.68	21.4	583.6	605.0	0.0497	1.3774
19.7	-17.20	0.02375	13.70	24.4	581.6	606.0	0.0560	1.3710
20.0	-16.64	0.02378	13.50	25.0	581.2	606.2	0.0578	1.3700
20.88	-15.0	0.02381	12.97	26.7	580.0	606.7	0.0618	1.3664
21.0	-14.78	0.02382	12.90	27.0	579.8	606.8	0.0623	1.3659
23.74	-10.0	0.02393	11.50	32.1	576.4	608.5	0.0738	1.3558
24.7	-8.40	0.02397	11.086	33.8	575.2	609.0	0.0772	1.3525
25.0	-7.96	0.02398	10.96	34.3	574.8	609.1	0.0787	1.3516
26.92	-5.0	0.02406	10.23	37.5	572.6	610.1	0.0857	1.3454
28.0	-3.40	0.02410	9.853	39.3	571.4	610.7	0.0895	1.3421
30.0	-0.57	0.02418	9.236	42.3	569.3	611.6	0.0962	1.3364
30.42	0.0	0.02419	9.116	42.9	568.9	611.8	0.0975	1.3352
34.27	5.0	0.02432	8.150	48.3	565.0	613.3	0.1092	1.3253
34.7	5.52	0.02433	8.067	48.9	564.6	613.5	0.1104	1.3243
35.0	5.89	0.02434	7.991	49.3	564.3	613.6	0.1113	1.3236
37.7	9.07	0.02443	7.452	52.8	561.8	614.6	0.1187	1.3175
38.51	10.0	0.02446	7.304	53.8	561.1	614.9	0.1208	1.3157
40.0	11.66	0.02451	7.047	56.6	559.8	615.4	0.1246	1.3125
43.14	15.00	0.02460	6.562	59.2	557.1	616.3	0.1323	1.3062
48.21	20.0	0.02474	5.910	64.7	553.1	617.8	0.1437	1.2969
50.0	21.67	0.02479	5.710	66.5	551.7	618.2	0.1475	1.2939
53.73	25.00	0.02488	5.334	70.2	548.9	619.1	0.1551	1.2879
73.32	40.0	0.02533	3.971	86.8	536.2	623.0	0.1885	1.2618
100.0	56.05	0.02584	2.952	104.7	521.8	626.5	0.2237	1.2356
107.0	60.0	0.02597	2.751	109.2	518.1	627.3	0.2322	1.2294
120.0	66.02	0.02618	2.476	116.0	512.4	628.4	0.2452	1.2201
124.3	68.00	0.02625	2.393	118.3	510.5	628.8	0.2494	1.2170
128.8	70.0	0.02632	2.312	120.5	508.6	629.1	0.2537	1.2140
130.0	70.53	0.02634	2.291	121.1	508.1	629.2	0.2548	1.2132
150.0	78.81	0.02664	1.904	130.6	499.9	630.5	0.2724	1.2009
153.0	80.0	0.02668	1.955	132.0	498.7	630.7	0.2749	1.1991
166.4	85.0	0.02687	1.801	137.8	493.6	631.4	0.2854	1.1918
169.2	86.00	0.02691	1.772	138.9	492.6	631.5	0.2875	1.1904
174.8	88.00	0.02698	1.716	141.2	490.6	631.8	0.2917	1.1875
180.0	89.78	0.02706	1.667	143.3	488.7	632.0	0.2954	1.1850
180.6	90.0	0.02707	1.661	143.5	488.5	632.0	0.2958	1.1846
186.6	92.00	0.02715	1.609	145.8	486.4	632.2	0.3000	1.1818
190.0	93.13	0.02720	1.581	147.2	485.2	632.4	0.3024	1.1802
200.0	96.34	0.02732	1.502	150.9	481.8	632.7	0.3090	1.1756
205.0	97.90	0.02738	1.466	152.7	480.1	632.8	0.3122	1.1734
214.7	100.90	0.02751	1.400	156.2	476.9	633.1	0.3180	1.1690
247.0	110.00	0.02790	1.217	167.0	466.7	633.7	0.3372	1.1566

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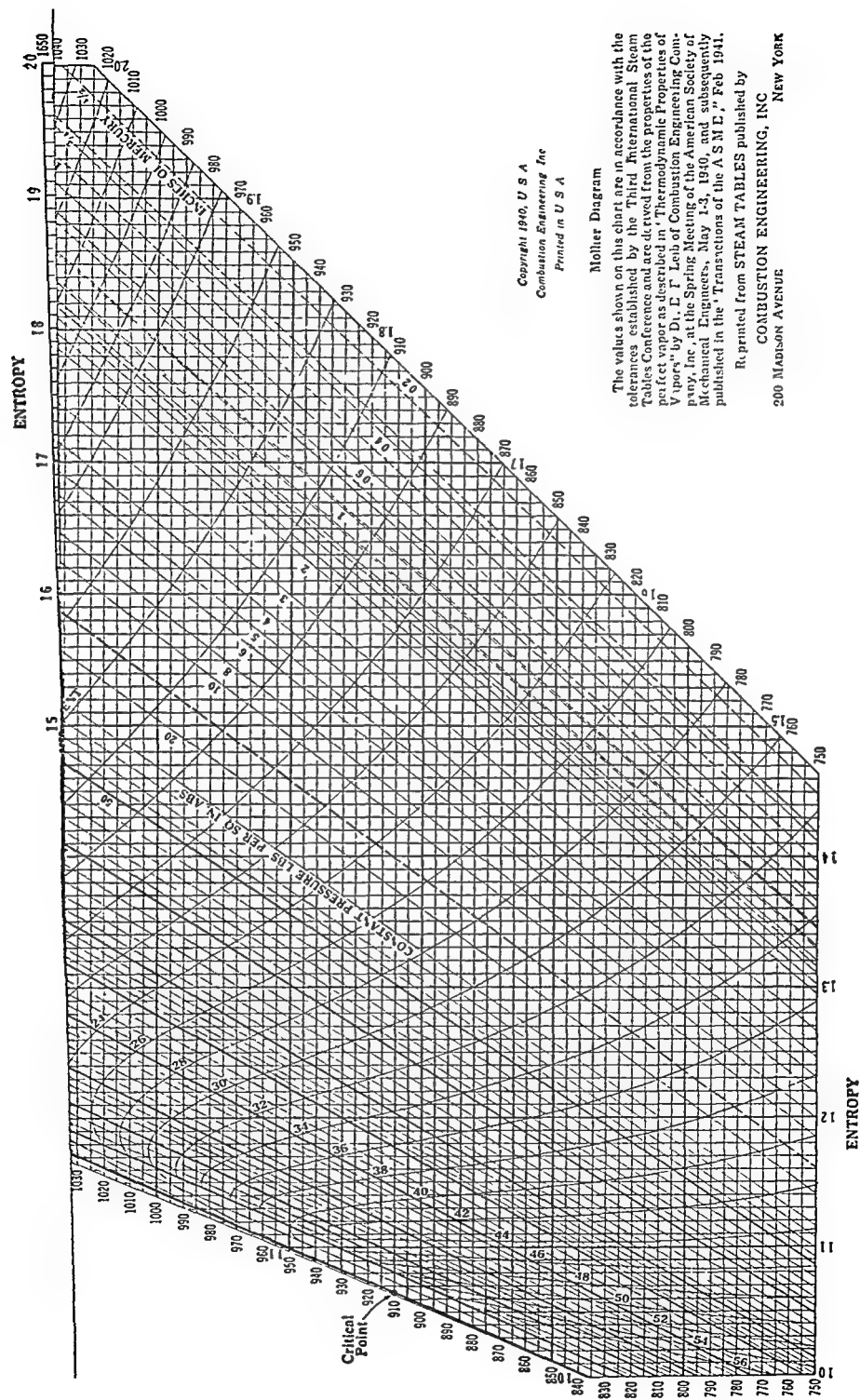
Table XVIII. SUPERHEATED AMMONIA*

Temp. ° F.	Absolute Pressure, psi. (Saturation temperature in italics)								
	15 -27.29°			20 -16.64°			25 -7.96°		
	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>
<i>Sat.</i>	17.67	602.4	1.5938	13.50	608.2	1.3700	10.96	603.1	1.3516
-20	18.01	606.4	1.4031
-10	18.47	611.9	.4154	13.74	610.0	1.3784
0	18.92	617.2	1.4272	14.09	615.5	1.3907	11.19	613.8	1.3616
10	19.37	622.5	.4386	14.44	621.0	.4025	11.47	619.4	.3738
20	19.82	627.8	.4497	14.78	626.4	.4138	11.75	625.0	.3855
30	20.26	633.0	.4604	15.11	631.7	.4248	12.03	630.4	.3967
40	20.70	638.2	.4709	15.45	637.0	.4356	12.30	635.8	.4077
50	21.14	643.4	1.4812	15.78	642.3	1.4460	12.57	641.2	1.4183
60	21.58	648.5	.4912	16.12	647.5	.4562	12.84	646.5	.4287
70	22.01	653.7	.5011	16.45	652.8	.4662	13.11	651.8	.4388
80	22.44	658.9	.5108	16.78	658.0	.4760	13.37	657.1	.4487
90	22.88	664.0	.5203	17.10	663.2	.4856	13.64	662.4	.4584
100	23.31	669.2	1.5296	17.43	668.5	1.4950	13.90	667.7	1.4679
110	23.74	674.4	.5388	17.76	673.7	.5042	14.17	673.0	.4772
120	24.17	679.6	.5478	18.08	678.9	.5133	14.43	678.2	.4864
130	24.60	684.8	.5567	18.41	684.2	.5223	14.69	683.5	.4954
140	25.03	690.0	.5655	18.73	689.4	.5312	14.95	688.8	.5043
150	25.46	695.3	1.5742	19.05	694.7	1.5399	15.21	694.1	1.5131
160	25.88	700.5	.5827	19.37	700.0	.5485	15.47	699.4	.5217
170	26.31	705.8	.5911	19.70	705.3	.5569	15.73	704.7	.5303
180	26.74	711.1	.5995	20.02	710.6	.5653	15.99	710.1	.5387
190	27.16	716.4	.6077	20.34	715.9	.5736	16.25	715.4	.5470
200	27.59	721.7	1.6158	20.66	721.2	1.5817	16.50	720.8	1.5552
220	28.44	732.4	.6318	21.30	732.0	.5978	17.02	731.6	.5713
240	21.94	742.8	.6135	17.53	742.5	.5870
260	18.04	753.4	.6025
	30 -0.57°			35 5.89°			40 11.66°		
	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>
	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>	<i>v</i>	<i>h</i>	<i>s</i>
<i>Sat.</i>	9.236	611.6	1.5364	7.991	613.6	1.3236	7.047	616.4	1.3125
10	9.492	617.8	1.3497	8.078	616.1	1.3289
20	9.731	623.5	.3618	8.287	622.0	.3413	7.203	620.4	1.3231
30	9.966	629.1	.3733	8.493	627.7	.3532	7.387	626.3	.3353
40	10.20	634.6	.3845	8.695	633.4	.3646	7.568	632.1	.3470
50	10.43	640.1	1.3953	8.895	638.9	1.3756	7.746	637.8	1.3583
60	10.65	645.5	.4059	9.093	644.4	.3863	7.922	643.4	.3692
70	10.88	650.9	.4161	9.289	649.9	.3967	8.096	648.9	.3797
80	11.10	656.2	.4261	9.484	655.3	.4069	8.268	654.4	.3900
90	11.33	661.6	.4359	9.677	660.7	.4168	8.439	659.9	.4000
100	11.55	666.9	1.4456	9.869	666.1	1.4265	8.609	665.3	1.4098
110	11.77	672.2	.4550	10.06	671.5	.4360	8.777	670.7	.4194
120	11.99	677.5	.4642	10.25	676.8	.4453	8.945	676.1	.4288
130	12.21	682.9	.4733	10.44	682.2	.4545	9.112	681.5	.4381
140	12.43	688.2	.4823	10.63	687.6	.4635	9.278	686.9	.4471
150	12.65	693.5	1.4911	10.82	692.9	1.4724	9.444	692.3	1.4561
160	12.87	698.8	.4998	11.00	698.3	.4811	9.609	697.7	.4648
170	13.08	704.2	.5083	11.19	703.7	.4897	9.774	703.1	.4735
180	13.30	709.6	.5168	11.38	709.1	.4982	9.938	708.5	.4820
190	13.52	714.9	.5251	11.56	714.5	.5066	10.10	714.0	.4904
200	13.73	720.3	1.5334	11.75	719.9	1.5148	10.27	719.4	1.4987
220	14.16	731.1	.5495	12.12	730.7	.5311	10.59	730.3	.5150
240	14.59	742.0	.5653	12.49	741.7	.5469	10.92	741.3	.5309
260	15.02	753.0	.5808	12.86	752.7	.5624	11.24	752.3	.5465
280	15.45	764.1	.5960	13.23	763.7	.5776	11.56	763.4	.5617
300	11.83	774.6	.5766

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Table XVIII SUPERHEATED AMMONIA (Continued)

Temp °F	Absolute Pressures (Saturation temperature in °F)								
	110 62.1°			120 68.02°			140 74.79°		
	v	h	s	v	h	s	v	h	s
Sat	2.653	6.75	12.75	2.48	6.284	12.01	2.152	6.299	12.068
70	2.761	633.7	12392	2.505	631.3	12.755			
80	2.837	640.5	2519	2.56	638.3	2386	2.166	633.8	12140
90	2.910	647.0	2610	2.645	645.0	2510	2.278	640.9	2272
100	2.981	653.4	12755	2.712	651.6	12678	2.288	647.8	12396
110	3.051	659.7	2866	2.778	658.0	2741	2.317	654.5	2515
120	3.120	665.8	2972	2.842	661.2	2850	2.404	661.1	2678
130	3.188	671.9	3076	2.905	664.0	2956	2.460	667.4	2738
140	3.255	677.8	3176	2.967	676.5	3058	2.515	673.7	2843
150	3.321	683.7	13274	3.029	682.5	13157	2.569	679.9	12945
160	3.386	689.6	3370	3.089	688.4	3.54	2.622	686.0	3045
170	3.451	695.4	3463	3.149	694.3	3348	2.675	692.0	3141
180	3.515	701.2	3555	3.209	700.2	3441	2.727	698.0	3236
190	3.579	707.0	3644	3.268	706.0	3531	2.779	704.0	3328
200	3.642	712.8	13737	3.326	711.8	13620	2.830	709.9	13418
210	3.705	718.5	3819	3.385	717.6	3707	2.880	715.8	3507
220	3.768	724.3	3904	3.442	723.4	3793	2.931	721.6	3594
230	3.830	730.0	3988	3.500	729.2	3877	2.981	727.5	3679
240	3.892	735.7	4073	3.557	734.9	3960	3.030	733.3	3763
250	3.954	741.5	14151	3.614	740.7	14042	3.080	739.2	13846
260	4.015	747.2	4232	3.671	746.5	4193	3.129	745.0	3928
270	4.076	752.9	4311	3.727	752.2	4202	3.179	750.8	4008
280	4.137	758.7	4389	3.783	758.0	4281	3.227	756.7	4088
290	4.198	764.5	4466	3.839	763.8	4359	3.275	762.5	4166
300	4.259	770.2	14413	3.895	769.6	14435	3.323	768.3	14743
310							3.420	780.0	4395
Sat	100 54°			180 89.76°			200 96.5°		
	v	h	s	v	h	s	v	h	s
	v	h	s	v	h	s	v	h	s
90	1.872	631.1	11502	1.667	63.0	11850	1.608	638.7	11758
100	1.914	636.6	12055	1.668	63.2	11853			
110	1.969	643.9	12186	1.720	639.9	11997	1.520	635.6	11809
120	2.023	651.0	2311	1.770	647.3	2123	1.567	643.4	1947
130	2.075	657.8	2429	1.818	654.4	2.47	1.612	650.9	2077
140	2.125	664.4	2542	1.865	661.3	2364	1.656	658.1	2.00
150	2.175	670.9	2657	1.910	668.0	2477	1.698	665.0	2317
160	2.224	677.2	12767	1.955	674.6	12586	1.740	671.8	12429
170	2.272	683.5	2859	1.999	681.0	2691	1.780	678.4	2537
180	2.319	689.7	2958	2.042	687.3	279	1.820	684.9	2641
190	2.365	695.8	3054	2.084	693.6	2891	1.859	691.3	2742
200	2.411	701.9	3148	2.126	699.8	2987	1.897	697.7	2840
210	2.457	707.9	13740	2.167	705.9	13081	1.935	703.9	12935
220	2.502	713.9	3331	2.208	712.0	3172	1.972	710.1	3029
230	2.547	719.9	3419	2.248	718.1	3262	2.009	716.3	3120
240	2.591	725.8	3506	2.288	724.1	3350	2.046	722.4	3209
250	2.635	731.7	3591	2.328	730.1	3436	2.082	728.4	3296
260	2.679	737.6	13675	2.367	736.1	13571	2.118	734.5	13382
270	2.723	743.5	3757	2.407	742.0	3605	2.154	740.5	3467
280	2.766	749.4	3838	2.446	748.0	3687	2.189	746.5	3550
290	2.809	755.3	3919	2.484	753.9	3763	2.225	752.5	3631
300	2.852	761.2	3998	2.523	759.9	3847	2.260	758.5	3712
310	2.895	767.1	14076	2.561	765.8	13926	2.295	764.5	13791
320	2.938	773.0	4279	2.607	771.7	4081	2.364	776.5	3947
330	2.984	779.7	4379	2.713	789.6	4231	2.432	788.5	4099



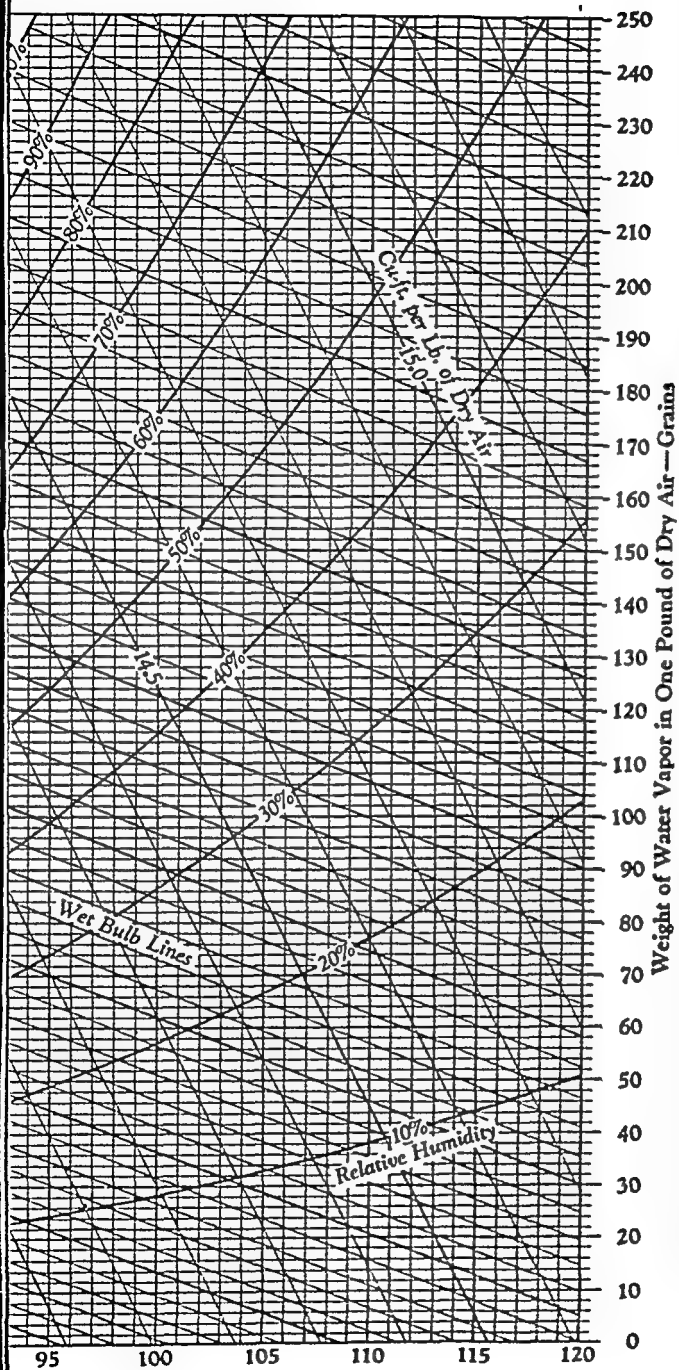
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Mollier Diagram

The values shown on this chart are in accordance with the tolerances established by the Third International Steam Tables Conference and are derived from the properties of the perfect vapor as described in "Thermodynamic Properties of Vapors" by Dr. E. F. Leub of Combustion Engineering Company, Inc. at the Spring Meeting of the American Society of Mechanical Engineers, May 1-3, 1940, and subsequently published in the "Transactions of the A S M E," Feb 1941.

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